Introduction to Proof

2

2.1 **A Little Dash of Logic**
Foundations for Proof 119

2.2 **And Now From a New Angle**
Special Angles and Postulates 135

2.3 **Forms of Proof**
Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof 153

2.4 **What's Your Proof?**
Angle Postulates and Theorems 175

2.5 **A Reversed Condition**
Parallel Line Converse Theorems 185

Chapter 2 Overview

This chapter focuses on the foundations of proof. Paragraph, two-column, construction, and flow chart proofs are presented. Proofs involving angles and parallel lines are completed.

Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
2.1	Foundations for Proof	G.CO.9	2	This lesson introduces key components of proof. Questions focus on identifying, comparing, and contrasting inductive and deductive reasoning. Students identify the hypothesis and conclusion of conditional statements and explore their truth values.	X	X	X	X	
2.2	Special Angles and Postulates	G.CO.9	2	This lesson addresses complementary, supplementary, adjacent, linear pair, and vertical angles. Questions focus on calculating angle measures and identifying types of angles. The lesson also addresses the difference between postulates and theorems, and the difference between Euclidean and non-Euclidian geometries.	X	X			
2.3	Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof	G.CO.9	2	This lesson presents the various forms of proof. Questions ask students to sketch examples of the properties of equality. Then, students use the various forms of proof to prove theorems involving right, supplementary, complementary, and vertical angles.	X	X	X	X	
2.4	Angle Postulates and Theorems	G.CO.9	2	This lesson focuses on proving postulates and theorems involving angles formed by parallel lines and a transversal. Questions ask students to complete proofs that involve corresponding, alternate interior, alternate exterior, same-side interior, and same-side exterior angles.	X		X	X	

Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
2.5	Parallel Line Converse Theorems	G.CO.9	1	This lesson focuses on proving converse postulates and theorems involving angles formed by parallel lines and a transversal. Questions ask students to complete proofs that involve corresponding, alternate interior, alternate exterior, same-side interior, and same-side exterior angles.	X			X	

Skills Practice Correlation for Chapter 2

Lesson		Problem Set	Objectives
2.1	Foundations for Proof		Vocabulary
		1 – 6	Identify specific and general information and conclusions in problem situations
		7 – 12	Determine whether inductive or deductive reasoning is used in situations
		13 – 18	Identify, compare, and contrast inductive and deductive reasoning in situations
		19 – 24	Write statements in propositional form
		25 – 30	Identify the hypothesis and conclusion of conditional statements
		31 – 34	Answer questions about conditional statements
		35 – 38	Draw diagrams to represent conditional statements and write the hypothesis and conclusion
2.2	Special Angles and Postulates		Vocabulary
		1 – 4	Draw angles supplementary to given angles with a common side
		5 – 8	Draw angles supplementary to given angles without a common side
		9 – 12	Draw angles complementary to given angles with a common side
		13 – 16	Draw angles complementary to given angles without a common side
		17 – 22	Solve for unknown angle measures
		23 – 26	Determine the measures of two angles given descriptions
		27 – 30	Determine whether angles are adjacent angles
		31 – 34	Determine whether two angles form a linear pair
		35 – 38	Identify pairs of vertical angles
		39 – 44	Identify the postulate that confirms statements about angles and segments
		45 – 50	Complete statements about angles and segments using postulates
2.3	Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof		Vocabulary
		1 – 12	Identify properties of equality in examples
		13 – 18	Write example statements using properties of equality
		19 – 22	Rewrite conditional statements by separating the hypothesis and conclusion
		23 – 28	Prove statements using construction, paragraph proofs, flow chart proofs, and two-column proofs
		29 – 34	Rewrite proofs as another type of proof

Lesson		Problem Set	Objectives
2.4	Angle Postulates and Theorems		Vocabulary
		1 – 4	Identify pairs of corresponding angles
		5 – 12	Write conjectures
		13 – 16	Draw and label diagrams to illustrate theorems
		17 – 20	Write "Given" and "Prove" statements using diagrams and descriptions
		21 – 24	Create proofs of statements about angle relationships
		25 – 28	Write the theorem illustrated by diagrams
2.5	Parallel Line Converse Theorems		Vocabulary
		1 – 4	Write the converse of postulates and theorems
		5 – 10	Write the converse of statements
		11 – 14	Draw and label diagrams to illustrate theorems
		15 – 18	Write "Given" and "Prove" statements using diagrams and descriptions
		19 – 22	Create proofs of statements

2

2.1

A Little Dash of Logic

Foundations for Proof

LEARNING GOALS

In this lesson, you will:

- Define inductive and deductive reasoning.
- Identify methods of reasoning.
- Compare and contrast methods of reasoning.
- Create examples using inductive and deductive reasoning.
- Identify the hypothesis and conclusion of a conditional statement.
- Explore the truth values of conditional statements.
- Use a truth table.

KEY TERMS

- induction
- deduction
- counterexample
- conditional statement
- propositional form
- propositional variables
- hypothesis
- conclusion
- truth value
- truth table

ESSENTIAL IDEAS

- Inductive reasoning moves from specific information or examples to a conclusion. You use inductive reasoning when you observe data, recognize patterns, make generalizations about the observations or patterns, and reapply those generalizations to unfamiliar situations.
- Deductive reasoning moves from a general rule or premise to a conclusion. It is the process of showing that certain statements follow logically from some proven facts or accepted rules. It is important that all conclusions are tracked back to given truths.
- A conditional statement is a statement that can be written in the form "if p, then q." Written as p ® q, it is read "p implies q." The variable p represents the hypothesis and the variable q represents the conclusion.
- The truth value of a conditional statement is whether the statement is true or false.
- Truth tables are used to organize truth values of conditional statements.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

G-CO Congruence

Prove geometric theorems

9. Prove theorems about lines and angles.

Overview

Students use inductive reasoning and deductive reasoning to solve problems. Conditional statements are analyzed and associated with truth value. Truth tables are used to help students organize information. Geometric statements are given and students draw diagrams and identify the given and prove statements associated with the conditional statements.

2

Warm Up

Decide whether each statement is true of false and explain your reasoning.

1. *All rectangles are quadrilaterals*.

   The statement is true because all rectangles have 4 sides.

2. *All rectangles are squares.*

   The statement is false because rectangles are not required to have 4 equal side lengths.

3. *If it rains today, then it will not rain tomorrow.*

   The statement is false because there is no guarantee that there will be no rain the day after it rains.

2

2

2.1

2

A Little Dash of Logic

Foundations for Proof

LEARNING GOALS

In this lesson, you will:

- Define inductive and deductive reasoning.
- Identify methods of reasoning.
- Compare and contrast methods of reasoning.
- Create examples using inductive and deductive reasoning.
- Identify the hypothesis and conclusion of a conditional statement.
- Explore the truth values of conditional statements.
- Use a truth table.

KEY TERMS

- induction
- deduction
- counterexample
- conditional statement
- propositional form
- propositional variables
- hypothesis
- conclusion
- truth value
- truth table

One of the most famous literary detectives is Sherlock Holmes and his trusted assistant Watson. Created by author Sir Arthur Conan Doyle, Sherlock Holmes first appeared in print in 1887 in the novel *A Study in Scarlet*. The character has gone on to appear in four novels, 56 short stories, and over 200 films. The Guinness Book of World Records lists Holmes as the most portrayed movie character, with more than 70 different actors playing the part.

Holmes is most famous for his keen powers of observation and logical reasoning, which always helped him solve the case. In many literary and film adaptations, Holmes is known to remark, "Elementary, my dear Watson," after explaining how he solved the mystery. However, this well-known phrase doesn't actually appear in any of the stories written by Doyle. It first appeared in the 1915 novel *Psmith, Journalist* by P.G.: Wodehouse and also appeared at the end of the 1929 film *The Return of Sherlock Holmes*. Regardless, this phrase will probably always be associated with the famous detective.

Problem 1

Students differentiate between the types of reasoning used in three scenarios before they are given the terminology related to inductive and deductive reasoning.

2

Grouping

Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 5

- Did Emma use a rule or observe a pattern to reach her conclusion?
- Did Ricky use a rule or observe a pattern to reach his conclusion?
- How did you determine Jennifer's charge per hour?
- How did you determine how much Aaron earned?

PROBLEM 1 How Do You Figure?

1. Emma considered the following statements.
   - $4^2 = 4 \times 4$
   - Nine cubed is equal to nine times nine times nine.
   - 10 to the fourth power is equal to four factors of 10 multiplied together.

   Emma concluded that raising a number to a power is the same as multiplying the number as many times as indicated by the exponent. How did Emma reach this conclusion?

   Emma looked at several specific examples, noticed a pattern, and generalized the pattern with a rule.

2. Ricky read that raising a number to a power is the same as multiplying that number as many times as indicated by the exponent. He had to determine seven to the fourth power using a calculator. So, he entered $7 \times 7 \times 7 \times 7$. How did Ricky reach this conclusion?

   Ricky learned a rule and applied that rule to a specific example.

3. Compare Emma's reasoning to Ricky's reasoning.

   Emma looked at specific examples and came up with a rule. Ricky learned a rule and then applied it to a specific example.

4. Jennifer is a writing consultant. She is paid \$900 for a ten-hour job and \$1980 for a twenty-two-hour job.

   a. How much does Jennifer charge per hour?

   Jennifer charges \$90 per hour.

   $$\frac{\$900}{10 \text{ hours}} = \frac{\$90}{\text{hour}}$$

   $$\frac{\$1980}{22 \text{ hours}} = \frac{\$90}{\text{hour}}$$

   b. To answer Question 4, part (a), did you start with a general rule and make a conclusion, or did you start with specific information and create a general rule?

   I started with specific information and created a general rule.

5. Your friend Aaron tutors elementary school students. He tells you that the job pays \$8.25 per hour.

   a. How much does Aaron earn from working 4 hours?

   Aaron earns 8.25×4, or \$33 for working 4 hours.

   b. To answer Question 5, part (a), did you start with a general rule and make a conclusion, or did you start with specific information and create a general rule?

   I started with a general rule and made a conclusion.

Problem 2

Terminology related to inductive reasoning and deductive reasoning is introduced and students revisit the scenarios in Problem 1 to determine which type of reasoning was used in each situation.

Grouping

Ask a student to read the definitions and information and complete Questions 1 and 2 as a class.

Guiding Questions for Discuss Phase

- What is an example of using deductive reasoning in everyday life?
- What is an example of using inductive reasoning in everyday life?

Grouping

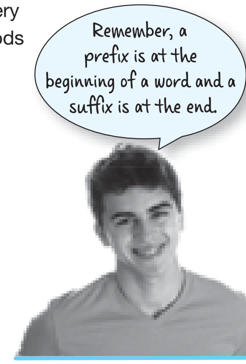
Have students complete Question 3 with a partner. Then share their responses as a class.

Guiding Questions for Discuss Phase, Question 3

- Does each scenario fit one of the two types of reasoning?
- Is there more than one correct answer?
- Can both types of reasoning be applied to a single situation? Why or why not?
- Do you suppose there are other types of reasoning?

PROBLEM 2 Is This English Class or Algebra?

The ability to use information to reason and make conclusions is very important in life and in mathematics. There are two common methods of reasoning. You can construct the name for each method of reasoning using your knowledge of prefixes, root words, and suffixes.



Word Fragment	Prefix, Root Word, or Suffix	Meaning
in-	Prefix	*toward* or *up to*
de-	Prefix	*down from*
-duc-	Root Word	*to lead* and often *to think*, from the Latin word *duco*
-tion	Suffix	*the act of*

1. Form a word that means "the act of thinking down from."
   Deduction

2. Form a word that means "the act of thinking toward or up to."
   Induction

Induction is reasoning that uses specific examples to make a conclusion. Sometimes you will make generalizations about observations or patterns and apply these generalizations to new or unfamiliar situations. For example, you may notice that when you don't study for a test, your grade is lower than when you do study for a test. You apply what you learned from these observations to the next test you take.

Deduction is reasoning that uses a general rule to make a conclusion. For example, you may learn the rule for which direction to turn a screwdriver: "righty tighty, lefty loosey." If you want to remove a screw, you apply the rule and turn the screwdriver counterclockwise.

3. Consider the reasoning used by Emma, Ricky, Jennifer, and Aaron in Problem 1.
   a. Who used inductive reasoning?
      Emma and Jennifer used inductive reasoning.
   b. Who used deductive reasoning?
      Ricky and Aaron used deductive reasoning.

Problem 3

Students identify the general information, specific information, and the conclusion in several situations and determine the type of reasoning used in each scenario.

2

Grouping

- Have students read the worked example. Complete and discuss Question 1 as a class.
- Have students complete Questions 2 through 6 with a partner. Then share the responses as a class.

PROBLEM 3 Coming to Conclusions

A problem situation can provide you with a great deal of information that you can use to make conclusions. It is important to identify specific and general information in a problem situation to reach appropriate conclusions. Some information may be irrelevant to reach the appropriate conclusion.

Ms. Ross teaches an Economics class every day from 1:00 PM to 2:15 PM. Students' final grade is determined by class participation, homework, quizzes, and tests. She noticed that Andrew has not turned in his homework 3 days this week. She is concerned that Andrew's grade will fall if he does not turn in his homework.

Irrelevant Information:
Ms. Ross teaches an Economics class every day from 1:00 PM to 2:15 PM.

General information:
Students' final grade is determined by class participation, homework, quizzes, and tests.

Specific information:
Andrew has not turned in his homework 3 days this week.

Conclusion:
Andrew's grade will fall if he does not turn in his homework.

1. Did Ms. Ross use induction or deduction to make this conclusion? Explain your answer.
Ms. Ross used deduction to come to this conclusion because she used a rule about how grades were calculated to make a conclusion.

Guiding Questions for Share Phase, Questions 2 through 6

- Is Ms. Ross's conclusion based on specific observations or general information?
- Is the information that Matilda smokes considered specific information or general information?
- Is the information that tobacco use increases the risk of getting cancer considered specific information or general information?
- Did Conner use a rule or observation to reach his conclusion?
- Is the information that it rained each day Molly was in England considered specific information or general information?
- Did Molly use a rule or observation to reach her conclusion?
- Is the information that Dontrell takes detailed notes in history class and math class considered specific information or general information?
- Did Trang and his teacher use the same type of reasoning to reach their conclusions?
- Do you notice a pattern in the sequence of numbers? If so, describe the pattern.

2. Conner read an article that claimed that tobacco use greatly increases the risk of getting cancer. He then noticed that his neighbor Matilda smokes. Conner is concerned that Matilda has a high risk of getting cancer.

   a. Which information is specific and which information is general in this problem situation?

   The general information is that tobacco use increases the risk of getting cancer.
   The specific information is that Matilda smokes.

   b. What is the conclusion in this problem?

   The conclusion is that Matilda has a high risk of getting cancer.

   c. Did Conner use inductive or deductive reasoning to make the conclusion? Explain your reasoning.

   He used deductive reasoning because he used a rule about tobacco use to make a conclusion.

   d. Is Conner's conclusion correct? Explain your reasoning.

   Conner's conclusion is correct.
   Matilda may not get cancer, but smoking increases her risk of getting cancer.

- Did you determine a general rule that fit the pattern? If so, what is it?
- Did Marie and Jose use the same type of reasoning?
- Is there more than one correct rule for this sequence of numbers?

2

2

3. Molly returned from a trip to England and tells you, "It rains every day in England!" She explains that it rained each of the five days she was there.

a. Which information is specific and which information is general in this problem situation?

The problem does not include any general information.

The specific information is that it rained each of the five days Molly was in England.

b. What is the conclusion in this problem?

The conclusion is that it rains every day in England.

c. Did Molly use inductive or deductive reasoning to make the conclusion? Explain your answer.

Molly used inductive reasoning because she used her specific experiences to make a general conclusion.

d. Is Molly's conclusion correct? Explain your reasoning.

Molly's conclusion is not correct.

There have been many days in the past in which it did not rain in England.

4. Dontrell takes detailed notes in history class and math class. His classmate Trang will miss biology class tomorrow to attend a field trip. Trang's biology teacher asks him if he knows someone who always takes detailed notes. Trang tells his biology teacher that Dontrell takes detailed notes. Trang's biology teacher suggests that Trang should borrow Dontrell's notes because he concludes that Dontrell's notes will be detailed.

   a. What conclusion did Trang make? What information supports this conclusion?

   Trang concluded that Dontrell's biology notes will be detailed because Dontrell's history and math notes are detailed.

   b. What type of reasoning did Trang use? Explain your reasoning.

   Trang used inductive reasoning because he used specific observations about Dontrell taking good history and math notes to make a general conclusion that Dontrell always takes good notes.

   c. What conclusion did the biology teacher make? What information supports this conclusion?

   The biology teacher concludes that Dontrell's biology notes tomorrow will be detailed because Trang said Dontrell takes detailed notes in other classes.

   d. What type of reasoning did the biology teacher use? Explain your reasoning.

   The teacher used deductive reasoning because the teacher used a general rule supplied by Trang to make a conclusion that Dontrell's notes tomorrow will be detailed.

   e. Will Trang's conclusion always be true? Will the biology teacher's conclusion always be true? Explain your reasoning.

   Answers may vary.

   Neither conclusion is always true. For example, Dontrell may take detailed notes because he likes history and math. If he doesn't like biology, he may not keep detailed notes. Dontrell may usually keep detailed notes in biology but if he is sick tomorrow, he may not take detailed notes that one day.

2

2

5. The first four numbers in a sequence are 4, 15, 26, and 37.

   a. What is the next number in the sequence? How did you calculate the next number?

   The next number in the sequence is 48. Every number is 11 more than the previous number.

   b. Describe how you used both induction and deduction, and what order you used these reasonings to make your conclusion.

   First, I used inductive reasoning to detect the pattern. Once I knew the rule, I used deductive reasoning to calculate the next number.

6. The first three numbers in a sequence are 1, 4, 9 . . . Marie and Jose both determined that the fourth number in the sequence is 16. Marie's rule involved multiplication whereas Jose's rule involved addition.

   a. What types of reasoning did Marie and Jose use to determine the fourth number in the sequence?

   Marie and Jose used inductive reasoning to describe the pattern. Then they used deductive reasoning to use their pattern to calculate the next number.

   b. What rule did Marie use to determine the fourth number in the sequence?

   Marie noticed that the list showed perfect squares in order: 1^2, 2^2, 3^2. So, the next number in the sequence would be 4^2, or 16.

   c. What rule did Jose use to determine the fourth number in the sequence?

   Jose noticed that he could add 3 to get from 1 to 4 and then add 5 to get from 4 to 9. He reasoned that he could keep adding odd integers. The next odd integer is 7, so he added 9 and 7 to get 16.

   d. Who used the correct rule? Explain your reasoning.

   Both rules are correct. The exact same sequence can be built using both rules.

Problem 4

Students explore two reasons why a conclusion may be false. They identify instances where either the assumed information is false or the argument is not valid in given situations. They also create their own examples. The term counterexample is defined and students provide counterexamples to demonstrate the general statements are not true.

Grouping

Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 5

- Is it possible to know everything?
- If two lines do not intersect, do they have to be parallel to each other? Why or why not?
- Is 2 a prime number? Is 2 an odd number?
- What is an example where the sum of the measures of two acute angles is greater than 90 degrees?
- What is an example where the sum of the measures of two acute angles is less than 90 degrees?

PROBLEM 4 Why Is This False?

There are two reasons why a conclusion may be false. Either the assumed information is false, or the argument is not valid.

1. Derek tells his little brother that it will not rain for the next 30 days because he "knows everything." Why is this conclusion false?

   The conclusion is false because the assumed information is false.

   No one knows everything. So, the conclusion that it will not rain for the next 30 days is most likely false.

2. Two lines are not parallel, so the lines must intersect. Why is this conclusion false?

   The conclusion is false because the argument is not valid.

   Two lines that are not parallel do not have to intersect. They could be skew lines.

3. Write an example of a conclusion that is false because the assumed information is false.

   Answers will vary.

