Probability 19

Chapter 19 Overview

This chapter investigates compound probability with an emphasis toward modeling and analyzing sample spaces to determine rules for calculating probabilities in different situations. Students explore various probability models and calculate compound probabilities with independent and dependent events in a variety of problem situations. Students use technology to run experimental probability simulations.

Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
19.1	Modeling Probability	S.CP.1	1	This lesson explores modeling probability situations with sample spaces and uniform and non-uniform probability models. Questions ask students to determine probabilities of events and their complements using probability notation.	X		X	X	
19.2	Compound Sample Spaces	S.CP.1	2	In this lesson, students use tree diagrams and organized lists to represent sample spaces. Students model compound sample spaces using disjoint and intersecting sets and identify independent and dependent events. The Counting Principle is introduced.	X	X	X	X	
19.3	Compound Probability with "And"	S.CP.2 S.CP.8	1	This lesson explores determining the probability of two or more independent events and two or more dependent events. Questions guide students to analyze various compound probability situations and develop the Rule for Compound Probability involving "and."	X		X	X	

Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
19.4	Compound Probability with "Or"	S.CP.7	1	This lesson explores determining the probability of one or another independent events and one or another dependent events. Questions guide students to analyze various compound probability situations and develop the Addition Rule for Probability. At the end, students organize what they have learned about compound events.	X		X		
19.5	Calculating Compound Probability	S.CP.2 S.CP.8	1	In this lesson, students apply what they have learned to calculate compound probabilities with replacement and without replacement. Questions ask students to determine compound probabilities in a variety of problem situations.					
19.6	Investigate Magnitude through Theoretical Probability and Experimental Probability	S.IC.2	1	This lesson explores the distinction between theoretical probability and experimental probability. Students use technology to generate random numbers, simulating an experimental probability situation.				X	X

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Skills Practice Correlation for Chapter 19

Lesson		Problem Set	Objectives
19.1	Modeling Probability		Vocabulary
		1 – 6	Identify the sample space for situations
		7 – 12	Construct uniform and non-uniform probability models for situations
		13 – 18	Determine the probability of events and their complements
19.2	Compound Sample Spaces		Vocabulary
		1 – 8	Identify the actions, outcomes, disjoint sets, intersecting sets, independent events, and dependent events in probability situations
		9 – 14	Sketch tree diagrams and write organized lists to represent sample spaces
		15 – 22	Use the Counting Principle to determine the number of possible outcomes for probability situations
19.3	Compound Probability with "And"		Vocabulary
		1 – 6	Determine the probability of events and compound events
		7 – 12	Determine the probability of events and dependent events
19.4	Compound Probability with "Or"		Vocabulary
		1 – 6	Use the Addition Rule for Probability to determine the probability of independent events
		7 – 14	Use the Addition Rule for Probability to determine the probability of dependent events
19.5	Calculating Compound Probability	1 – 6	Determine the probability of compound events with replacement
		7 – 14	Determine the probability of compound events without replacement
19.6	Investigate Magnitude through Theoretical Probability and Experimental Probability		Vocabulary
		1 – 6	Use the multiplication rule of probability for compound independent events to solve problems
		7 – 12	Determine experimental probabilities using a random number generator to solve problems
		13 – 18	Compare theoretical and experimental probabilities in situations

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19.1

These Are a Few of My Favorite Things

Modeling Probability

LEARNING GOALS

In this lesson, you will:

- List the sample space for situations involving probability.
- Construct a probability model for a situation.
- Differentiate between uniform and non-uniform probability models.

KEY TERMS

- outcome
- sample space
- event
- probability
- probability model
- uniform probability model
- complement of an event
- non-uniform probability model

ESSENTIAL IDEAS

- The probability of an event is the ratio of the number of desired outcomes to the total number of possible outcomes.
- A sample space is all of the possible outcomes in a probability situation.
- An event is an outcome or set of outcomes in a sample space.
- A probability model lists the possible outcomes and each outcome's probability. The sum of the probabilities in the model must equal one.
- The complement of an event is an event which contains all the outcomes in the sample space that are not outcomes in the event.
- A non-uniform probability model is a model in which all of the outcomes are not equal.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

S-CP Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Overview

The terms associated with probability such as sample space, event, and probability model are defined. Students list the sample space for different situations, construct a probability model for different situations and differentiate between uniform and non-uniform probability models. They conclude that a probability of 0 represents an impossible event and a probability of 1 represents an event that is certain to occur. Probability notation is used.