   All boys like football. John is a boy. So, John likes football.

4. Write an example of a conclusion that is false because the argument is not valid.

   Answers will vary.

   Kayleigh doesn't like green fruit. An apple is a green fruit. So, she doesn't like apples.

To show that a statement is false, you can provide a *counterexample*. A **counterexample** is a specific example that shows that a general statement is not true.

5. Provide a counterexample for each of these statements to demonstrate that they are not true.

   a. All prime numbers are odd.

   The only counterexample is 2, which is a prime number that is even.

   b. The sum of the measures of two acute angles is always greater than 90°.

   Answers will vary.

   Two angles that each measure 35° are acute angles, but the sum of their measures is 70°, which is less than 90°.

2

Problem 5

The terminology conditional statement, propositional form, propositional variables, hypothesis, conclusion, and truth value are introduced. Students explore conditional statements, their components, and their truth values. A truth table for a conditional statement is defined and applied to a given situation. Students also create a truth table for a conditional statement.

Guiding Questions for Discuss Phase

- Share a true conditional statement that applies to everyday life.
- Share a false conditional statement that applies to everyday life.
- Is it possible for a conditional statement to be true sometimes and false sometimes? If yes, share an example of true or false conditional statement that applies to everyday life.

Grouping

Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 6

- Can all statements be rewritten as conditional statements? Why or why not?

PROBLEM 5 You Can't Handle the Truth Value

A **conditional statement** is a statement that can be written in the form "If p, then q." This form is the **propositional form** of a conditional statement. It can also be written using symbols as $p \rightarrow q$, which is read as "p implies q." The variables p and q are **propositional variables**. The **hypothesis** of a conditional statement is the variable p. The **conclusion** of a conditional statement is the variable q.

The **truth value** of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then the truth value of the statement is considered true. The truth value of a conditional statement is either true or false, but not both.

You can identify the hypothesis and conclusion from a conditional statement.

Conditional Statement
If $x^2 = 36$, then $x = 6$ or $x = -6$.

Hypothesis of the Conditional Statement
$x^2 = 36$

Conclusion of the Conditional Statement
$x = 6$ or $x = -6$.

Consider the conditional statement: If the measure of an angle is 32°, then the angle is acute.

1. What is the hypothesis p?
   The measure of an angle is 32°.

2. What is the conclusion q?
   The angle is acute.

3. If p is true and q is true, then the truth value of a conditional statement is true.
   a. What does the phrase "If p is true" mean in terms of the conditional statement?
      "If p is true" means that the measure of the angle is 32°.
   b. What does the phrase "If q is true" mean in terms of the conditional statement?
      "If q is true" means that the angle is acute.
   c. Explain why the truth value of the conditional statement is true if both p and q are true.
      The truth value of the conditional statement is true because, by definition, an acute angle is an angle whose measure is less than 90°. An angle whose measure is 32° has a measure less than 90° must be an acute angle.

- Which propositional variable represents the hypothesis of a conditional statement?
- Which propositional variable represents the conclusion of a conditional statement?
- How can you differentiate between the hypothesis and the conclusion in a conditional statement?
- Do all conditional statements have a truth value? Why or why not?

- If the hypothesis and the conclusion of a conditional statement are both true, is the truth value of the conditional statement true or false?
- If the hypothesis and the conclusion of a conditional statement are both false, is the truth value of the conditional statement true or false?
- Under what circumstances is the truth value of a conditional statement false?
- Under what circumstances is the truth value of a conditional statement true?
- What is the purpose of a truth table?
- Why do you suppose truth tables are used?
- How can a truth table be used to answer questions?

4. If p is true and q is false, then the truth value of a conditional statement is false.

a. What does the phrase "If p is true" mean in terms of the conditional statement?

"If p is true" means that the measure of the angle is 32°.

b. What does the phrase "If q is false" mean in terms of the conditional statement?

"If q is false" means that the angle is not acute.

c. Explain why the truth value of the conditional statement is false if p is true and q is false.

The truth value of the conditional statement is false because, by definition, an acute angle is an angle whose measure is less than 90°. The angle is 32°, so the statement that the angle is not acute is a false statement.

5. If p is false and q is true, then the truth value of a conditional statement is true.

a. What does the phrase "If p is false" mean in terms of the conditional statement?

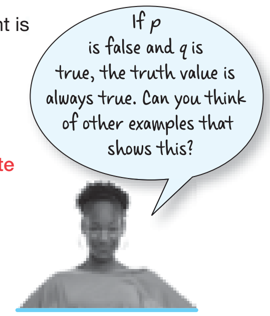
"If p is false" means that the measure of the angle is not 32°.

b. What does the phrase "If q is true" mean in terms of the conditional statement?

"If q is true" means that the angle is acute.

c. Explain why the truth value of the conditional statement is true if p is false and q is true.

The truth value of the conditional statement is true because, by definition, an acute angle is an angle whose measure is less than 90°. The angle could be less than 90°, so the statement that the angle is acute is a true statement.



6. If p is false and q is false, then the truth value of a conditional statement is true.

a. What does the phrase "If p is false" mean in terms of the conditional statement?

"If p is false" means that the measure of the angle is not 32°.

b. What does the phrase "If q is false" mean in terms of the conditional statement?

"If q is false" means that the angle is not acute.

c. Explain why the truth value of the conditional statement is true if both p and q are false.

The truth value of the conditional statement is true because, by definition, an acute angle is an angle whose measure is less than 90°. The angle could be greater than 90°, so the statement that the angle is not acute is a true statement.

Guiding Question for Discuss Phase

Share conditional statements from everyday life that model each row of the truth table.

Grouping

Have students complete Question 7 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 7

- What is the difference between the hypothesis and the conclusion?
- What patterns to do you notice for the truth values in the truth table?
- Share a strategy that helps you learn and understand the truth values in the truth table.
- Share a scenario with a real-world hypothesis and conclusion that models the truth values in the truth table.

A **truth table** is a table that summarizes all possible truth values for a conditional statement $p \rightarrow q$. The first two columns of a truth table represent all possible truth values for the propositional variables p and q. The last column represents the truth value of the conditional statement $p \rightarrow q$.

The truth values for the conditional statement "If the measure of an angle is 32°, then the angle is acute" is shown.

The truth value of the conditional statement $p \rightarrow q$ is determined by the truth value of p and the truth value of q.

- If p is true and q is true, then $p \rightarrow q$ is true.
- If p is true and q is false, then $p \rightarrow q$ is false.
- If p is false and q is true, then $p \rightarrow q$ is true.
- If p is false and q is false, then $p \rightarrow q$ is true.

p	q	$p \rightarrow q$
the measure of an angle is 32°	**the angle is acute**	**If the measure of an angle is 32°, then the angle is acute.**
T	T	T
T	F	F
F	T	T
F	F	T

7. Consider the conditional statement: If $m\overline{AB} = 6$ inches and $m\overline{BC} = 6$ inches, then $\overline{AB} \cong \overline{BC}$.

   a. What is the hypothesis p?

   $m\overline{AB} = 6$ inches and $m\overline{BC} = 6$ inches

   b. What is the conclusion q?

   $\overline{AB} \cong \overline{BC}$

   c. If both p and q are true, what does that mean? What is the truth value of the conditional statement if both p and q are true?

   Both p and q are true means that the measure of line segment AB is 6 inches and the measure of line segment BC is 6 inches; therefore, line segment AB is congruent to line segment BC.

   If both p and q are true, then the truth value of the conditional statement is true.

d. If p is true and q is false, what does that mean? What is the truth value of the conditional statement if p is true and q is false?

p is true and q is false means that the measure of line segment AB is 6 inches and the measure of line segment BC is 6 inches; therefore, line segment AB is not congruent to line segment BC.

If p is true and q is false, then the truth value of the conditional statement is false.

e. If p is false and q is true, what does that mean? What is the truth value of the conditional statement if p is false and q is true?

p is false and q is true means that the measure of line segment AB is not 6 inches and the measure of line segment BC is not 6 inches; therefore, line segment AB is congruent to line segment BC.

If p is false and q is true, then the truth value of the conditional statement is true. This statement could be true if line segment AB and line segment BC have the same measure, but a value different than 6 inches.

f. If both p and q are false, what does that mean? What is the truth value of the conditional statement if both p and q are false?

Both p and q are false means that the measure of line segment AB is not 6 inches and the measure of line segment BC is not 6 inches; therefore, line segment AB is not congruent to line segment BC.

If both p and q are false, then the truth value of the conditional statement is true. This statement could be true if line segments AB and BC do not have the same measure.

g. Summarize your answers to parts (a) through (f) by completing a truth table for the conditional statement.

p	q	$p \rightarrow q$
$m\overline{AB} = 6$ inches and $m\overline{BC} = 6$ inches	$\overline{AB} \cong \overline{BC}$	If $m\overline{AB} = 6$ inches and $m\overline{BC} = 6$ inches, then $\overline{AB} \cong \overline{BC}$.
T	T	T
T	F	F
F	T	T
F	F	T

2

Problem 6

Geometric conditional statements are given. Students draw the appropriate diagrams and identify the given and prove statements for each situation. This will prepare them for reading a theorem, expressing the theorem mathematically, and proving the theorem.

Grouping

Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 5

- Is there more than one correct diagram that exemplifies this conditional statement?
- What is the hypothesis in this conditional statement? Should that be written as the given or prove statement?
- What is the conclusion in this conditional statement? Should that be written as the given or prove statement?
- Is the hypothesis always written first in a conditional statement?
- Is the conclusion always written last in a conditional statement?
- Is there more than one way to write the given statement?

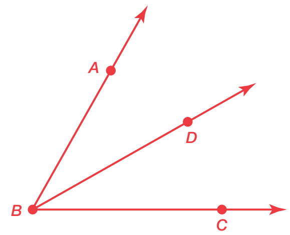
PROBLEM 6 Rewriting Conditional Statements

For each conditional statement, draw a diagram and then write the hypothesis as the "Given" and the conclusion as the "Prove."

1. If $\overrightarrow{BD}$ bisects $\angle ABC$, then $\angle ABD \cong \angle CBD$.

   Given: $\overrightarrow{BD}$ bisects $\angle ABC$

   Prove: $\angle ABD \cong \angle CBD$



2. $\overline{AM} \cong \overline{MB}$, if M is the midpoint of $\overline{AB}$.

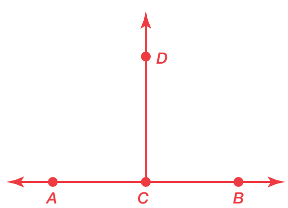
   Given: M *is the midpoint of* $\overline{AB}$

   Prove: $\overline{AM} \cong \overline{MB}$

3. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point C, then $\angle ACD$ is a right angle and $\angle BCD$ is a right angle.

   Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point C.

   Prove: $\angle ACD$ is a right angle and $\angle BCD$ is a right angle

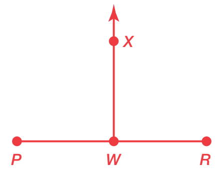


- Is there more than one way to write the prove statement?
- What counterexample can be drawn to show Susan she is incorrect?

4. $\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{PR}$, if $\overleftrightarrow{WX} \perp \overline{PR}$ and $\overleftrightarrow{WX}$ bisects $\overline{PR}$.

Given: $\overleftrightarrow{WX} \perp \overline{PR}$ and $\overleftrightarrow{WX}$ bisects $\overline{PR}$

Prove: $\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{PR}$



5. Mr. David wrote the following information on the board.

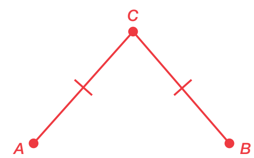
If $\overline{AC} \cong \overline{BC}$, then C is the midpoint of $\overline{AB}$.

He asked his students to discuss the truth of this conditional statement.

Susan said she believed the statement to be true in all situations. Marcus disagreed with Susan and said that the statement was not true all of the time.

What is Marcus thinking and who is correct?

Marcus is correct. If point C is not located on $\overline{AB}$, the hypothesis may be true but the conclusion is false as shown.



Talk the Talk

Students write a short note to a friend explaining the difference between inductive reasoning and deductive reasoning.

2

Talk the Talk

1. Write a short note to a friend explaining induction and deduction. Include definitions of both terms and examples that are very easy to understand.

Answers will vary.

Dear Effie,

Induction is when you observe some specific things and think that something general is always true because of what you saw. For example, when you see a couple of green frogs, you might think that all frogs are green.

Deduction is when you learn a general rule and then you apply it to a situation. For example, you probably were taught to never get into a stranger's car. The next day when a stranger invites you into her car, you know that you should say "No!"

Have a nice day,

Albert

Be prepared to share your solutions and methods.

Check for Students' Understanding

1. Scenario: If I throw a coin into the fountain, it will sink to the bottom.

   a. Write a statement about this scenario using inductive reasoning.

   Statement: Jeff sees all of the coins on the bottom of the fountain, so he concludes that the coin he throws into the fountain will also sink.

   b. Write a statement about this scenario using deductive reasoning.

   Statement: Jeff knows the laws of physics concerning mass and gravity, so he concludes that the coin will sink to the bottom of the fountain.

2. An old wives' tale is "*A watched pot will never boil*."

   a. Rewrite as a Conditional Statement:

   If a pot is watched, then the water will never boil.

   b. Identify the hypothesis:

   A pot is watched.

   c. Identify the conclusion:

   The water will never boil.

   d. If the hypothesis is true and the conclusion is true, what does this mean?

   It means the pot was watched and the water never boiled.

   e. If the hypothesis is true and the conclusion is false, what does this mean?

   It means the pot was watched and the water boiled.

   f. If the hypothesis is false and the conclusion is true, what does this mean?

   It means the pot was not watched and the water never boiled.

   g. If the hypothesis is false and the conclusion is false, what does this mean?

   It means the pot was not watched and the water boiled.

   h. Is this old wives' tale true? Explain.

   Answers will vary.

   Possible Response: It is not true. It may seem like it takes forever to boil if you are watching and waiting, but it will eventually boil whether you are watching it or not watching it.

2

And Now From a New Angle

Special Angles and Postulates

2.2

LEARNING GOALS

In this lesson, you will:

- Calculate the complement and supplement of an angle.
- Classify adjacent angles, linear pairs, and vertical angles.
- Differentiate between postulates and theorems.
- Differentiate between Euclidean and non-Euclidean geometries.

KEY TERMS

- supplementary angles
- complementary angles
- adjacent angles
- linear pair
- vertical angles
- postulate
- theorem
- Euclidean geometry
- Linear Pair Postulate
- Segment Addition Postulate
- Angle Addition Postulate

ESSENTIAL IDEAS

- Two angles are supplementary angles if the sum of their measures is equal to 180°.
- Two angles are complementary angles if the sum of their measures is equal to 90°.
- Adjacent angles are angles that share a common vertex and share a common side.
- A linear pair is two angles that are both adjacent and supplementary.
- Vertical angles are a pair of non-adjacent angles formed by two intersecting lines.
- A postulate is a statement that is accepted without proof.
- A theorem is a statement that can be proven.
- The Linear Pair Postulate states: "If two angles are a linear pair, then the angles are supplementary."
- The Segment Addition Postulate states: "If point B is on $\overline{AC}$ and between points A and C, the $AB + BC = AC$."
- The Angle Addition Postulate states:" If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."

COMMON CORE STATE STANDARDS FOR MATHEMATICS

G-CO Congruence

Prove geometric theorems

9. Prove theorems about lines and angles.

Overview

Students algebraically solve for supplements and complements of angles. They answer questions related to adjacent angles, linear pairs, and vertical angles. Euclidean geometry is distinguished from non-Euclidean geometry and Euclid's first five postulates and Euclid's Elements are mentioned. The terms postulate and theorem are defined, and students use the Linear Pair Postulate, the Segment Addition Postulate, and the Angle Addition Postulate to answer related questions.

Warm Up

1. If $x + 4x = 90$, solve for x.

   $5x = 90$

   $x = 18$

2. If $0.5x + 2x = 90$, solve for x.

   $2.5x = 90$

   $x = 36$

3. If $x + 2x = 180$, solve for x.

   $3x = 60$

   $x = 20$

4. If $0.5x + 4x = 180$, solve for x.

   $4.5x = 180$

   $x = 40$

2

2

2.2

And Now From a New Angle

Special Angles and Postulates

2

LEARNING GOALS

In this lesson, you will:

- Calculate the complement and supplement of an angle.
- Classify adjacent angles, linear pairs, and vertical angles.
- Differentiate between postulates and theorems.
- Differentiate between Euclidean and non-Euclidean geometries.

KEY TERMS

- supplementary angles
- complementary angles
- adjacent angles
- linear pair
- vertical angles
- postulate
- theorem
- Euclidean geometry
- Linear Pair Postulate
- Segment Addition Postulate
- Angle Addition Postulate

A compliment is an expression of praise, admiration, or congratulations. Often when someone does something noteworthy, you may "pay them a compliment" to recognize the person's accomplishments.

Even though they are spelled similarly, the word "complement" means something very different. To complement something means to complete or to make whole. This phrase is used in mathematics, linguistics, music, and art. For example, complementary angles have measures that sum to 180 degrees—making the straight angle "whole." In music, a complement is an interval that when added to another spans an octave—makes it "whole."

The film *Jerry McGuire* features the famous line "You complete me," meaning that the other person complements them or that together they form a whole. So, a complement can be quite a compliment indeed!

Problem 1

Supplementary and complementary angles are defined. Students use a compass to draw angles of specified measures. Then they use algebra to calculate the complement and supplement of different angles.

Grouping

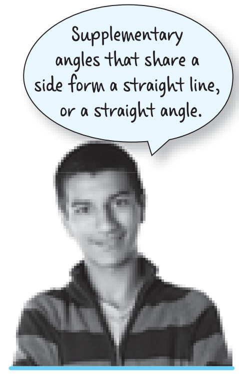
Have students complete Questions 1 through 7 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 7

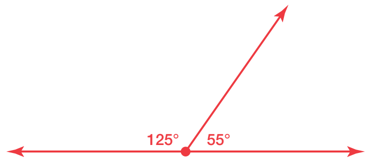
- What are the measures of your angles in the supplementary pair?
- How did you calculate the measure of the supplement of the 22° angle?
- What are the measures of your angles in the complementary pair?
- How did you calculate the measure of the complement of the 622° angle?
- Is it possible for two angles to be both congruent and supplementary?
- Is it possible for two angles to be both congruent and complementary?

PROBLEM 1 Supplements and Complements

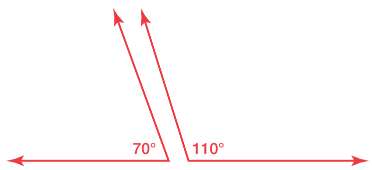
Two angles are **supplementary angles** if the sum of their angle measures is equal to 180°.



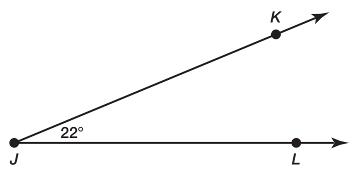
1. Use a protractor to draw a pair of supplementary angles that share a common side, and then measure each angle.



2. Use a protractor to draw a pair of supplementary angles that do not share a common side, and then measure each angle.



3. Calculate the measure of an angle that is supplementary to $\angle KJL$.

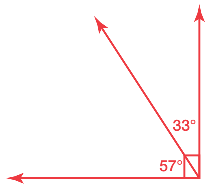


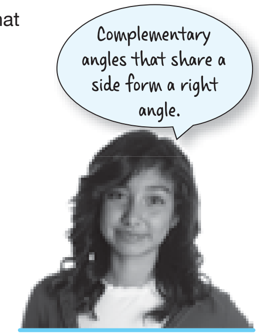
An angle that is supplementary to $\angle KJL$ has a measure of 158°.

- How do you know when to set the sum of the measures of the two angles equal to 90°?
- How do you know when to set the sum of the measures of the two angles equal to 180°?

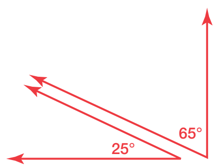
Two angles are **complementary angles** if the sum of their angle measures is equal to 90°.

4. Use a protractor to draw a pair of complementary angles that share a common side, and then measure each angle.

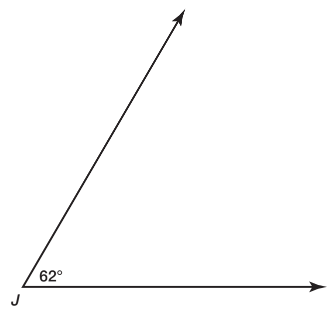




5. Use a protractor to draw a pair of complementary angles that do not share a common side, and then measure each angle.



6. Calculate the measure of an angle that is complementary to $\angle J$.



An angle that is complementary to $\angle J$ has a measure of 28°.

2

7. Determine the measure of each angle. Show your work and explain your reasoning.

a. Two angles are congruent and supplementary.

$x + x = 180$

$2x = 180$

$x = 90$

The measure of each angle is 90°.

The sum of the angle measures is 180°.

b. Two angles are congruent and complementary.

$x + x = 90$

$2x = 90$

$x = 45$

The measure of each angle is 45°.

The sum of the angle measures is 90°.

c. The complement of an angle is twice the measure of the angle.

$x + 2x = 90$

$3x = 90$

$x = 30$

The measure of the angle is 30°.

The measure of the complement of the angle is $2 \times 30°$, or 60°.

The sum of the angle measures is 90°.

d. The supplement of an angle is half the measure of the angle.

$x + 0.5x = 180$

$1.5x = 180$

$x = 120$

The measure of the angle is 120°.

The measure of the supplement of the angle is $\frac{1}{2}(120°)$, or 60°.

The sum of the angle measures is 180°.

Grouping

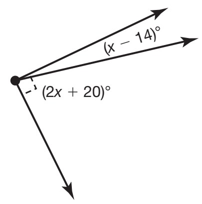
Have students complete Question 8 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Question 8

- What do parts (a) and (b) have in common?
- What is different parts (a) and (b)?
- Share a general description for the solution methods of parts (a) and (b).

8. Determine the angle measures in each diagram.

a.



$(x - 14) + (2x + 20) = 90$

$3x + 6 = 90$

$3x = 84$

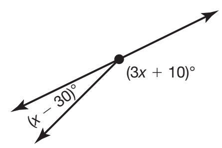
$x = 28$

Measure of 1st angle $= x - 14$
$= 28 - 14$
$= 14$

Measure of 2nd angle $= 2x + 20$
$= 2(28) + 20$
$= 56 + 20$
$= 76$

The measures of the two angles are 14° and 76°.

b.



$(3x + 10) + (x - 30) = 180$

$4x - 20 = 180$

$4x = 200$

$x = 50$

Measure of 1st angle $= x - 30$
$= 50 - 30$
$= 20$

Measure of 2nd angle $= 3x + 10$
$= 3(50) + 10$
$= 150 + 10$
$= 160$

The measures of the two angles are 20° and 160°.

Problem 2

Students explore adjacent angles, linear pairs, and vertical angles.

Grouping

Have students complete Question 1 with a partner. Then have students share their responses as a class.

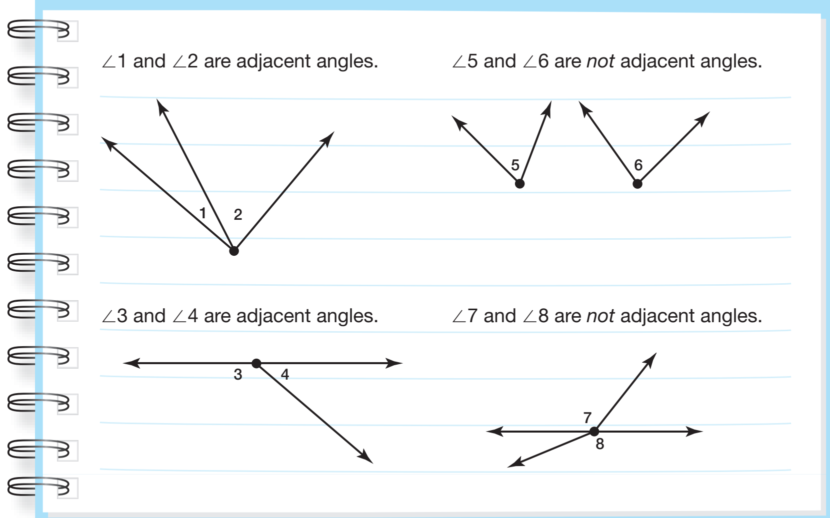
Guiding Questions for Share Phase, Question 1

- Do adjacent angles share a common vertex?
- Do adjacent angles share a common side?
- Is there more than one way to correctly draw angle 2 so that it is adjacent to angle 1? Explain.

2

PROBLEM 2 **Angle Relationships**

You have learned that angles can be supplementary or complementary. Let's explore other angle relationships.

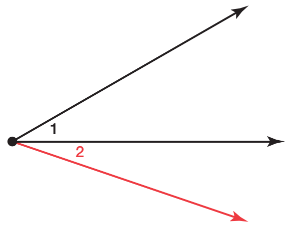


1. Analyze the worked example. Then answer each question.

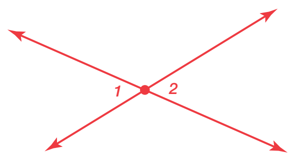
   a. Describe adjacent angles.

   Two angles are adjacent if they share a common vertex and share a common side.

b. Draw ∠2 so that it is adjacent to ∠1.



c. Is it possible to draw two angles that share a common vertex but do not share a common side? If so, draw an example. If not, explain why not.



d. Is it possible to draw two angles that share a common side, but do not share a common vertex? If so, draw an example. If not, explain why not.
It is not possible. If two angles share a side, then they must be adjacent on the same line, and therefore share a common vertex.

Adjacent angles are two angles that share a common vertex and share a common side.

2

Grouping

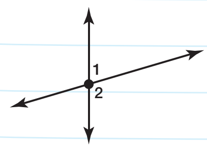
Have students complete Question 2 with a partner. Then share their responses as a class.

2

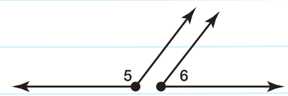
Guiding Questions for Share Phase, Question 2

- Does a linear pair of angles always form a line?
- Does a line have a degree measure?
- What is the sum of the measures of the two angles that form a linear pair?
- Are the two angles forming a linear pair of angles always adjacent angles?
- Are the two non-common sides of a linear pair always opposite rays?

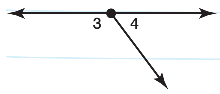
$\angle 1$ and $\angle 2$ form a linear pair.



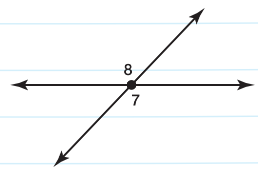
$\angle 5$ and $\angle 6$ do *not* form a linear pair.



$\angle 3$ and $\angle 4$ form a linear pair.



$\angle 7$ and $\angle 8$ do *not* form a linear pair.

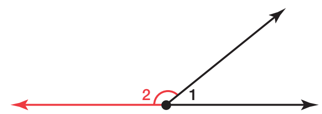


2. Analyze the worked example. Then answer each question.

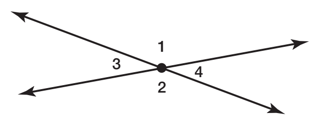
a. Describe a linear pair of angles.

Two angles are a linear pair if they are adjacent and their noncommon sides form a line.

b. Draw ∠2 so that it forms a linear pair with ∠1.



c. Name all linear pairs in the figure shown.



The linear pairs are ∠1 and ∠4, ∠1 and ∠3, ∠2 and ∠3, ∠2 and ∠4.

d. If the angles that form a linear pair are congruent, what can you conclude?

If the angles that form a linear pair are congruent, then the intersecting lines, line segments, or rays forming the linear pair are perpendicular.

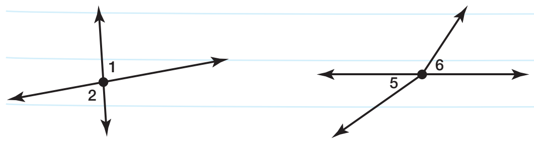
A **linear pair** of angles are two adjacent angles that have noncommon sides that form a line.

Guiding Questions for Share Phase, Question 3

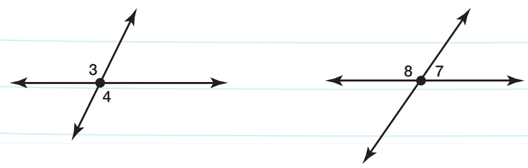
- Are vertical angles adjacent or non-adjacent angles?
- Do intersecting lines always form vertical angles?
- Two intersecting lines form how many different pairs of vertical angles?
- Three intersecting lines form how many different pairs of vertical angles?
- Why do you suppose vertical angles always congruent?

2

$\angle 1$ and $\angle 2$ are vertical angles. $\angle 5$ and $\angle 6$ are *not* vertical angles.



$\angle 3$ and $\angle 4$ are vertical angles. $\angle 7$ and $\angle 8$ are *not* vertical angles.

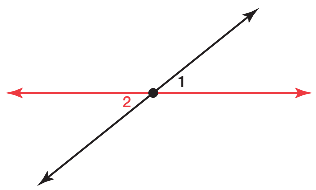


3. Analyze the worked example. Then answer each question.

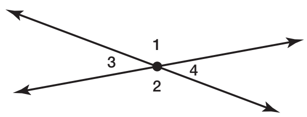
a. Describe vertical angles.

Vertical angles are nonadjacent angles formed by two intersecting lines.

b. Draw ∠2 so that it forms a vertical angle with ∠1.



c. Name all vertical angle pairs in the diagram shown.



∠1 and ∠2 are vertical angles. ∠3 and ∠4 are also vertical angles.

d. Measure each angle in part (c). What do you notice?

Vertical angles are always congruent.

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

Grouping

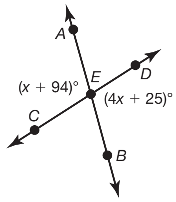
- Have students complete Questions 4 and 5 with a partner. Then have students share their responses as a class.

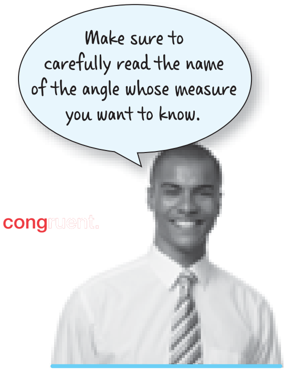
2

Guiding Questions for Share Phase, Questions 4 and 5

- Is there more than one way to draw a correct diagram to represent the conditional statements in Question 5?
- Identify the hypothesis of the conditional statements in Question 5.
- Identify the conclusion of the conditional statements in Question 5.

4. Determine $m\angle AED$. Explain how you determined the angle measure.





Angles DEB and AEC are vertical angles, so they are congruent.

$(x + 94) = (4x + 25)$

$94 = 3x + 25$

$69 = 3x$

$23 = x$

$m\angle AEC = 23 + 94 = 117°$

$m\angle DEB = 117°$

Angles AEC and AED are supplementary.

$m\angle AEC + m\angle AED = 180$

$117 + m\angle AED = 180$

$m\angle AED = 63$

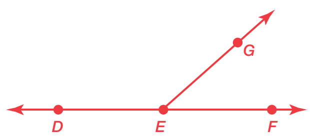
So, the measure of angle AED is 63°.

5. For each conditional statement, draw a diagram and then write the hypothesis as the "Given" and the conclusion as the "Prove."

a. $m\angle DEG + m\angle GEF = 180°$, if $\angle DEG$ and $\angle GEF$ are a linear pair.

Given: $\angle DEG$ and $\angle GEF$ are a linear pair

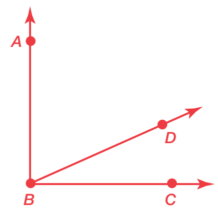
Prove: $m\angle DEG + m\angle GEF = 180°$



b. If $\angle ABD$ and $\angle DBC$ are complementary, then $\overrightarrow{BA} \perp \overrightarrow{BC}$.

Given: $\angle ABD$ and $\angle DBC$ are complementary

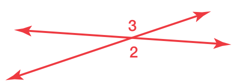
Prove: $\overrightarrow{BA} \perp \overrightarrow{BC}$



c. If $\angle 2$ and $\angle 3$ are vertical angles, then $\angle 2 \cong \angle 3$.

Given: $\angle 2$ and $\angle 3$ are vertical angles

Prove: $\angle 2 \cong \angle 3$



2

2

Problem 3

The terms postulate and theorem are defined. Euclid's first five postulates and Euclid's Elements are mentioned as the foundation of Euclidean Geometry. Non-Euclidean Geometry is also mentioned. The Linear Pair Postulate, the Segment Addition Postulate, and the Angle Addition Postulate are stated. Students use each postulate to complete statements and answer questions.

Guiding Questions for Discuss Phase

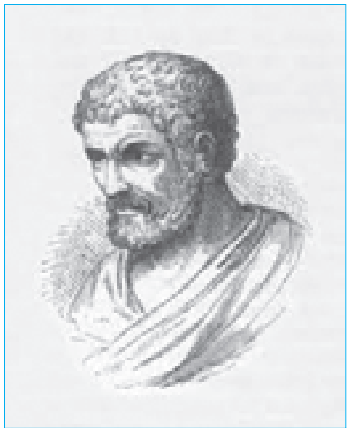
- Who was Euclid and what is he known for?
- What is the key difference between Euclidean and non-Euclidean geometry?

PROBLEM 3 Postulates and Theorems

A **postulate** is a statement that is accepted without proof.

A **theorem** is a statement that can be proven.

The Elements is a book written by the Greek mathematician Euclid. He used a small number of undefined terms and postulates to systematically prove many theorems. As a result, Euclid was able to develop a complete system we now know as **Euclidean geometry**.



Greek mathematician Euclid is sometimes referred to as the Father of Geometry.

Euclid's first five postulates are:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn that has the segment as its radius and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (This postulate is equivalent to what is known as the parallel postulate.)

Euclid used only the first four postulates to prove the first 28 propositions or theorems of *The Elements*, but was forced to use the fifth postulate, the parallel postulate, to prove the 29th theorem.

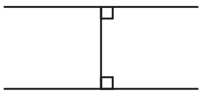
The *Elements* also includes five "common notions":

1. Things that equal the same thing also equal one another.
2. If equals are added to equals, then the wholes are equal.
3. If equals are subtracted from equals, then the remainders are equal.
4. Things that coincide with one another equal one another.
5. The whole is greater than the part.

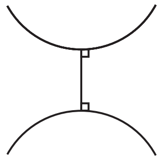
It is important to note that Euclidean geometry is not the only system of geometry. Examples of non-Euclidian geometries include hyperbolic and elliptic geometry. The essential difference between Euclidean and non-Euclidean geometry is the nature of parallel lines.

Another way to describe the differences between these geometries is to consider two lines in a plane that are both perpendicular to a third line.

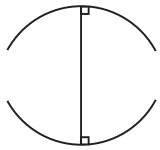
- In Euclidean geometry, the lines remain at a constant distance from each other and are known as parallels.



- In hyperbolic geometry, the lines "curve away" from each other.



- In elliptic geometry, the lines "curve toward" each other and eventually intersect.



Using this textbook as a guide, you will develop your own system of geometry, just like Euclid. You already used the three undefined terms *point, line,* and *plane* to define related terms such as *line segment* and *angle*.

Your journey continues with the introduction of three fundamental postulates:

- The Linear Pair Postulate
- The Segment Addition Postulate
- The Angle Addition Postulate

You will use these postulates to make various conjectures. If you are able to prove your conjectures, then the conjectures will become theorems. These theorems can then be used to make even more conjectures, which may also become theorems. Mathematicians use this process to create new mathematical ideas.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

2

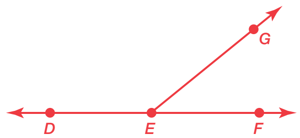
Guiding Questions for Share Phase, Questions 1 through 3

- What is the hypothesis in the Linear Pair Postulate?
- What is the conclusion in the Linear Pair Postulate?
- Does the Linear Pair Postulate tell you the sum of the measures of the two angles forming the linear pair is equal to 180°?
- What is the definition of supplementary angles?
- Does the definition of supplementary angles tell you the sum of the measures of the two angles forming the linear pair is equal to 180°?
- What is the hypothesis in the Segment Addition Postulate?
- What is the conclusion in the Segment Addition Postulate?
- Would the Segment Addition Postulate hold true if point *B* was not located between points *A* and *C*?
- What is the hypothesis in the Angle Addition Postulate?
- What is the conclusion in the Angle Addition Postulate?
- Would the Angle Addition Postulate hold true if point *D* did not lie in the interior of angle *ABC*?

The **Linear Pair Postulate** states: "If two angles form a linear pair, then the angles are supplementary."

1. Use the Linear Pair Postulate to complete each representation.

   a. Sketch and label a linear pair.



   b. Use your sketch and the Linear Pair Postulate to write the hypothesis.

   $\angle DEG$ and $\angle GEF$ form a linear pair.

   c. Use your sketch and the Linear Pair Postulate to write the conclusion.

   $\angle DEG$ and $\angle GEF$ are supplementary angles.

   d. Use your conclusion and the definition of supplementary angles to write a statement about the angles in your figure.

   $m\angle DEG + m\angle GEF = 180°$

The **Segment Addition Postulate** states: "If point B is on $\overline{AC}$ and between points A and C, then $AB + BC = AC$."

2. Use the Segment Addition Postulate to complete each representation.

   a. Sketch and label collinear points D, E, and F with point E between points D and F.

   b. Use your sketch and the Segment Addition Postulate to write the hypothesis.

   Point E is on $\overline{DF}$ and between points D and F.

   c. Use your sketch and the Segment Addition Postulate to write the conclusion.

   $DE + EF = DF$

   d. Write your conclusion using measure notation.

   $m\overline{DE} + m\overline{EF} = m\overline{DF}$

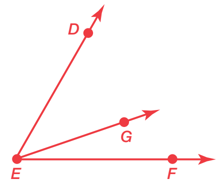
2

2

The **Angle Addition Postulate** states: "If point *D* lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."

3. Use the Angle Addition Postulate to complete each representation.

a. Sketch and label $\angle DEF$ with $\overrightarrow{EG}$ drawn in the interior of $\angle DEF$.



b. Use your sketch and the Angle Addition Postulate to write the hypothesis.

Point *D* lies in the interior of $\angle ABC$.

c. Use your sketch and the Angle Addition Postulate to write the conclusion.

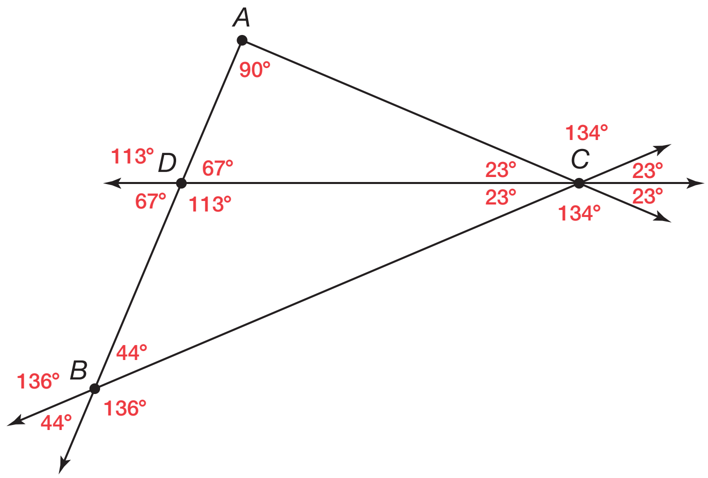
$m\angle DEG + m\angle GEF = m\angle DEF$

Be prepared to share your solutions and methods.