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Warm Up

Consider the 7 days of a week. Suppose the name of each week is written on a separate piece of folded paper and placed into a bag. You reach into the bag and choose one piece of paper.

1. What is the probability of randomly choosing a day that begins with the letter *s*?
   The probability of randomly choosing a day that begins with the letter *s* is $\frac{2}{7}$.

2. What is the probability of randomly choosing a weekday?
   The probability of randomly choosing a weekday is $\frac{5}{7}$.

3. What is the probability of randomly choosing Wednesday?
   The probability of randomly choosing a Wednesday is $\frac{1}{7}$.

4. What is the probability of randomly choosing a day that is not Sunday?
   The probability of randomly choosing a day that is not Sunday is $\frac{6}{7}$.

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19.1

These Are a Few of My Favorite Things

Modeling Probability

LEARNING GOALS

In this lesson, you will:

- List the sample space for situations involving probability.
- Construct a probability model for a situation.
- Differentiate between uniform and non-uniform probability models.

KEY TERMS

- outcome
- sample space
- event
- probability
- probability model
- uniform probability model
- complement of an event
- non-uniform probability model

The meteorologist forecasts a 60% chance of rain. A new drug is reported to have a 0.5% chance of causing headaches. You have a 1 out of 4 chance of guessing the answer to a multiple-choice question with four possible answers. All of these statements have one thing in common—they attempt to predict the future. In mathematics, you can use probability to determine what *may* happen in the future. How else do you encounter probability in the real world?

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Problem 1

In this problem, flipping a coin, rolling a number cube, and spinning a spinner are used to generate probabilities. Students review probability terms, identify the sample space, answer probability questions, and construct uniform and non-uniform probability models.

Grouping

- Ask students to read the information. Discuss as a class.
- Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2

- When flipping a fair coin, what are the possible outcomes?
- With regard to a coin flip, what does $P(H)$ mean?
- With regard to a coin flip, what does $P(T)$ mean?
- If the coin is flipped again, does $P(T)$ change?
- When flipping a fair coin, what is the probability of flipping heads or tails?
- What is the decimal equivalent of $\frac{1}{2}$?

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PROBLEM 1 What Are the Chances?

An **outcome** is a result of an experiment. The **sample space** is the set of all of the possible outcomes of an experiment. An **event** is an outcome or set of outcomes in a sample space.

The **probability** of an event is the ratio of the number of desired outcomes to the total number of possible outcomes. The probability of event A is $P(A) = \frac{\text{desired outcomes}}{\text{possible outcomes}}$.

1. Suppose a fair coin is flipped.

   a. What is the sample space?

   For flipping a fair coin, the sample space is heads and tails. The sample space is the set of all possible outcomes.

   b. Let H represent the event of flipping a coin that results in heads up. Let T represent the event of flipping a coin that results in tails up.

   What is the probability that a coin flip lands heads up, $P(\text{H})$? Tails up, $P(\text{T})$?

   There are two possible outcomes for flipping a coin. It lands with heads up or tails up.

   $P(\text{H}) = \frac{1}{2}$

   $P(\text{T}) = \frac{1}{2}$

   c. If the coin is flipped again, does the probability of it landing heads up change? Explain your reasoning.

   No. As long as the coin is fair, the probability of a heads up result on any one flip is always $\frac{1}{2}$. There are two possible outcomes for each flip, either heads up or tails up.

It is often helpful to construct a model when analyzing situations involving probability. A **probability model** lists the possible outcomes and the probability for each outcome. In a probability model, the sum of the probabilities must equal 1.