Check for Students' Understanding

Given: $\overrightarrow{AB} \perp \overrightarrow{AC}$, $m\angle ABC = 44°$, and $\overleftrightarrow{CD}$ bisects $\angle ACB$

1. Determine the measure of each angle in the diagram.



2. List all of the definitions, theorems, or reasons you used to determine the measures of the angles in the diagram.

1. The sum of the measures of the interior angles of a triangle is equal to 180°
2. Vertical angles are congruent
3. Definition of linear pair
4. Definition of bisect
5. Definition of perpendicular
6. Definition of right angle
7. Definition of supplementary angles

2

2.3

Forms of Proof

Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof

LEARNING GOALS

In this lesson, you will:

- Use the addition and subtraction properties of equality.
- Use the reflexive, substitution, and transitive properties.
- Write a paragraph proof.
- Prove theorems involving angles.
- Complete a two-column proof.
- Perform a construction proof.
- Complete a flow chart proof.

KEY TERMS

- Addition Property of Equality
- Subtraction Property of Equality
- Reflexive Property
- Substitution Property
- Transitive Property
- flow chart proof
- two-column proof
- paragraph proof
- construction proof
- Right Angle Congruence Theorem
- Congruent Supplement Theorem
- Congruent Complement Theorem
- Vertical Angle Theorem

ESSENTIAL IDEAS

- The Addition Property of Equality states: if a, b, and c are real numbers and $a = b$, then $a + c = b + c$.
- The Subtraction Property of Equality states: if a, b, and c are real numbers and $a = b$, then $a - c = b - c$.
- The Reflexive Property is used to conclude an angle is congruent to itself, or to conclude a line segment is congruent to itself.
- The Substitution Property is used to conclude two angles are equal in measure resulting from the measure of each angle equal to the same value, or to conclude two line segments are equal in measure resulting from the measure of each line segment equal to the same value.
- The Transitive Property is used to conclude two angles are congruent resulting from each angle congruent to the same angle, or to conclude two line segments are congruent resulting from each line segment congruent to the same line segment.
- A paragraph proof is a proof in which the steps and their corresponding reasons are written in complete sentences.
- A two-column proof is a proof in which the statements appear in the left column and their corresponding reasons appear in the right column.
- A construction proof is a proof that results from creating an object with specific properties using only a compass and straightedge.
- A flow chart proof is a proof that is similar to the two-column proof but uses boxes to write each statement and reason. Arrows are used to connect the boxes that denote how each statement and reason is generated from one or several other statements and reasons.
- If two angles are supplements of the same angle (or congruent angles), then they are congruent.
- If two angles are complements of the same angle (or congruent angles), then they are congruent.
- Vertical angles are congruent.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

G-CO Congruence

Prove geometric theorems

9. Prove theorems about lines and angles.

2

Overview

Students apply the Addition Property of Equality, the Subtraction Property of Equality, the Identity Property, the Reflexive Property, the Substitution Property, and the Transitive Property to line segments, distances, angles or angle measures. Four models of proof are introduced: paragraph proof, two-column proof, construction proof, and flow chart proof. Students prove the Congruent Supplement Theorem, the Congruent Complement Theorem, and the Vertical Angle Theorem using the various proof models.

Warm Up

1. If $m\angle 5 + m\angle 8 = m\angle 1 + m\angle 8$, what information can be concluded?

It can be concluded that $m\angle 5 = m\angle 1$ because when the same angle measure is subtracted from both sides of the equation, the remaining angle measures are equal.

2

2. If $m\overline{CD} + m\overline{RP} = m\overline{FG} + m\overline{RP}$, what information can be concluded?

It can be concluded that measure of segment *CD* is equal to the measure of segment *FG* because when the same length is subtracted from both sides of the equation, the remaining lengths are equal.

2

Forms of Proof

2.3

Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof

LEARNING GOALS

In this lesson, you will:

- Use the addition and subtraction properties of equality.
- Use the reflexive, substitution, and transitive properties.
- Write a paragraph proof.
- Prove theorems involving angles.
- Complete a two-column proof.
- Perform a construction proof.
- Complete a flow chart proof.

KEY TERMS

- Addition Property of Equality
- Subtraction Property of Equality
- Reflexive Property
- Substitution Property
- Transitive Property
- flow chart proof
- two-column proof
- paragraph proof
- construction proof
- Right Angle Congruence Theorem
- Congruent Supplement Theorem
- Congruent Complement Theorem
- Vertical Angle Theorem

Have you ever heard the famous phrase, "The proof is in the pudding"? If you stop to think about what this phrase means, you might be left scratching your head!

The phrase used now is actually a shortened version of the original phrase, "The proof of the pudding is in the eating." This phrase meant that the pudding recipe may appear to be delicious, include fresh ingredients, and may even look delicious after it is made. However, the only way to really "prove" that the pudding is delicious is by eating it!

Today it is used to imply that the quality or truth of something that only be determined by putting it into action. For example, you don't know how good an idea is until you actually test the idea.

Can you think of any other popular phrases that don't seem to make sense? Perhaps you should do a little research to find out where these phrases came from.

Problem 1

Students will apply the Addition Property of Equality, the Subtraction Property of Equality, the Reflexive Property, the Substitution Property, and the Transitive Property to line segments, distances, angles, and angle measures.

Grouping

Have students complete Questions 1 through 10 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 10

- In Question 1, did you add the same angle to both sides of the equation?
- In Question 2, did you add the same line segment to both sides of the equation?
- In Question 3, did you subtract the same angle from both sides of the equation?
- In Question 4, did you subtract the same line segment from both sides of the equation?
- In Question 5, how many angles are involved when you apply the Reflexive Property?
- In Question 6, how many line segments are involved when you apply the Reflexive Property?
- In Question 7, what did you substitute for what?

PROBLEM 1 Properties of Real Numbers in Geometry

Many properties of real numbers can be applied in geometry. These properties are important when making conjectures and proving new theorems.

The **Addition Property of Equality** states: "If a, b, and c are real numbers and $a = b$, then $a + c = b + c$."

The Addition Property of Equality can be applied to angle measures, segment measures, and distances.

Angle measures:

If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$.

Segment measures:

If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} + m\overline{EF} = m\overline{CD} + m\overline{EF}$.

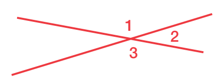
Distances:

If $AB = CD$, then $AB + EF = CD + EF$.

1. Sketch a diagram and write a statement that applies the Addition Property of Equality to angle measures.

   Answers will vary.

   If $m\angle 1 = m\angle 3$, then $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$.



2. Sketch a diagram and write a statement that applies the Addition Property of Equality to segment measures.

   Answers will vary.

   If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$.

- In Question 8, what did you substitute for what?
- In Question 9, how many angles are involved when you apply the transitive property? What angle was used twice?
- In Question 10, how many line segments are involved when you apply the transitive property? What line segment was used twice?
- How is the using the transitive property different than using substitution?

Note

Emphasize which properties are associated with an equal sign and which properties are associated with a congruent sign.

The **Subtraction Property of Equality** states: "If a, b, and c are real numbers and $a = b$, then $a - c = b - c$."

The Subtraction Property of Equality can be applied to angle measures, segment measures, and distances.

Angle measures:

If $m\angle 1 = m\angle 2$, then $m\angle 1 - m\angle 3 = m\angle 2 - m\angle 3$.

Segment measures:

If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} - m\overline{EF} = m\overline{CD} - m\overline{EF}$.

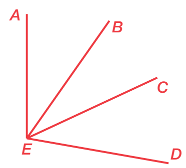
Distances:

If $AB = CD$, then $AB - EF = CD - EF$.

3. Sketch a diagram and write a statement that applies the Subtraction Property of Equality to angle measures.

Answers will vary.

If $m\angle AEC = m\angle BED$, then $m\angle AEC - m\angle BEC = m\angle BED - m\angle BEC$.
So, $m\angle AEB = m\angle CED$.



4. Sketch a diagram and write a statement that applies the Subtraction Property of Equality to segment measures.

Answers will vary.

If $m\overline{AC} = m\overline{BD}$, then $m\overline{AC} - m\overline{BC} = m\overline{BD} - m\overline{BC}$.
So, $m\overline{AB} = m\overline{CD}$.

2

The **Reflexive Property** states: "If *a* is *a* real number, then $a = a$."

The Reflexive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

Angle measures:

$m\angle 1 = m\angle 1$

Segment measures:

$m\overline{AB} = m\overline{AB}$

Distances:

$AB = AB$

Congruent angles:

$\angle 1 \cong \angle 1$

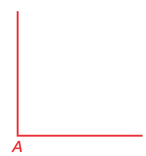
Congruent segments:

$\overline{AB} \cong \overline{AB}$

5. Sketch a diagram and write a statement that applies the Reflexive Property to angles.

Answers will vary.

$\angle A \cong \angle A$



6. Sketch a diagram and write a statement that applies the Reflexive Property to segments.

Answers will vary.

$\overline{CD} \cong \overline{CD}$

The **Substitution Property** states: "If a and b are real numbers and $a = b$, then a can be substituted for b."

The Substitution Property can be applied to angle measures, segment measures, and distances.

Angle measures:

If $m\angle 1 = 56°$ and $m\angle 2 = 56°$, then $m\angle 1 = m\angle 2$.

Segment measures:

If $m\overline{AB} = 4$ mm and $m\overline{CD} = 4$ mm, then $m\overline{AB} = m\overline{CD}$.

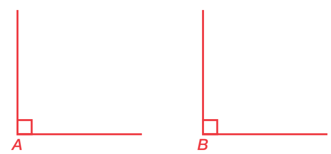
Distances:

If $AB = 12$ ft and $CD = 12$ ft, then $AB = CD$.

7. Sketch a diagram and write a statement that applies the Substitution Property to angles.

Answers will vary.

If $m\angle A = 90°$ and $m\angle B = 90°$, then $m\angle A = m\angle B$.



8. Sketch a diagram and write a statement that applies the Substitution Property to segments.

Answers will vary.

If $m\overline{CD} = 8$ inches and $m\overline{EF} = 8$ inches, then $m\overline{CD} = m\overline{EF}$.

2

2

The **Transitive Property** states: "If a, b, and c are real numbers, $a = b$, and $b = c$, then $a = c$."

The Transitive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

Angle measures:

If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Segment measures:

If $m\overline{AB} = m\overline{CD}$ and $m\overline{CD} = m\overline{EF}$, then $m\overline{AB} = m\overline{EF}$.

Distances:

If $AB = CD$ and $CD = EF$, then $AB = EF$.

Congruent angles:

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

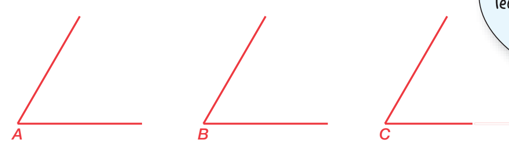
Congruent segments:

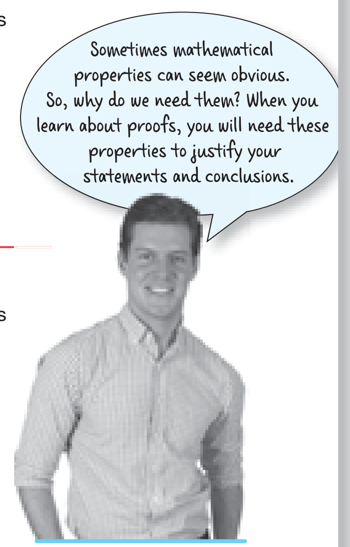
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

9. Sketch a diagram and write a statement that applies the Transitive Property to angles.

Answers will vary.

If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

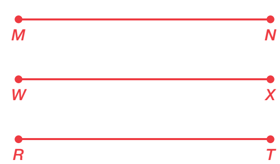




10. Sketch a diagram and write a statement that applies the Transitive Property to congruent segments.

Answers will vary.

If $\overline{MN} \cong \overline{WX}$ and $\overline{WX} \cong \overline{RT}$, then $\overline{MN} \cong \overline{RT}$.



Problem 2

Students explore four different models of proof: Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof. They will need to use scissors in Question 1.

Grouping

Have students complete Questions 1 parts (a) through (c) with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 parts (a) through (c)

- Which statement and reason should be in the first position in the flow chart? Why?
- Which statement and reason should be in the last position in the flow chart? Why?
- How do you know which statement and reason should follow the previous statement and reason?
- Is there more than one correct way to order these statements and reasons? Explain.
- What is an example of one statement that must follow another statement?
- What is an example of one statement that must come before another statement?

PROBLEM 2 Various Forms of Proof

A **proof** is a logical series of statements and corresponding reasons that starts with a hypothesis and arrives at a conclusion. In this course, you will use four different kinds of proof.

1. The diagram shows four collinear points A, B, C, and D such that point B lies between points A and C, point C lies between points B and D, and $\overline{AB} \cong \overline{CD}$.

Consider the conditional statement: If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

a. Write the hypothesis as the "Given" and the conclusion as the "Prove."

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$

A **flow chart proof** is a proof in which the steps and reasons for each step are written in boxes. Arrows connect the boxes and indicate how each step and reason is generated from one or more other steps and reasons.

b. Cut out the steps on the flow chart proof.

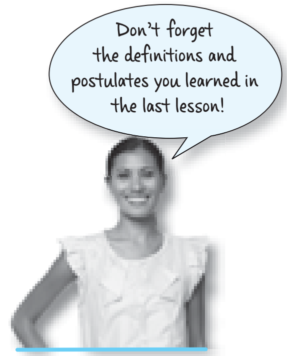
$\overline{AB} \cong \overline{CD}$ Given	$\overline{AC} \cong \overline{BD}$ Definition of congruent segments
$m\overline{AB} = m\overline{CD}$ Definition of congruent segments	$m\overline{BC} = m\overline{BC}$ Reflexive Property
$m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$ Addition Property of Equality	$m\overline{AB} + m\overline{BC} = m\overline{AC}$ Segment Addition Postulate
$m\overline{AC} = m\overline{BD}$ Substitution Property	$m\overline{BC} + m\overline{CD} = m\overline{BD}$ Segment Addition Postulate

2

c. Complete the flow chart proof of the conditional statement in Question 1 by assembling your cutout steps in order. Use arrows to show the order of the flow chart proof.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$



$\overline{AB} \cong \overline{CD}$
Given

$m\overline{AB} = m\overline{CD}$
Definition of congruent segments

$m\overline{BC} = m\overline{BC}$
Reflexive Property

$m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$
Addition Property of Equality

$m\overline{AB} + m\overline{BC} = m\overline{AC}$
Segment Addition Postulate

$m\overline{BC} + m\overline{CD} = m\overline{BD}$
Segment Addition Postulate

$m\overline{AC} = m\overline{BD}$
Substitution Property

$\overline{AC} \cong \overline{BD}$
Definition of congruent segments

Grouping

Have students complete Question 1 part (d) with a partner. Then have students share their responses as a class.

2

Guiding Questions for Share Phase, Question 1 part (d)

- What was required to take the flow chart and rewrite it as a two-column proof?
- Did the order of the steps change? If so, how?
- Did the first and last step remain the same?

Grouping

Have students complete Question 1 part (e) with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1 part (e)

- What was required to take the flow chart or two-column proof and rewrite it as a paragraph proof?
- Did the order of the steps change? If so, how?
- Did the first and last step remain the same?

A **two-column proof** is a proof in which the steps are written in the left column and the corresponding reasons are written in the right column. Each step and corresponding reason are numbered.

d. Create a two-column proof of the conditional statement in Question 1. Each box of the flow chart proof in Question 1, part (c) should appear as a row in the two-column proof.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $m\overline{AB} = m\overline{CD}$	2. Definition of congruent segments
3. $m\overline{BC} = m\overline{BC}$	3. Reflexive Property
4. $m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$	4. Addition Property of Equality
5. $m\overline{AB} + m\overline{BC} = m\overline{AC}$	5. Segment Addition Postulate
6. $m\overline{CD} + m\overline{BC} = m\overline{BD}$	6. Segment Addition Postulate
7. $m\overline{AC} = m\overline{BD}$	7. Substitution Property
8. $\overline{AC} \cong \overline{BD}$	8. Definition of congruent segments

A **paragraph proof** is a proof in which the steps and corresponding reasons are written in complete sentences.

e. Write a paragraph proof of the conditional statement in Question 1. Each row of the two-column proof in Question 1, part (d) should appear as a sentence in the paragraph proof.

If $AB \cong CD$, then $m\overline{AB} = m\overline{CD}$ by the definition of congruent segments. Add the same line segment measure, $m\overline{BC}$, to both segments. By the Addition Property of Equality, $m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$. By segment addition, the segments can be renamed such that $m\overline{AB} + m\overline{BC} = m\overline{AC}$ and $m\overline{BC} + m\overline{CD} = m\overline{BD}$. Then $m\overline{AC} = m\overline{BD}$ because if you add the same segment (BC) to two segments of equal measure, the resulting segments remain equal in measure. Therefore, $\overline{AC} \cong \overline{BD}$.

Grouping

Have students complete Question 1 part (f) with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 2 part (f)

- To compare the lengths of the segments, did you place segments *AB* and *CD* on the same starter line?
- To compare the lengths of the segments, did you place segments *AC* and *BD* on the same starter line?
- In this construction proof, how many times did you have to duplicate a segment?
- How did you begin the construction proof?
- Did your partner complete the construction proof using the same steps? If not, what was different?

Problem 3

Students will prove the Right Angle Congruence Theorem using a flow chart model.

Grouping

Have students complete the problem with a partner. Then share the responses as a class.

Guiding Questions for Share Phase

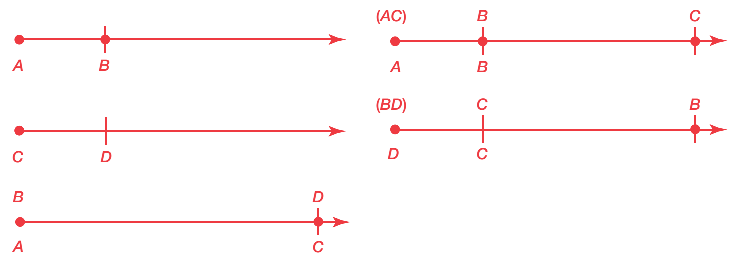
- Can Karl and Roy both be correct? How is this possible?

A **construction proof** is a proof that results from creating an object with specific properties using only a compass and a straightedge.

f. Create a proof by construction of the conditional statement in Question 2.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$



PROBLEM 3 Proof of the Right Angle Congruence Theorem

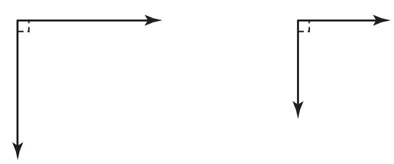


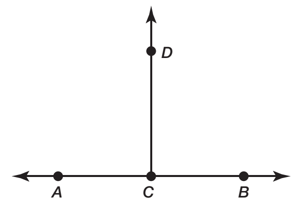
Fig. 1 Fig. 2

1. Karl insists the angle in Figure 1 is larger than the angle in Figure 2. Roy disagrees and insists both angles are the same size. Who is correct? What is your reasoning?

   Karl is correct if he is referring to the dimensions of the actual drawing of the angles when the rays representing the sides are not extended. Roy is correct if he is referring to the measures of the angles because both angle measures are 90°.

- What is the definition of a right angle?
- Which property equates two angles that are the same measure?
- What is the definition of congruent angles?

The **Right Angle Congruence Theorem** states: "All right angles are congruent."



Given: $\angle ACD$ and $\angle BCD$ are right angles.

Prove: $\angle ACD \cong \angle BCD$

Complete the flow chart of the Right Angle Congruence Theorem by writing the statement for each reason in the boxes provided.

$\angle ACD$ is a right angle
Given

$\angle BCD$ is a right angle
Given

$m\angle ACD = 90°$
Definition of right angles

$m\angle BCD = 90°$
Definition of right angles

$m\angle ACD = m\angle BCD$
Transitive Property of Equality

$\angle ACD \cong \angle BCD$
Definition of congruent angles

Problem 4

Students will prove the Congruent Supplement Theorem using both a flow chart model and a two-column model.

Grouping

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4

- Which statement and reason should be in the first position in the flow chart? Why?
- Which statement and reason should be in the last position in the flow chart? Why?
- How do you know which statement and reason should follow the previous statement and reason?
- Is there more than one correct way to order these statements and reasons? Explain.
- What is an example of one statement that must follow another statement?
- What is an example of one statement that must come before another statement?
- What is the definition of supplementary angles?
- What is the definition of congruent angles?
- What are you substituting in this proof?

PROBLEM 4 Proofs of the Congruent Supplement Theorem

The **Congruent Supplement Theorem** states: "If two angles are supplements of the same angle or of congruent angles, then the angles are congruent."

1. Use the diagram to write the "Given" statements for the Congruent Supplement Theorem. The "Prove" statement is provided.

Given: $\angle 1$ is supplementary to $\angle 2$

Given: $\angle 3$ is supplementary to $\angle 4$

Given: $\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$

2. Cut out the steps of the flow chart proof.

$\angle 1$ is supplementary to $\angle 2$
Given

$\angle 3$ is supplementary to $\angle 4$
Given

$m\angle 1 + m\angle 2 = 180°$
Definition of supplementary angles

$\angle 2 \cong \angle 4$
Given

$m\angle 3 + m\angle 4 = 180°$
Definition of supplementary angles

$m\angle 1 = m\angle 3$
Subtraction Property of Equality

$m\angle 2 = m\angle 4$
Definition of congruent angles

$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$
Substitution Property

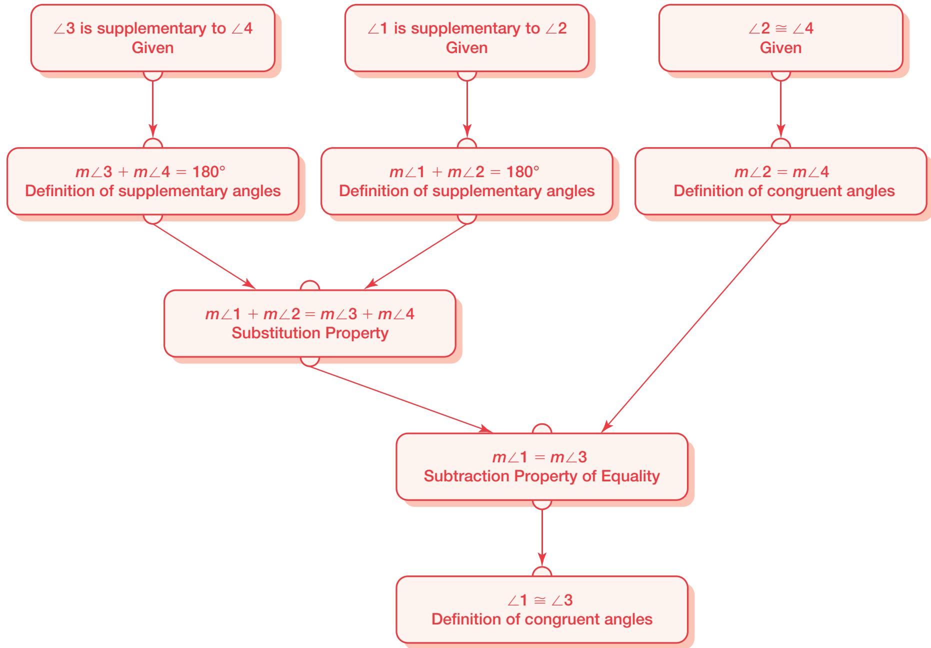
$\angle 1 \cong \angle 3$
Definition of congruent angles

- Which property allows you to subtract angles of equal measure from both sides of the equation?
- What was required to take the flow chart or two-column proof and rewrite it as a paragraph proof?
- Did the order of the steps change? If so, how?
- Did the first and last step remain the same?

2

2

3. Complete the flow chart proof of the Congruent Supplements Theorem by assembling your cutout steps in order. Use arrows to show the order of the flow chart proof.



2

Problem 5

Students will prove the Congruent Complement Theorem using both a flow chart model and a two-column model.

2

Grouping

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4

- What is the hypothesis in the Congruent Complement Theorem?
- What is the conclusion in the Congruent Complement Theorem?
- How many given statements are involved in proving this theorem?
- How many prove statements are involved in proving this theorem?
- Do you suppose there is ever more than one prove statement in a proof?
- How many steps are in your flow chart proof?
- How many steps are in your two-column proof?
- Which statement and reason should be in the first position in the flow chart? Why?
- Which statement and reason should be in the last position in the flow chart? Why?

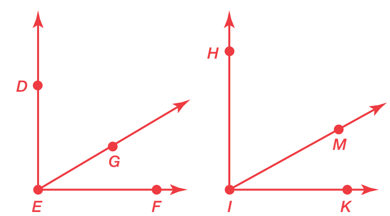
4. Create a two-column proof of the Congruent Supplement Theorem. Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

Statements	Reasons
1. $\angle 1$ is supplementary to $\angle 2$	1. Given
2. $\angle 3$ is supplementary to $\angle 4$	2. Given
3. $\angle 2 \cong \angle 4$	3. Given
4. $m\angle 2 = m\angle 4$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 2 = 180°$	5. Definition of supplementary angles
6. $m\angle 3 + m\angle 4 = 180°$	6. Definition of supplementary angles
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	7. Substitution Property
8. $m\angle 1 = m\angle 3$	8. Subtraction Property of Equality
9. $\angle 1 \cong \angle 3$	9. Definition of congruent angles

PROBLEM 5 Proofs of the Congruent Complement Theorem

The **Congruent Complement Theorem** states: "If two angles are complements of the same angle or of congruent angles, then they are congruent."

1. Draw and label a diagram illustrating this theorem.



2. Use your diagram to write the "Given" and "Prove" statements for the Congruent Complement Theorem.

Given: $\angle DEG$ is a complement of $\angle GEF$

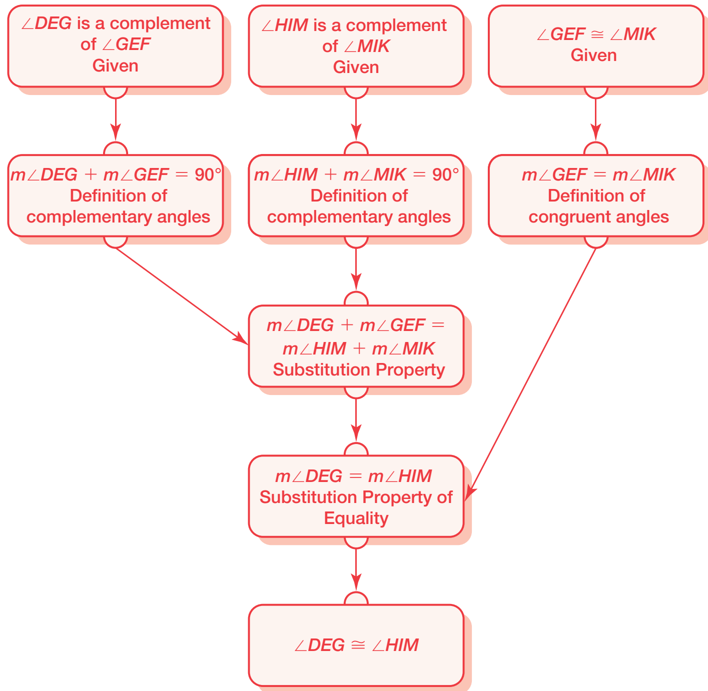
Given: $\angle HIM$ is a complement of $\angle MIK$

Given: $\angle GEF \cong \angle MIK$

Prove: $\angle DEG \cong \angle HIM$

- How do you know which statement and reason should follow the previous statement and reason?
- Is there more than one correct way to order these statements and reasons? Explain.
- What is an example of one statement that must follow another statement?
- What is an example of one statement that must come before another statement?

3. Create a flow chart proof of the Congruent Complement Theorem.



Problem 6

Students will prove the Vertical Angle Theorem using a flow chart proof and a two-column proof.

Grouping

2

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4

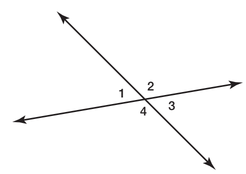
- What is the hypothesis in the Vertical Angle Theorem?
- What is the conclusion in the Vertical Angle Theorem?
- How many given statements are involved in proving this theorem?
- How many prove statements are involved in proving this theorem?
- How many steps are in your flow chart proof?
- How many steps are in your two-column proof?
- Which statement and reason should be in the first position in the flow chart? Why?
- Which statement and reason should be in the last position in the flow chart? Why?
- How do you know which statement and reason should follow the previous statement and reason?
- Is there more than one correct way to order these statements and reasons? Explain.

4. Create a two-column proof of the Congruent Complement Theorem. Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

Statements	Reasons
1. $\angle DEG$ is a complement of $\angle GEF$	1. Given
2. $\angle HIM$ is a complement of $\angle MIK$	2. Given
3. $\angle GEF \cong \angle MIK$	3. Given
4. $m\angle DEG + m\angle GEF = 90°$	4. Definition of complementary angles
5. $m\angle HIM + m\angle MIK = 90°$	5. Definition of complementary angles
6. $m\angle GEF = m\angle MIK$	6. Definition of congruent angles
7. $m\angle DEG + m\angle GEF = m\angle HIM + m\angle MIK$	7. Substitution Property
8. $m\angle DEG = m\angle HIM$	8. Subtraction Property of Equality
9. $\angle DEG \cong \angle HIM$	9. Definition of congruent angles

PROBLEM 6 Proofs of the Vertical Angle Theorem

1. The **Vertical Angle Theorem** states: "Vertical angles are congruent."



2. Use the diagram to write the "Prove" statements for the Vertical Angle Theorem. The "Given" statements are provided.

Given: $\angle 1$ and $\angle 2$ are a linear pair

Given: $\angle 2$ and $\angle 3$ are a linear pair

Given: $\angle 3$ and $\angle 4$ are a linear pair

Given: $\angle 4$ and $\angle 1$ are a linear pair

Prove: $\angle 1 \cong \angle 3$

Prove: $\angle 2 \cong \angle 4$

- What is an example of one statement that must follow another statement?
- What is an example of one statement that must come before another statement?

3. Create a flow chart proof of the first "Prove" statement of the Vertical Angle Theorem.

$\angle 1$ and $\angle 2$ are a linear pair
Given

$\angle 1$ and $\angle 2$ are supplementary angles
Linear Pair Postulate

$m\angle 1 + m\angle 2 = 180°$
Definition of supplementary angles

$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$
Substitution Property

$\angle 2$ and $\angle 3$ are a linear pair
Given

$\angle 2$ and $\angle 3$ are supplementary angles
Linear Pair Postulate

$m\angle 2 + m\angle 3 = 180°$
Definition of supplementary angles

$m\angle 2 = m\angle 2$
Reflexive Property

$m\angle 1 = m\angle 3$
Subtraction Property of Equality

$\angle 1 \cong \angle 3$
Definition of congruent angles

2

4. Create a two-column proof of the second "Prove" statement of the Vertical Angle Theorem.

Given: $\angle 4$ and $\angle 1$ are a linear pair

Given: $\angle 1$ and $\angle 2$ are a linear pair

Prove: $\angle 2 \cong \angle 4$

Statements	Reasons
1. $\angle 4$ and $\angle 1$ are a linear pair	1. Given
2. $\angle 1$ and $\angle 2$ are a linear pair	2. Given
3. $\angle 4$ and $\angle 1$ are supplementary angles	3. Linear Pair Postulate
4. $\angle 1$ and $\angle 2$ are supplementary angles	4. Linear Pair Postulate
5. $m\angle 4 + m\angle 1 = 180°$	5. Definition of supplementary angles
6. $m\angle 2 + m\angle 1 = 180°$	6. Definition of supplementary angles
7. $m\angle 4 + m\angle 1 = m\angle 2 + m\angle 1$	7. Substitution
8. $m\angle 1 = m\angle 1$	8. Reflexive Property
9. $m\angle 4 = m\angle 2$	9. Subtraction Property of Equality
10. $\angle 4 \cong \angle 2$	10. Definition of congruent angles

Problem 7

Students will complete a proof using the Angle Addition Postulate using a proof model of their choice.

Grouping

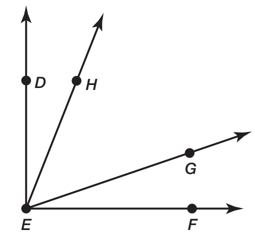
Have students complete the problem with a partner. Then share the responses as a class.

Guiding Questions for Share Phase

- Which method of proof did you choose? Why?
- Which method of proof is easiest to use? Why?
- Which method of proof is the most difficult? Why?
- Can all four methods of proof be used to prove this conditional statement? Why or why not?
- What definitions were used to prove this conditional statement?
- What properties were used to prove this conditional statement?

PROBLEM 7 Proofs Using the Angle Addition Postulate

Given: $\angle DEG \cong \angle HEF$
Prove: $\angle DEH \cong \angle GEF$



1. Prove the conditional statement using any method you choose.

It is given that angles *DEG* and *HEF* are congruent, so by the definition of congruent angles we can say the measures of angles *DEG* and *HEF* are equal. By the angle addition postulate we know that the measure of angle *DEH* plus the measure of angle *HEG* must be equal to the measure of angle *DEG*. We also know by the angle addition postulate that the measure of angles *GEF* plus the measure of angle *HEG* must be equal to the measure of angle *HEF*. Angle *HEG* is equal to itself because of the Reflexive Property. By substitution, we know the measures of angle *GEF* plus *HEG* must be equal to the measure of angle *DEH* plus the measure of angle *HEG*. Using the subtraction property of equality we can conclude the measure of angle *GEF* must equal the measure of angle *DEH*. If the measures of two angles are equal, then by the definition of congruent angles, they must also be congruent, so angle *DEH* is congruent to angle *GEF*.

Statements	Reasons
1. $\angle DEG \cong \angle HEF$	1. Given
2. $m\angle DEG = m\angle HEF$	2. Definition of congruent angles
3. $m\angle DEH + m\angle HEG = m\angle DEG$	3. Angle Addition Postulate
4. $m\angle GEF + m\angle HEG = m\angle HEF$	4. Angle Addition Postulate
5. $m\angle HEG = m\angle HEG$	5. Reflexive Property
6. $m\angle DEH + m\angle HEG = m\angle GEF + m\angle HEG$	6. Substitution Property
7. $m\angle DEH = m\angle GEF$	7. Subtraction Property of Equality
8. $\angle DEH \cong \angle GEF$	8. Definition of congruent angles

2

Talk the Talk

Students will have the opportunity to reflect on their experiences of using four different proof models. Also, the new theorems introduced in this lesson are summarized.

2

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Talk the Talk

1. List the advantages and disadvantages of each form of proof.
   a. flow chart proof
      Answers will vary.
   b. two-column proof
      Answers will vary.
   c. paragraph proof
      Answers will vary.
   d. construction proof
      Answers will vary.

2. Which form of proof do you prefer? Explain.
   Answers will vary.

Once a theorem has been proven, it can be used as a reason in another proof. Using theorems that have already been proven allows you to write shorter proofs.

In this chapter, you proved these theorems:

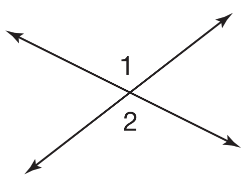
- The Right Angle Congruence Theorem: All right angles are congruent.
- The Congruent Supplement Theorem: Supplements of congruent angles, or of the same angle, are congruent.
- The Congruent Complement Theorem: Complements of congruent angles, or of the same angle, are congruent.
- The Vertical Angle Theorem: Vertical angles are congruent.

A list of theorems that you prove throughout this course will be an excellent resource as you continue to make new conjectures and expand your system of geometry.

Be prepared to share your solutions and methods.

Check for Students' Understanding

Match each scenario with the correct property or theorem.

1. (E)	$\angle H \cong \angle K$ $\angle K \cong \angle M$ Therefore $\angle H \cong \angle M$	A. Substitution
2. (F)	$m\overline{MN} = m\overline{OP}$ $m\overline{MN} + m\overline{RS} = m\overline{OP} + m\overline{RS}$	B. Congruent Complement Theorem
3. (A)	$m\angle T = 34°$ $m\angle W = 34°$ Therefore $m\angle T = m\angle W$	C. Vertical Angle Theorem
4. (G)	$m\angle A + m\angle B = 180°$ $m\angle A + m\angle C = 180°$ Therefore $\angle B \cong \angle C$	D. Identity Property
5. (H)	$\overline{WX} \cong \overline{WX}$	E. Transitive Property
6. (D)	$m\angle V = m\angle V$	F. Addition Property of Equality
7. (C)	$\angle 1 \cong \angle 2$ 	G. Congruent Supplement Theorem
8. (B)	$m\angle A + m\angle B = 90°$ $m\angle A + m\angle C = 90°$ Therefore $\angle B \cong \angle C$	H. Reflexive Property

2

2.4

What's Your Proof?

Angle Postulates and Theorems

LEARNING GOALS

In this lesson, you will:

- Use the Corresponding Angle Postulate.
- Prove the Alternate Interior Angle Theorem.
- Prove the Alternate Exterior Angle Theorem.
- Prove the Same-Side Interior Angle Theorem.
- Prove the Same-Side Exterior Angle Theorem.

KEY TERMS

- Corresponding Angle Postulate
- conjecture
- Alternate Interior Angle Theorem
- Alternate Exterior Angle Theorem
- Same-Side Interior Angle Theorem
- Same-Side Exterior Angle Theorem

ESSENTIAL IDEAS

- The Corresponding Angle Postulate states: If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.
- The Alternate Interior Angle Theorem (AIA) states: If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.
- The Alternate Exterior Angle Theorem (AEA) states: If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.
- The Same-Side Interior Angle Theorem (SSIA) states: If two parallel lines are cut by a transversal, then the pairs of same-side interior angles are supplementary.
- The Same-Side Exterior Angle Theorem (SSEA) states: If two parallel lines are cut by a transversal, then the pairs of same-side exterior angles are supplementary.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

G-CO Congruence

Prove geometric theorems

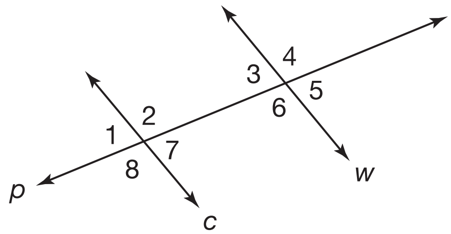
9. Prove theorems about lines and angles.

Overview

The Corresponding Angle Postulate is stated and using this postulate, students make several conjectures about the relationship of other pairs of angles formed by a transversal intersecting two parallel lines. Those conjectures are written as theorems and proven using different methods of proof.

Warm Up

Use the numbered angles in the diagram to answer the questions.



1. Identify all pairs of corresponding angles.

   The pairs of corresponding angles are ∠1 and ∠3, ∠2 and ∠4, ∠8 and ∠6, and ∠7 and ∠5.

2. Which pairs of angles do you *know* to be congruent? How do you know?

   The pairs of congruent angles are ∠1 and ∠7, ∠2 and ∠8, ∠3 and ∠5, and ∠4 and ∠6. Each pair of angles is congruent because they are vertical angles.

3. Suppose you are given $c \parallel w$, which pairs of angles do you *know* to be congruent? How do you know?

   If $c \parallel w$, then ∠1, ∠7, ∠3, and ∠5 are congruent because vertical angles and alternate interior angles are congruent. For the same reason, ∠2, ∠8, ∠6, and ∠4 are congruent.

2

2

2.4

What's Your Proof?

Angle Postulates and Theorems

2

LEARNING GOALS

In this lesson, you will:

- Use the Corresponding Angle Postulate.
- Prove the Alternate Interior Angle Theorem.
- Prove the Alternate Exterior Angle Theorem.
- Prove the Same-Side Interior Angle Theorem.
- Prove the Same-Side Exterior Angle Theorem.

KEY TERMS

- Corresponding Angle Postulate
- conjecture
- Alternate Interior Angle Theorem
- Alternate Exterior Angle Theorem
- Same-Side Interior Angle Theorem
- Same-Side Exterior Angle Theorem

You are constantly bombarded with information through magazines, newspaper, television, and the Internet. However, not all "facts" that you read about are actually true! If you want to be an educated consumer of information, you should always be looking for the argument, or proof, to back up a statement. If you can't find such information then you should be skeptical.

Sometimes you need to carefully examine the evidence. For example, say someone claims that 4 out of 5 dentists recommend a certain toothpaste. Sounds pretty impressive, right? However, what if you learned that only five dentists were asked their opinions? You might start to question the claim. What if you also learned that the dentists were paid by the toothpaste company for their opinions? As you can see, sometimes the "truth" isn't always what it appears to be.

2

Problem 1

The Corresponding Angle Postulate is stated. Students use the postulate to form conjectures regarding pairs of alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles. These conjectures will then be proven and renamed as theorems.

Grouping

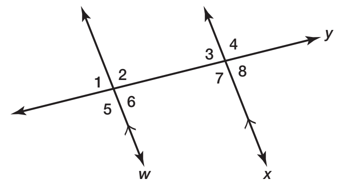
Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3

- Is the Corresponding Angle Postulate a conditional statement?
- What is the hypothesis of the Corresponding Angle Postulate?
- What is the conclusion of the Corresponding Angle Postulate?
- How would you describe the location of a pair of corresponding angles?
- How many pairs of congruent corresponding angles are formed when two parallel lines are intersected by a transversal?
- How would you describe the location of a pair of alternate interior angles?

PROBLEM 1 The Corresponding Angle Postulate

The **Corresponding Angle Postulate** states: "If two parallel lines are intersected by a transversal, then corresponding angles are congruent."



1. Name all pairs of angles that are congruent using the Corresponding Angle Postulate.

   I can use the Corresponding Angle Postulate to state $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$, $\angle 5 \cong \angle 7$, and $\angle 6 \cong \angle 8$.

A **conjecture** is a hypothesis that something is true. The hypothesis can later be proved or disproved.

2. Write a conjecture about each pair of angles formed by parallel lines cut by a transversal. Explain how you made each conjecture.

   a. alternate interior angles.

   The conjecture is that if two parallel lines are intersected by a transversal, then alternate interior angles are congruent.

   In the figure, $\angle 2$ and $\angle 7$ are alternate interior angles. By the Corresponding Angle Postulate, $\angle 2 \cong \angle 4$. By the Vertical Angle Theorem, $\angle 4 \cong \angle 7$. Therefore, by the Transitive Property, $\angle 2 \cong \angle 7$.

   Similarly, $\angle 3$ and $\angle 6$ are alternate interior angles. By the Corresponding Angle Postulate, $\angle 3 \cong \angle 1$. By the Vertical Angle Theorem, $\angle 1 \cong \angle 6$. Therefore, by the Transitive Property, $\angle 3 \cong \angle 6$.

   b. alternate exterior angles.

   The conjecture is that if two parallel lines are intersected by a transversal, then alternate exterior angles are congruent.

   In the figure, $\angle 4$ and $\angle 5$ are alternate exterior angles. By the Corresponding Angle Postulate, $\angle 4 \cong \angle 2$. By the Vertical Angle Theorem, $\angle 2 \cong \angle 5$. Therefore, by the Transitive Property, $\angle 4 \cong \angle 5$.

   Similarly, $\angle 1$ and $\angle 8$ are alternate exterior angles. By the Corresponding Angle Postulate, $\angle 1 \cong \angle 3$. By the Vertical Angle Theorem, $\angle 3 \cong \angle 8$. Therefore, by the Transitive Property, $\angle 1 \cong \angle 8$.

- How many pairs of congruent alternate interior angles are formed when two parallel lines are intersected by a transversal?
- How would you describe the location of a pair of alternate exterior angles?
- How many pairs of congruent alternate exterior angles are formed when two parallel lines are intersected by a transversal?
- How would you describe the location of a pair of same-side interior angles?

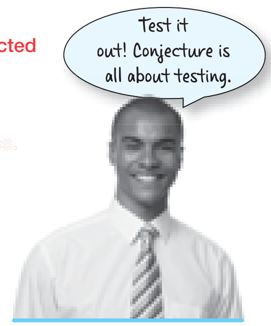
- How many pairs of same-side interior angles are formed when two parallel lines are intersected by a transversal?
- How would you describe the location of a pair of same-side exterior angles?
- How many pairs of same-side exterior angles are formed when two parallel lines are intersected by a transversal?

Note

Emphasize the difference between a conjecture and a theorem.

c. same-side interior angles

The conjecture is that if two parallel lines are intersected by a transversal, then same-side interior angles are supplementary.



In the figure, $\angle 2$ and $\angle 3$ are same-side interior angles. By the Corresponding Angle Postulate, $\angle 2 \cong \angle 4$. By the Linear Pair Postulate, $\angle 3$ and $\angle 4$ are supplementary. Therefore, by the Substitution Property, $\angle 2$ and $\angle 3$ are supplementary.

Similarly, $\angle 6$ and $\angle 7$ are same-side interior angles. By the Corresponding Angle Postulate, $\angle 6 \cong \angle 8$. By the Linear Pair Postulate, $\angle 7$ and $\angle 8$ are supplementary. Therefore, by the Substitution Property, $\angle 6$ and $\angle 7$ are supplementary.

d. same-side exterior angles

The conjecture is that if two parallel lines are intersected by a transversal, then same-side exterior angles are supplementary.

In the figure, $\angle 1$ and $\angle 4$ are same-side exterior angles. By the Corresponding Angle Postulate, $\angle 1 \cong \angle 3$. By the Linear Pair Postulate, $\angle 3$ and $\angle 4$ are supplementary. Therefore, by the Substitution Property, $\angle 1$ and $\angle 4$ are supplementary.

Similarly, $\angle 5$ and $\angle 8$ are same-side exterior angles. By the Corresponding Angle Postulate, $\angle 5 \cong \angle 7$. By the Linear Pair Postulate, $\angle 7$ and $\angle 8$ are supplementary. Therefore, by the Substitution Property, $\angle 5$ and $\angle 8$ are supplementary.

3. Did you use inductive or deductive reasoning to make each conjecture?

I used inductive reasoning because I used postulates and properties to make these conjectures.

Problem 2

Students use a variety of methods of proof to prove the Alternate Interior Angle Theorem, the Alternate Exterior Angle Theorem, the Same-Side Interior Angle Theorem, and the Same-Side Exterior Angle Theorem.

2

Grouping

Have students complete Question 1 with a partner. Then have students share their responses as a class.

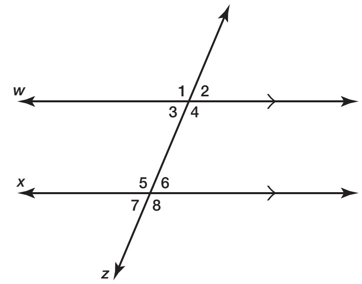
Guiding Questions for Share Phase, Question 1

- How is the Corresponding Angle Postulate used to help prove the Alternate Interior Angle Theorem?
- Which pair of alternate interior angles did you prove congruent?
- Can you prove a different pair of alternate interior angles congruent using the same strategy?
- Do you need to prove both pairs of alternate interior angles congruent to prove this theorem?
- Why is it necessary to prove a second pair of alternate interior angles congruent to prove this theorem?
- Did you use the flow chart proof to create the two-column proof? Was it helpful?
- How is the flow chart proof similar to the two-column proof?
- How is the flow chart proof different than the two-column proof?
- Which type of proof is easier to create? Why?

PROBLEM 2 Conjecture or Theorem?

If you can prove that a conjecture is true, then it becomes a theorem.

1. The Alternate Interior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then alternate interior angles are congruent."



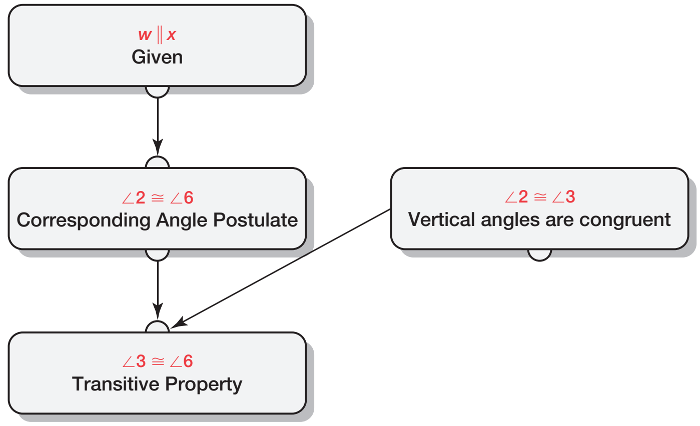
a. Use the diagram to write the "Given" and "Prove" statements for the Alternate Interior Angle Conjecture.

Given: $w \parallel x$, z is a transversal

Prove: $\angle 3 \cong \angle 6$

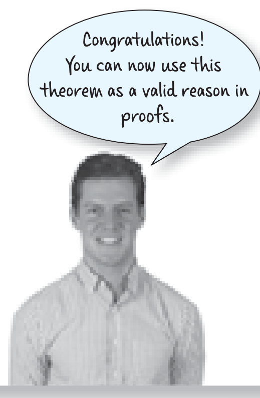
2

b. Complete the flow chart proof of the Alternate Interior Angle Conjecture by writing the reason for each statement in the boxes provided.



c. Create a two-column proof of the Alternate Interior Angle Theorem.

Statements	Reasons
1. $w \parallel x$	1. Given
2. $\angle 1 \cong \angle 5$	2. Corresponding Angle Postulate
3. $\angle 1 \cong \angle 4$	3. Vertical angles are congruent
4. $\angle 4 \cong \angle 5$	4. Transitive Property



You have just proven the Alternate Interior Angle Conjecture. It is now known as the **Alternate Interior Angle Theorem**.

Grouping

Have students complete Question 2 with a partner. Then have students share their responses as a class.

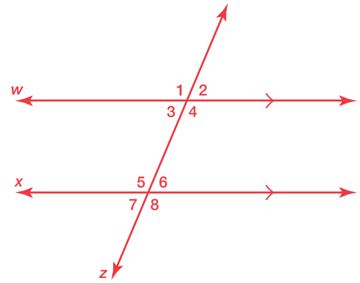
Guiding Questions for Share Phase, Question 2

2

- How is the Corresponding Angle Postulate used to help prove the Alternate Exterior Angle Theorem?
- Which pair of alternate exterior angles did you prove congruent?
- Can you prove a different pair of alternate exterior angles congruent using the same strategy?
- Do you need to prove both pairs of alternate exterior angles congruent to prove this theorem?

2. The Alternate Exterior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then alternate exterior angles are congruent."

a. Draw and label a diagram illustrating the Alternate Exterior Angle Conjecture. Then, write the given and prove statements.

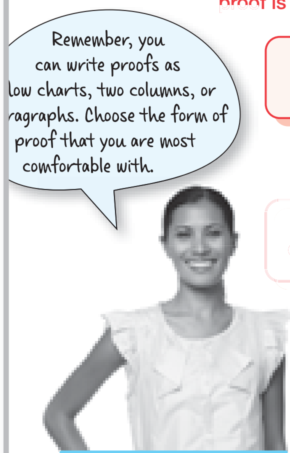


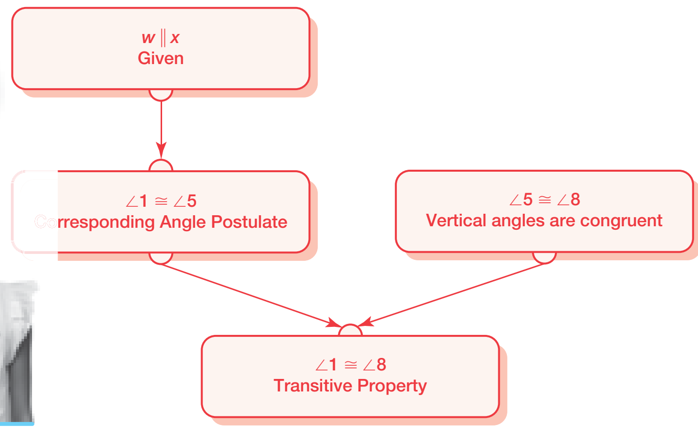
Given: $w \parallel x$, z is a transversal

Prove: $\angle 1 \cong \angle 8$ or $\angle 2 \cong \angle 7$

b. Prove the Alternate Exterior Angle Conjecture.

Students may use a flow chart, two-column, or paragraph proof. The flow chart proof is provided. Students only need to use one method.





You have just proven the Alternate Exterior Angle Conjecture. It is now known as the **Alternate Exterior Angle Theorem**.

Grouping

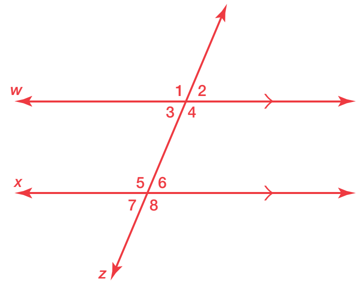
Have students complete Question 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 3

- How is the Corresponding Angle Postulate used to help prove the Same-Side Interior Angle Theorem?
- Which pair of Same-Side interior angles did you prove supplementary?
- Can you prove a different pair of Same-Side interior angles supplementary using the same strategy?
- Do you need to prove both pairs of Same-Side interior angles supplementary to prove this theorem?

3. The Same-Side Interior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are supplementary."

a. Draw and label a diagram illustrating the Same-Side Interior Angle Conjecture. Then, write the given and prove statements.



Given: $w \parallel x$, z is a transversal

Prove: $\angle 3$ and $\angle 5$ are supplementary angles or $\angle 4$ and $\angle 6$ are supplementary angles

b. Prove the Same-Side Interior Angle Conjecture.

Students may use a flow chart, two-column, or paragraph proof. The two-column proof is provided. Students only need to use one method.

Statements	Reasons
1. $w \parallel x$	1. Given
2. $\angle 1$ and $\angle 3$ are a linear pair	2. Definition of linear pair
3. $\angle 1$ and $\angle 3$ are supplementary	3. Linear Pair Postulate
4. $m\angle 1 + m\angle 3 = 180°$	4. Definition of supplementary angles
5. $\angle 1 \cong \angle 5$	5. Corresponding Angle Postulate
6. $m\angle 1 = m\angle 5$	6. Definition of congruent angles
7. $m\angle 3 + m\angle 5 = 180°$	7. Substitution Property
8. $\angle 1$ and $\angle 3$ are supplementary angles	8. Definition of supplementary angles

You have just proven the Same-Side Interior Angle Conjecture. It is now known as the **Same-Side Interior Angle Theorem**.

Grouping

Have students complete Question 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 4

- How is the Corresponding Angle Postulate used to help prove the Same-Side Exterior Angle Theorem?
- Which pair of Same-Side exterior angles did you prove supplementary?
- Can you prove a different pair of Same-Side exterior angles supplementary using the same strategy?
- Do you need to prove both pairs of Same-Side exterior angles supplementary to prove this theorem?

Note:

Suggest that students use abbreviations for the theorem names such as;

Alternate Interior Angle Theorem: AIA Theorem

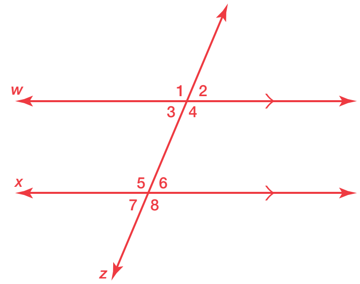
Alternate Exterior Angle Theorem: AEA Theorem

Same-Side Interior Angle Theorem: SSIA Theorem

Same-Side Exterior Angle Theorem: SSEA Theorem

4. The Same-Side Exterior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then exterior angles on the same side of the transversal are supplementary."

a. Draw and label a diagram illustrating the Same-Side Exterior Angle Conjecture. Then, write the given and prove statements.



Given: $w \parallel x$, z is a transversal

Prove: $\angle 1$ and $\angle 7$ are supplementary angles or $\angle 2$ and $\angle 8$ are supplementary angles

b. Prove the Same-Side Exterior Angle Conjecture.

Students may use a flow chart, two-column, or paragraph proof. The flow chart proof is provided. Students only need to use one method.

$w \parallel x$
Given

$\angle 3 \cong \angle 7$
Corresponding Angle Postulate

$m\angle 3 \cong m\angle 7$
Definition of congruent angles

$\angle 1$ and $\angle 3$ are a linear pair
Definition of linear pair

$\angle 1$ and $\angle 3$ are supplementary
Linear Pair Postulate

$m\angle 1 + m\angle 3 = 180°$
Definition of supplementary angles

$m\angle 1 + m\angle 7 = 180°$
Substitution Property

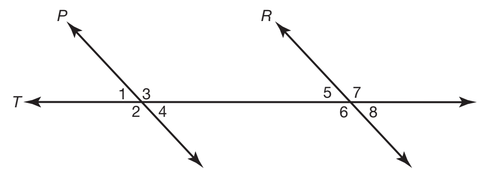
$\angle 1$ and $\angle 7$ are supplementary angles
Definition of supplementary angles

You have just proven the Same-Side Exterior Angle Conjecture. It is now known as the **Same-Side Exterior Angle Theorem**.

Talk the Talk

The theorems in this lesson are summarized in this activity. Encourage students to write these theorems in their notebooks. Suggest they sketch a small diagram beside each theorem highlighting at least one pair of angles depicted in each theorem.

Talk the Talk



Given: $m\angle 4 = 37°$

1. Gail determined the measures of all eight angles labeled using the given information. Stu said she could only calculate the measure of four angles with certainty. Who is correct? Explain your reasoning.

   Stu is correct. Gail cannot determine the measures of angles 5, 6, 7, and 8 unless it is stated that line *P* is parallel to line *R*.

Guiding Questions for Discuss Phase

- How many pairs of corresponding angles exist when two parallel lines are intersected by a transversal?
- How many pairs of alternate interior angles exist when two parallel lines are intersected by a transversal?
- How many pairs of alternate exterior angles exist when two parallel lines are intersected by a transversal?
- How many pairs of same-side interior angles exist when two parallel lines are intersected by a transversal?
- How many pairs of same-side exterior angles exist when two parallel lines are intersected by a transversal?

2

If two parallel lines are intersected by a transversal, then:

- corresponding angles are congruent.
- alternate interior angles are congruent.
- alternate exterior angles are congruent.
- same-side interior angles are supplementary.
- same-side exterior angles are supplementary.

Each of these relationships is represented by a postulate or theorem.

- **Corresponding Angle Postulate**: If two parallel lines are intersected by a transversal, then corresponding angles are congruent.
- **Alternate Interior Angle Theorem**: If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.
- **Alternate Exterior Angle Theorem**: If two parallel lines are intersected by a transversal, then alternate exterior angles are congruent.
- **Same-Side Interior Angle Theorem**: If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are supplementary.
- **Same-Side Exterior Angle Theorem**: If two parallel lines are intersected by a transversal, then exterior angles on the same side of the transversal are supplementary.

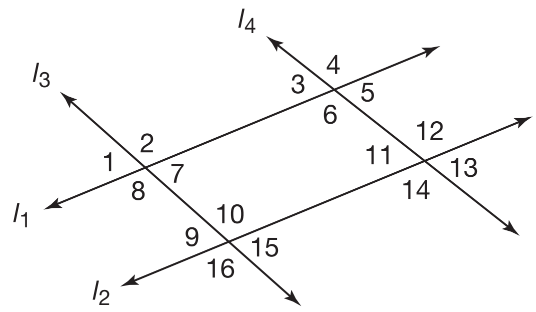
2. Did you use inductive or deductive reasoning to prove each theorem?

   I used deductive reasoning because I used general rules, definitions, and postulates about geometric relationships to make conclusions.

Be prepared to share your solutions and methods.

Check for Students' Understanding

Given: $l_1 \parallel l_2$ and $l_3 \parallel l_4$



Using the diagram, provide the appropriate theorem or postulate that supports each statement.

Statement	Theorem or Postulate
1. $\angle 3 \cong \angle 13$	Alternate Exterior Angle Theorem
2. $\angle 9 \cong \angle 11$	Corresponding Angle Postulate
3. $\angle 10 \cong \angle 14$	Alternated Interior Angle Theorem
4. $\angle 9$ and $\angle 10$ are supplementary angles.	Linear Pair Postulate
5. $\angle 6$ and $\angle 11$ are supplementary angles.	Same Side Interior Angles Theorem
6. $\angle 12 \cong \angle 14$	Vertical Angle Theorem

2

2

2.5

A Reversed Condition

Parallel Line Converse Theorems

LEARNING GOALS

In this lesson, you will:

- Write and prove parallel line converse conjectures.

KEY TERMS

- converse
- Corresponding Angle Converse Postulate
- Alternate Interior Angle Converse Theorem
- Alternate Exterior Angle Converse Theorem
- Same-Side Interior Angle Converse Theorem
- Same-Side Exterior Angle Converse Theorem

ESSENTIAL IDEAS

- The Corresponding Angle Converse Postulate states: If two lines cut by a transversal form congruent corresponding angles, then the lines are parallel.
- The Alternate Interior Angle Converse Theorem states: If two lines cut by a transversal form congruent alternate interior angles, then the lines are parallel.
- The Alternate Exterior Angle Converse Theorem states: If two lines cut by a transversal form congruent alternate exterior angles, then the lines are parallel.
- The Same-Side Interior Angle Converse Theorem states: If two lines cut by a transversal form same-side interior angles that are supplementary, then the lines are parallel.
- The Same-Side Exterior Angle Converse Theorem states: If two lines cut by a transversal form same-side exterior angles that are supplementary, then the lines are parallel.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

G-CO Congruence

Prove geometric theorems

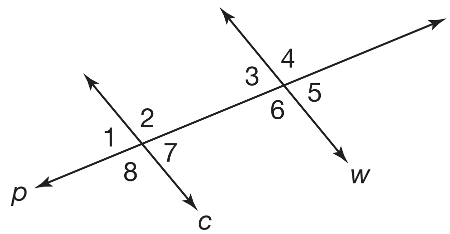
9. Prove theorems about lines and angles.

Overview

Student write the converses of the parallel line postulates and theorems for corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles. Then, each conjecture is proven.

2

Warm Up



1. If $\angle 3 \cong \angle 7$, then which theorem leads to the conclusion that $c \parallel w$?
The Alternate Interior Angle Converse Theorem leads to the conclusion that $c \parallel w$.

2. If $\angle 5$ and $\angle 8$ are supplementary, then which theorem leads to the conclusion that $c \parallel w$?
The Same-Side Exterior Angle Converse Theorem leads to the conclusion that $c \parallel w$.

3. If $\angle 4 \cong \angle 8$, then which theorem leads to the conclusion that $c \parallel w$?
The Alternate Exterior Angle Converse Theorem leads to the conclusion that $c \parallel w$.

4. If $\angle 2$ and $\angle 3$ are supplementary, then which theorem leads to the conclusion that $c \parallel w$?
The Same-Side Interior Angle Converse Theorem leads to the conclusion that $c \parallel w$.

2

2.5

A Reversed Condition

Parallel Line Converse Theorems

2

LEARNING GOALS

In this lesson, you will:

- Write and prove parallel line converse conjectures.

KEY TERMS

- converse
- Corresponding Angle Converse Postulate
- Alternate Interior Angle Converse Theorem
- Alternate Exterior Angle Converse Theorem
- Same-Side Interior Angle Converse Theorem
- Same-Side Exterior Angle Converse Theorem

Lewis Carroll is best known as the author of *Alice's Adventures in Wonderland* and its sequel *Through the Looking Glass*. However, Carroll also wrote several mathematics books, many of which focus on logic. In fact, Carroll included logic in many of his fictional books. Sometimes these took the form of "logical nonsense" such as the tea party scene with the Mad Hatter.

At one point of the scene, Alice proclaims that she says what she means, or at least, that she means what she says, insisting that the two statements are the same thing. The numerous attendees of the tea party then correct her with a series of flipped sentences which have totally different meanings. For example, "I like what I get" and "I get what I like".

Are these two sentences saying the same thing? Can you think of other examples of flipped sentences?

Problem 1

The term converse is defined and the Corresponding Angle Converse Postulate is stated. Students identify the hypothesis and conclusions in addition to writing the converse of the parallel line theorems. They also use the Corresponding Angle Converse Postulate to construct a pair of parallel lines.

2

Grouping

- Ask a student to read the definition and postulate. Discuss as a class.
- Have students complete Question 1 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- What do all of the conclusions in all of the parallel line theorems have in common?
- What do all of the hypotheses in all of the converse parallel line conjectures have in common?
- If a conditional statement is true, do you suppose the converse is always true? Explain.
- If a conditional statement is false, do you suppose the converse is always false? Explain.

PROBLEM 1 Converses

The **converse** of a conditional statement written in the form "If p, then q" is the statement written in the form "If q, then p." The converse is a new statement that results when the hypothesis and conclusion of the conditional statement are interchanged.

The Corresponding Angle Postulate states: "If two parallel lines are intersected by a transversal, then the corresponding angles are congruent."

The **Corresponding Angle Converse Postulate** states: "If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel."

The Corresponding Angle Converse Postulate is used to prove new conjectures formed by writing the converses of the parallel lines theorems.

1. For each theorem:
   - Identify the hypothesis p and conclusion q.
   - Write the converse of the theorem as a conjecture.

   a. Alternate Interior Angle Theorem: If two parallel lines are intersected by a transversal, then the alternate interior angles are congruent.

   Hypothesis p: Parallel lines are intersected by a transversal.

   Conclusion q: Alternate interior angles are congruent.

   Alternate Interior Angle Converse Conjecture: If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel.

   b. Alternate Exterior Angle Theorem: If two parallel lines are intersected by a transversal, then the alternate exterior angles are congruent.

   Hypothesis p: Parallel lines are intersected by a transversal.

   Conclusion q: Alternate exterior angles are congruent.

   Alternate Exterior Angle Converse Conjecture: If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel.

   c. Same-Side Interior Angle Theorem: If two parallel lines are intersected by a transversal, then the same-side interior angles are supplementary.

   Hypothesis p: Parallel lines are intersected by a transversal.

   Conclusion q: Same-side interior angles are supplementary.

   Same-Side Interior Angle Converse Conjecture: If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel.

Grouping

Have students complete Question 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 2

- How was the Corresponding Angle Converse Postulate helpful in constructing parallel lines?
- Would the Corresponding Angle Postulate be helpful in constructing parallel lines? Why or why not?

d. Same-Side Exterior Angle Theorem: If two parallel lines are intersected by a transversal, then the same-side exterior angles are supplementary.

Hypothesis p: Parallel lines are intersected by a transversal.

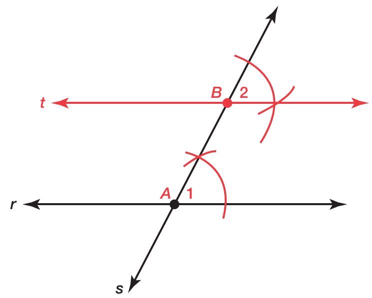
Conclusion q: Same-side exterior angles are supplementary.

Same-Side Exterior Angle Converse Conjecture: If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel.

2

2. Consider lines r and s.

a. Use the Corresponding Angle Converse Postulate to construct a line parallel to line r. Write the steps.



- Label the point at which line r and line s intersect as point A.
- Label one of the angles formed by the intersection of line r and line s as $\angle 1$.
- Locate and label point B on line s such that point B is above line r.
- Using point B as a vertex, duplicate $\angle 1$.
- Label the duplicate angle as $\angle 2$ and label the new line forming $\angle 2$ as line t.

b. Which line is a transversal?

Line s is a transversal.

c. Which lines are parallel?

Line r is parallel to line t.

Problem 2

Students use a variety of methods of proof to prove the Alternate Interior Angle Converse Theorem, the Alternate Exterior Angle Converse Theorem, the Same-Side Interior Angle Converse Theorem, and the Same-Side Exterior Angle Converse Theorem.

2

Grouping

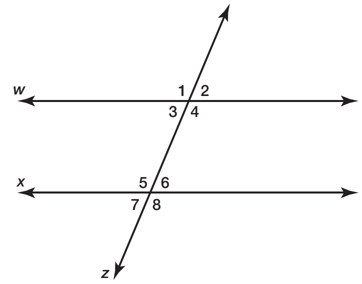
Have students complete Question 1 with a partner. Then have students share their responses as a class

Guiding Questions for Share Phase, Question 1

- What is the hypothesis of the Alternate Interior Angle Converse Conjecture?
- What is the conclusion of the Alternate Interior Angle Converse Conjecture?
- How can you use the Corresponding Angle Converse Postulate to help you prove this conjecture?

PROBLEM 2 Proving the Parallel Line Converse Conjectures

1. The Alternate Interior Angle Converse Conjecture states: "If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel."



a. Use the diagram to write the given and prove statements for the Alternate Interior Angle Converse Conjecture.

Given: $\angle 3 \cong \angle 6$ or $\angle 4 \cong \angle 5$

Prove: $w \parallel x$

b. Prove the Alternate Interior Angle Converse Conjecture.

Students may use a flow chart, two-column, or paragraph proof. The flow chart proof is provided. Students only need to use one method.

- $\angle 3 \cong \angle 6$ — Given
- $\angle 2 \cong \angle 3$ — Vertical angles are congruent
- $\angle 2 \cong \angle 6$ — Transitive Property
- $w \parallel x$ — Corresponding Angle Converse Postulate

You have just proven the Alternate Interior Angle Converse Conjecture. It is now known as the **Alternate Interior Angle Converse Theorem**.

Grouping

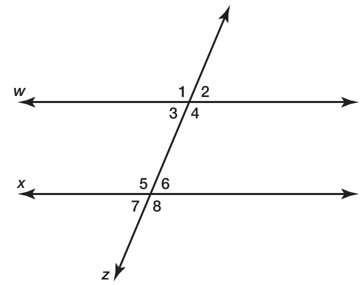
Have students complete Question 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 2

- What is the hypothesis of the Alternate Exterior Angle Converse Conjecture?
- What is the conclusion of the Alternate Exterior Angle Converse Conjecture?
- How can you use the Corresponding Angle Converse Postulate to help you prove this conjecture?
- How is the proof of this conjecture similar to the proof of the last conjecture?
- How is the proof of this conjecture different than the proof of the last conjecture?

2

2. The Alternate Exterior Angle Converse Conjecture states: "If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel."



a. Use the diagram to write the given and prove statements for the Alternate Exterior Angle Converse Conjecture.

Given: $\angle 1 \cong \angle 8$ or $\angle 2 \cong \angle 7$

Prove: $w \parallel x$

b. Prove the Alternate Exterior Angle Converse Conjecture.

Students may use a flow chart, two-column, or paragraph proof. The two-column proof is provided. Students only need to use one method.

Statements	Reasons
1. $\angle 1 \cong \angle 8$	1. Given
2. $\angle 1 \cong \angle 4$	2. Vertical angles are congruent
3. $\angle 4 \cong \angle 8$	3. Transitive Property
4. $w \parallel x$	4. Corresponding Angle Converse Postulate

You have just proven the Alternate Exterior Angle Converse Conjecture. It is now known as the **Alternate Exterior Angle Converse Theorem**.

Grouping

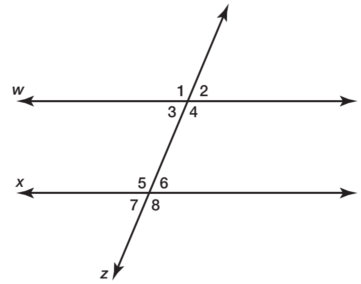
Have students complete Question 3 with a partner. Then have students share their responses as a class.

2

Guiding Questions for Share Phase, Question 3

- What is the hypothesis of the Same-Side Interior Angle Converse Conjecture?
- What is the conclusion of the Same-Side Interior Angle Converse Conjecture?
- How can you use the Corresponding Angle Converse Postulate to help you prove this conjecture?
- How is the proof of this conjecture similar to the proof of the last conjecture?
- How is the proof of this conjecture different than the proof of the last conjecture?
- What other Postulate is helpful in proving this conjecture?
- Did you use any theorems to help prove this conjecture?
- Did you use any definitions to help prove this conjecture?

3. The Same-Side Interior Angle Converse Conjecture states: "If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel."



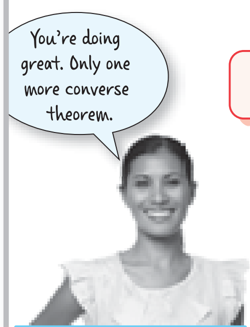
a. Use the diagram to write the given and prove statements for the Same-Side Interior Angle Converse Conjecture.

Given: $\angle 3$ and $\angle 5$ are supplementary or $\angle 4$ and $\angle 6$ are supplementary

Prove: $w \parallel x$

b. Prove the Same-Side Interior Angle Converse Conjecture.

Students may use a flow chart, two-column, or paragraph proof. The flow chart proof is provided. Students only need to use one method.



$\angle 1$ and $\angle 3$ are a linear pair
Definition of linear pair

$\angle 3$ and $\angle 5$ are supplementary
Given

$\angle 1$ and $\angle 3$ are supplementary
Linear Pair Postulate

$\angle 1 \cong \angle 5$
Supplements of the same angle are congruent

$w \parallel x$
Corresponding Angle Converse Postulate

You have just proven the Same-Side Interior Angle Converse Conjecture. It is now known as the **Same-Side Interior Angle Converse Theorem**.

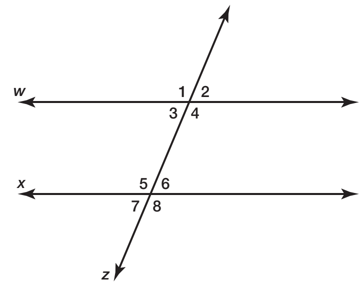
Grouping

Have students complete Question 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 4

- What is the hypothesis of the Same-Side Exterior Angle Converse Conjecture?
- What is the conclusion of the Same-Side Exterior Angle Converse Conjecture?
- How can you use the Corresponding Angle Converse Postulate to help you prove this conjecture?
- How is the proof of this conjecture similar to the proof of the last conjecture?
- How is the proof of this conjecture different than the proof of the last conjecture?
- What other Postulate is helpful in proving this conjecture?
- Did you use any theorems to help prove this conjecture?
- Did you use any definitions to help prove this conjecture?

4. The Same-Side Exterior Angle Converse Conjecture states: "If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel."



a. Use the diagram to write the given and prove statements for the Same-Side Exterior Angle Converse Conjecture.

Given: $\angle 1$ and $\angle 7$ are supplementary or $\angle 2$ and $\angle 8$ are supplementary

Prove: $w \parallel x$

b. Prove the Same-Side Exterior Angle Converse Conjecture.

Students may use a flow chart, two-column, or paragraph proof. The two-column proof is provided. Students only need to use one method.

Statements	Reasons
1. $\angle 1$ and $\angle 7$ are supplementary	1. Given
2. $\angle 1$ and $\angle 3$ are a linear pair	2. Definition of linear pair
3. $\angle 1$ and $\angle 3$ are supplementary	3. Linear Pair Postulate
4. $\angle 3 \cong \angle 7$	4. Supplements of the same angle are congruent
5. $w \parallel x$	5. Corresponding Angle Converse Postulate

You have just proven the Same-Side Exterior Angle Converse Conjecture. It is now known as the **Same-Side Exterior Angle Converse Theorem**.

Talk the Talk

The theorems in this lesson are summarized in this activity. Encourage students to write these theorems in their notebooks. Suggest they sketch a small diagram beside each theorem highlighting at least one pair of angles depicted in each theorem.

Grouping

- Ask students to review postulates and theorems, and refer to them while completing Questions 1 through 10. Discuss as a class.
- Have students complete Questions 1 through 10 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 10

- For which questions is it appropriate to use an original parallel line theorem, not a converse parallel line theorem?
- For which questions is it appropriate to use a converse parallel line theorem?
- In general, how can you determine when to use an original parallel line theorem or a converse parallel line theorem?

Talk the Talk

Here are all the converse postulates you have proven. Each converse conjecture you have proven is a new theorem.

Corresponding Angle Converse Postulate: If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel.

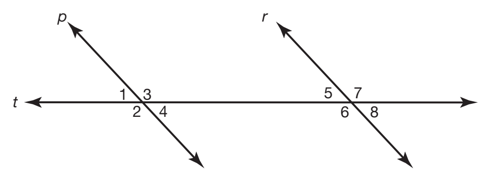
Alternate Interior Angle Converse Theorem: If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel.

Alternate Exterior Angle Converse Theorem: If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel.

Same-Side Interior Angle Converse Theorem: If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel.

Same-Side Exterior Angle Converse Theorem: If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel.

Use the diagram to answer the questions.



1. Which theorem or postulate would use $\angle 2 \cong \angle 7$ to justify line p is parallel to line r?
   Alternate Exterior Angle Converse Theorem

2. Which theorem or postulate would use $\angle 4 \cong \angle 5$ to justify line p is parallel to line r?
   Alternate Interior Angle Converse Theorem

3. Which theorem or postulate would use $\angle 1 \cong \angle 5$ to justify line p is parallel to line r?
   Corresponding Angle Converse Postulate

4. Which theorem or postulate would use $m\angle 4 + m\angle 6 = 180°$ to justify line p is parallel to line r?
Same-Side Interior Angle Converse Theorem

5. Which theorem or postulate would use $m\angle 1 + m\angle 7 = 180°$ to justify line p is parallel to line r?
Same-Side Exterior Angle Converse Theorem

6. Which theorem or postulate would use line p is parallel to line r to justify $\angle 2 \cong \angle 7$?
Alternate Exterior Angle Theorem

7. Which theorem or postulate would use line p is parallel to line r to justify $\angle 4 \cong \angle 5$?
Alternate Interior Angle Theorem

8. Which theorem or postulate would use line p is parallel to line r to justify $\angle 1 \cong \angle 5$?
Corresponding Angle Postulate

9. Which theorem or postulate would use line p is parallel to line r to justify $m\angle 4 + m\angle 6 = 180°$?
Same-Side Interior Angle Theorem

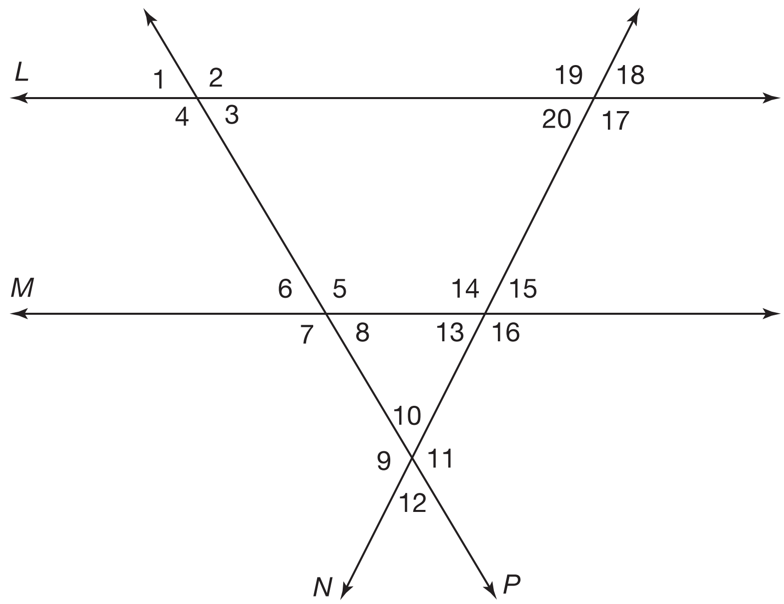
10. Which theorem or postulate would use line p is parallel to line r to justify $m\angle 1 + m\angle 7 = 180°$?
Same-Side Exterior Angle Theorem

Be prepared to share your methods and solutions.

Check for Students' Understanding

2

Given: $\angle 2 \cong \angle 7 \cong \angle 19$, $m\angle 2 = 125°$



1. Using the diagram in conjunction with postulates and theorems, determine the measure of all unknown angles.

   $m\angle 2 = m\angle 7 = m\angle 4 = m\angle 5 = m\angle 19 = m\angle 17 = m\angle 14 = m\angle 16 = 125°$

   $m\angle 1 = m\angle 3 = m\angle 6 = m\angle 8 = m\angle 18 = m\angle 20 = m\angle 15 = m\angle 13 = 55°$

   $m\angle 10 = m\angle 12 = 70°$

   $m\angle 9 = m\angle 11 = 110°$

2. List all of the known postulates and theorems that could have been used to solve for the measures of the unknown angles.

   - The Corresponding Angle Postulate
   - The Linear Pair Postulate
   - The Corresponding Angle Converse Postulate
   - The Alternate Interior Angle Theorem
   - The Alternate Exterior Angle Theorem
   - The Same Side Interior Angle Theorem
   - The Same Side Exterior Angle Theorem
   - The Triangle Sum Theorem
   - The Vertical Angle Theorem
   - The Alternate Interior Angle Converse Theorem
   - The Alternate exterior Angle Converse Theorem
   - The Same Side Interior Angle Converse Theorem
   - The Same Side Exterior Angle Converse Theorem

Chapter 2 Summary

KEY TERMS

- induction (2.1)
- deduction (2.1)
- counterexample (2.1)
- conditional statement (2.1)
- propositional form (2.1)
- propositional variables (2.1)
- hypothesis (2.1)
- conclusion (2.1)
- truth value (2.1)
- truth table (2.1)
- supplementary angles (2.2)
- complementary angles (2.2)
- adjacent angles (2.2)
- linear pair (2.2)
- vertical angles (2.2)
- postulate (2.2)
- theorem (2.2)
- Euclidean geometry (2.2)
- Addition Property of Equality (2.3)
- Subtraction Property of Equality (2.3)
- Reflexive Property (2.3)
- Substitution Property (2.3)
- Transitive Property (2.3)
- flow chart proof (2.3)
- two-column proof (2.3)
- paragraph proof (2.3)
- construction proof (2.3)
- conjecture (2.4)
- converse (2.5)

POSTULATES AND THEOREMS

- Linear Pair Postulate (2.2)
- Segment Addition Postulate (2.2)
- Angle Addition Postulate (2.2)
- Right Angle Congruence Theorem (2.3)
- Congruent Supplement Theorem (2.3)
- Congruent Complement Theorem (2.3)
- Vertical Angle Theorem (2.3)
- Corresponding Angle Postulate (2.4)
- Alternate Interior Angle Theorem (2.4)
- Alternate Exterior Angle Theorem (2.4)
- Same-Side Interior Angle Theorem (2.4)
- Same-Side Exterior Angle Theorem (2.4)
- Corresponding Angle Converse Postulate (2.5)
- Alternate Interior Angle Converse Theorem (2.5)
- Alternate Exterior Angle Converse Theorem (2.5)
- Same-Side Interior Angle Converse
- Theorem (2.5)
- Same-Side Exterior Angle Converse Theorem (2.5)

2.1 Identifying and Comparing Induction and Deduction

Induction uses specific examples to make a conclusion. Induction, also known as inductive reasoning, is used when observing data, recognizing patterns, making generalizations about the observations or patterns, and reapplying those generalizations to unfamiliar situations. Deduction, also known as deductive reasoning, uses a general rule or premise to make a conclusion. It is the process of showing that certain statements follow logically from some proven facts or accepted rules.

2

Example

Kyra sees coins at the bottom of a fountain. She concludes that if she throws a coin into the fountain, it too will sink. Tyler understands the physical laws of gravity and mass and decides a coin he throws into the fountain will sink.

The specific information is the coins Kyra and Tyler observed at the bottom of the fountain.
The general information is the physical laws of gravity and mass.
Kyra's conclusion that her coin will sink when thrown into the fountain is induction.
Tyler's conclusion that his coin will sink when thrown into the fountain is deduction.

2.1 Identifying False Conclusions

It is important that all conclusions are tracked back to given truths. There are two reasons why a conclusion may be false. Either the assumed information is false or the argument is not valid.

Example

Erin noticed that every time she missed the bus, it rained. So, she concludes that next time she misses the bus it will rain.

Erin's conclusion is false because missing the bus is not related to what makes it rain.

2.1 Writing a Conditional Statement

A conditional statement is a statement that can be written in the form "If *p*, then *q*." The portion of the statement represented by *p* is the hypothesis. The portion of the statement represented by *q* is the conclusion.

Example

If I plant an acorn, then an oak tree will grow.

A solid line is drawn under the hypothesis, and a dotted line is drawn under the conclusion.

2.1 Using a Truth Table to Explore the Truth Value of a Conditional Statement

The truth value of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then its truth value is considered "true." The first two columns of a truth table represent the possible truth values for p (the hypothesis) and q (the conclusion). The last column represents the truth value of the conditional statement ($p \rightarrow q$). Notice that the truth value of a conditional statement is either "true" or "false," but not both.

2

Example

Consider the conditional statement, "If I eat too much, then I will get a stomach ache."

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

When p is true, I ate too much. When q is true, I will get a stomach ache. It is true that when I eat too much, I will get a stomach ache. So, the truth value of the conditional statement is true.

When p is true, I ate too much. When q is false, I will not get a stomach ache. It is false that when I eat too much, I will not get a stomach ache. So, the truth value of the conditional statement is false.

When p is false, I did not eat too much. When q is true, I will get a stomach ache. It could be true that when I did not eat too much, I will get a stomach ache for a different reason. So, the truth value of the conditional statement in this case is true.

When p is false, I did not eat too much. When q is false, I will not get a stomach ache. It could be true that when I did not eat too much, I will not get a stomach ache. So, the truth value of the conditional statement in this case is true.

2.1 Rewriting Conditional Statements

A conditional statement is a statement that can be written in the form "If p, then q." The hypothesis of a conditional statement is the variable p. The conclusion of a conditional statement is the variable q.

Example

Consider the following statement: If two angles form a linear pair, then the sum of the measures of the angles is 180 degrees. The statement is a conditional statement. The hypothesis is "two angles form a linear pair," and the conclusion is "the sum of the measures of the angles is 180 degrees." The conditional statement can be rewritten with the hypothesis as the "Given" statement and the conclusion as the "Prove" statement.

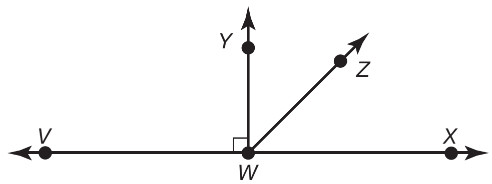
Given: Two angles form a linear pair.

Prove: The sum of the measures of the angles is 180 degrees.

2

2.2 Identifying Complementary and Supplementary Angles

Two angles are supplementary if the sum of their measures is 180 degrees.
Two angles are complementary if the sum of their measures is 90 degrees.



Example

In the diagram above, angles *YWZ* and *ZWX* are complementary angles.

In the diagram above, angles *VWY* and *XWY* are supplementary angles.
Also, angles *VWZ* and *XWZ* are supplementary angles.

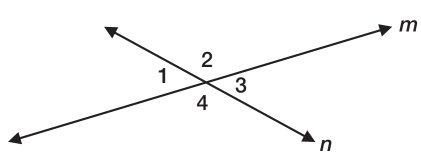
2.2 Identifying Adjacent Angles, Linear Pairs, and Vertical Angles

Adjacent angles are angles that share a common vertex and a common side.

A linear pair of angles consists of two adjacent angles that have noncommon sides that form a line.

Vertical angles are nonadjacent angles formed by two intersecting lines.

Example



Angles 2 and 3 are adjacent angles.

Angles 1 and 2 form a linear pair. Angles 2 and 3 form a linear pair. Angles 3 and 4 form a linear pair. Angles 4 and 1 form a linear pair.

Angles 1 and 3 are vertical angles. Angles 2 and 4 are vertical angles.

2.2 Determining the Difference Between Euclidean and Non-Euclidean Geometry

Euclidean geometry is a system of geometry developed by the Greek mathematician Euclid that included the following five postulates.

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn that has the segment as its radius and one point as the center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

Example

Euclidean geometry:

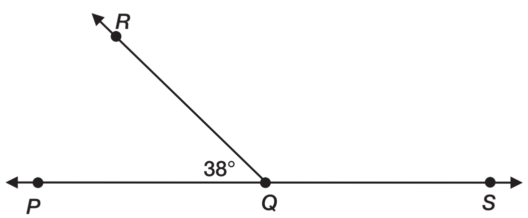
Non-Euclidean geometry:

2

2.2 Using the Linear Pair Postulate

The Linear Pair Postulate states: "If two angles form a linear pair, then the angles are supplementary."

Example



$$m\angle PQR + m\angle SQR = 180°$$

$$38° + m\angle SQR = 180°$$

$$m\angle SQR = 180° - 38°$$

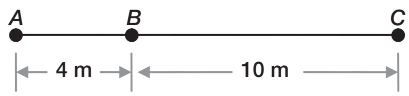
$$m\angle SQR = 142°$$

2

2.2 Using the Segment Addition Postulate

The Segment Addition Postulate states: "If point B is on segment AC and between points A and C, then $AB + BC = AC$."

Example



$$AB + BC = AC$$

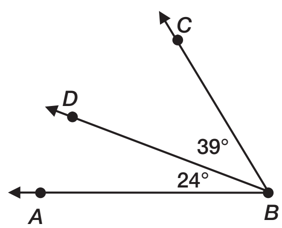
$$4 \text{ m} + 10 \text{ m} = AC$$

$$AC = 14 \text{ m}$$

2.2 Using the Angle Addition Postulate

The Angle Addition Postulate states: "If point D lies in the interior of angle ABC, then $m\angle ABD + m\angle DBC = m\angle ABC$."

Example



$$m\angle ABD + m\angle DBC = m\angle ABC$$

$$24° + 39° = m\angle ABC$$

$$m\angle ABC = 63°$$

2.3 Using Properties of Real Numbers in Geometry

The Addition Property of Equality states: "If a, b, and c are real numbers and $a = b$, then $a + c = b + c$."

The Subtraction Property of Equality states: "If a, b, and c are real numbers and $a = b$, then $a - c = b - c$."

The Reflexive Property states: "If a is a real number, then $a = a$."

The Substitution Property states: "If a and b are real numbers and $a = b$, then a can be substituted for b.

The Transitive Property states: "If a, b, and c are real numbers and $a = b$ and $b = c$, then $a = c$."

Example

Addition Property of Equality applied to angle measures: If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$.

Subtraction Property of Equality applied to segment measures: If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} - m\overline{EF} = m\overline{CD} - m\overline{EF}$.

Reflexive Property applied to distances: $AB = AB$

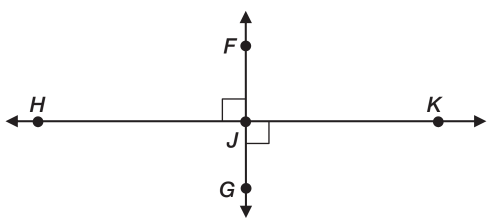
Substitution Property applied to angle measures: If $m\angle 1 = 20°$ and $m\angle 2 = 20°$, then $m\angle 1 = m\angle 2$.

Transitive Property applied to segment measures: If $m\overline{AB} = m\overline{CD}$ and $m\overline{CD} = m\overline{EF}$, then $m\overline{AB} = m\overline{EF}$.

2.3 Using the Right Angle Congruence Theorem

The Right Angle Congruence Theorem states: "All right angles are congruent."

Example



$\angle FJH \cong \angle GJK$

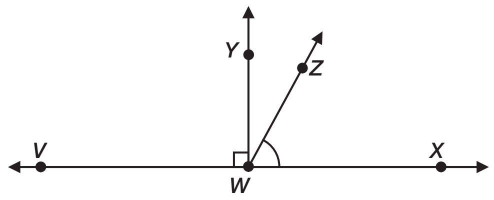
2

2

2.3 Using the Congruent Supplement Theorem

The Congruent Supplement Theorem states: "If two angles are supplements of the same angle or of congruent angles, then the angles are congruent."

Example

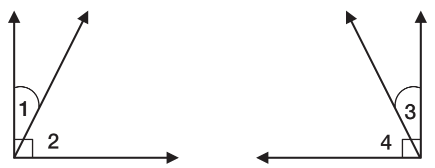


$\angle VWZ \cong \angle XWY$

2.3 Using the Congruent Complement Theorem

The Congruent Complement Theorem states: "If two angles are complements of the same angle or of congruent angles, then the angles are congruent."

Example

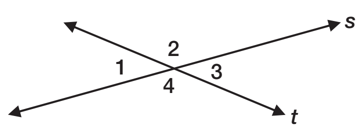


$\angle 2 \cong \angle 4$

2.3 Using the Vertical Angle Theorem

The Vertical Angle Theorem states: "Vertical angles are congruent."

Example

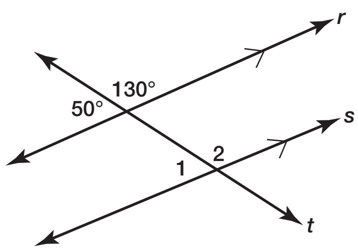


$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

2.4 Using the Corresponding Angle Postulate

The Corresponding Angle Postulate states: "If two parallel lines are intersected by a transversal, then corresponding angles are congruent."

Example



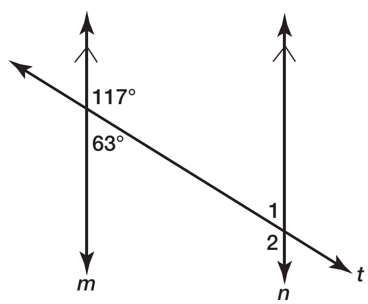
The angle that measures 50° and $\angle 1$ are corresponding angles.
So, $m\angle 1 = 50°$.

The angle that measures 130° and $\angle 2$ are corresponding angles.
So, $m\angle 2 = 130°$.

2.4 Using the Alternate Interior Angle Theorem

The Alternate Interior Angle Theorem states: "If two parallel lines are intersected by a transversal, then alternate interior angles are congruent."

Example



The angle that measures 63° and $\angle 1$ are alternate interior angles.
So, $m\angle 1 = 63°$.

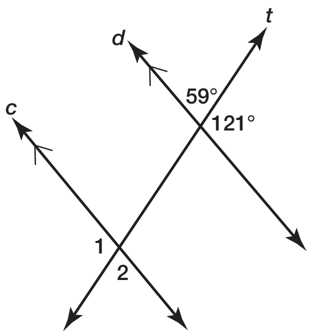
The angle that measures 117° and $\angle 2$ are alternate interior angles.
So, $m\angle 2 = 117°$.

2

2.4 Using the Alternate Exterior Angle Theorem

The Alternate Exterior Angle Theorem states: "If two parallel lines are intersected by a transversal, then alternate exterior angles are congruent."

Example



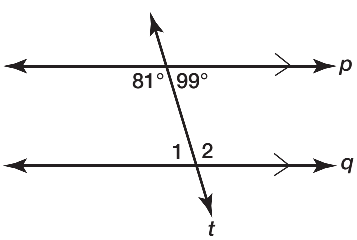
The angle that measures 121° and $\angle 1$ are alternate exterior angles. So, $m\angle 1 = 121°$.

The angle that measures 59° and $\angle 2$ are alternate exterior angles. So, $m\angle 2 = 59°$.

2.4 Using the Same-Side Interior Angle Theorem

The Same-Side Interior Angle Theorem states: "If two parallel lines are intersected by a transversal, then same-side interior angles are supplementary."

Example



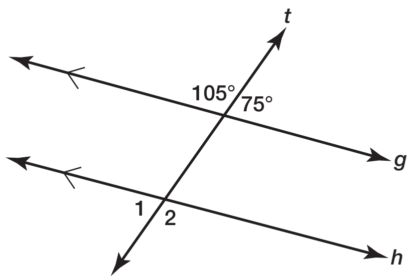
The angle that measures 81° and $\angle 1$ are same-side interior angles.
So, $m\angle 1 = 180° - 81° = 99°$.

The angle that measures 99° and $\angle 2$ are same-side interior angles.
So, $m\angle 2 = 180° - 99° = 81°$.

2.4 Using the Same-Side Exterior Angle Theorem

The Same-Side Exterior Angle Theorem states: “If two parallel lines are intersected by a transversal, then same-side exterior angles are supplementary.”

Example



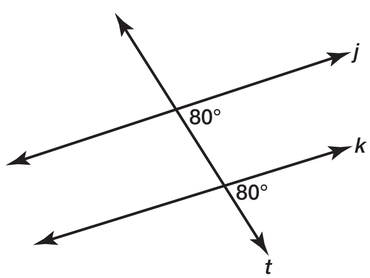
The angle that measures 105° and $\angle 1$ are same-side exterior angles.
So, $m\angle 1 = 180° - 105° = 75°$.

The angle that measures 75° and $\angle 2$ are same-side exterior angles.
So, $m\angle 2 = 180° - 75° = 105°$.

2.5 Using the Corresponding Angle Converse Postulate

The Corresponding Angle Converse Postulate states: “If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel.”

Example



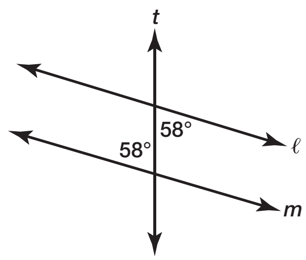
Corresponding angles have the same measure. So, $j \parallel k$.

2

2.5 Using the Alternate Interior Angle Converse Theorem

The Alternate Interior Angle Converse Theorem states: “If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel.”

Example

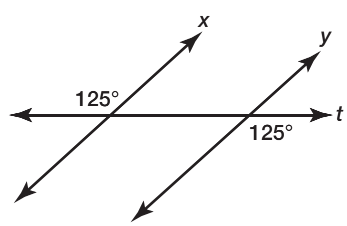


Alternate interior angles have the same measure. So, $\ell \parallel m$.

2.5 Using the Alternate Exterior Angle Converse Theorem

The Alternate Exterior Angle Converse Theorem states: “If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel.”

Example

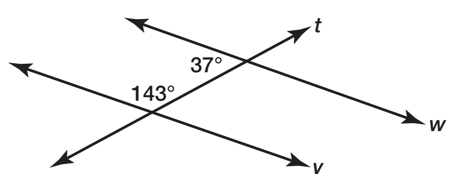


Alternate exterior angles have the same measure. So, $x \parallel y$.

2.5 Using the Same-Side Interior Angle Converse Theorem

The Same-Side Interior Angle Converse Theorem states: “If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel.”

Example

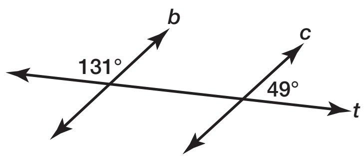


Same-side interior angles are supplementary: $37° + 143° = 180°$. So, $v \parallel w$.

2.5 Using the Same-Side Exterior Angle Converse Theorem

The Same-Side Exterior Angle Converse Theorem states: "If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel."

Example



Same-side exterior angles are supplementary: $131° + 49° = 180°$. So, $b \parallel c$.

2

2