2. Construct the probability model for flipping a coin.

Outcomes	Heads (H)	Tails (T)
Probability	$\frac{1}{2}$	$\frac{1}{2}$

The possible outcomes for flipping a coin are heads up and tails up.

Grouping

Have students complete Questions 3 and 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 3 and 4

- What numbers appear on a number cube?
- What is the probability of rolling an odd number, *P*(odd)?
- What is the probability of rolling a number less than 2, *P*(less than 2)?
- What is the decimal equivalent of $\frac{1}{6}$?

3. Suppose you roll a number cube once.

a. Identify the sample space.

The sample space for rolling a number cube once is 1, 2, 3, 4, 5, and 6.

b. What is the probability of rolling a 5, *P*(5)?

$P(5) = \frac{1}{5}$

There are six possible outcomes for rolling a number cube and one of the outcomes is a 5.

c. What is the probability of rolling an even number, *P*(even)?

$P(\text{even}) = \frac{3}{6}$

There are six possible outcomes for rolling a number cube and three of the outcomes are even. The even outcomes are 2, 4, and 6.

d. What is the probability of rolling a number greater than 2, *P*(greater than 2)?

$P(\text{greater than 2}) = \frac{4}{6}$

There are six possible outcomes for rolling a number cube and four of the outcomes are greater than 2. The outcomes greater than 2 are 3, 4, 5, and 6.

e. Determine the probability of rolling a number greater than 3, $P(> 3)$, and the probability of rolling a number less than or equal to 3, $P(\leq 3)$.

$P(> 3) = \frac{3}{6}$

$P(\leq 3) = \frac{3}{6}$

There are six possible outcomes for rolling a number cube. Three of the outcomes are greater than 3 and three of the outcomes are less than or equal to 3. The outcomes greater than 3 are 4, 5, and 6. The outcomes less than or equal to 3 are 1, 2, and 3.

f. What is the sum of the probabilities from part (e)?

$P(> 3) + P(\leq 3) = \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1$

4. Construct the probability model for rolling a number cube.

Outcomes	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

The possible outcomes for rolling a number cube are 1, 2, 3, 4, 5, and 6.

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Grouping

Have students complete Questions 5 through 7 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 5 through 7

- If $P(E) = 0.7$, then what is the probability of its complement?
- If $P(E) = 0.5$, then what is the probability of its complement?
- If $P(E) = 0$, then what is the probability of its complement?
- What is the highest possible value for the probability of an event?
- What is the lowest possible value for the probability of an event?

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In a **uniform probability model**, the probabilities for each outcome are equal. The model for rolling a number cube is a uniform probability model.

The **complement of an event** is an event that contains all the outcomes in the sample space that are not outcomes in the event. In mathematical notation, if E is an event, then the complement of E is often denoted as $\overline{E}$ or E^c. You already did some work with *complementary events* in Question 3, parts (e) and (f).

5. What is the relationship between the number of outcomes in an event E and the number of outcomes in the complement of event E, E^c?

The sum of the number of outcomes in E and E^c is equal to the total number of outcomes in the sample space.

6. What is the relationship between the probability of an event, $P(E)$, and the probability of its complement, $P(E^c)$? Explain your reasoning.

$P(E) + P(E^c) = 1$

The sum of the probability of an event and the probability of its complement is 1.

7. List three more complementary events in the number cube situation.

Answers will vary.

Examples of three complementary events:

- Rolling a 6 and not rolling a 6
- Rolling an even number and rolling an odd number
- Rolling a number less than 3 and rolling a number greater than or equal to 3

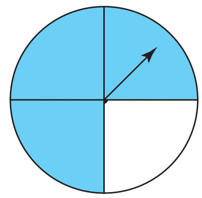
Grouping

Have students complete Questions 8 through 11 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 8 through 11

- Why is the probability model of the spinner considered non-uniform?
- If you spin the spinner multiple times, is the probability of landing on a shaded section the same for each spin, or is it different for each spin?
- If you spin the spinner multiple times, is the probability of landing on an unshaded section the same for each spin, or is it different for each spin?

8. The spinner shown is divided into 4 congruent sections and you spin the spinner 1 time.



George and Maria each determine the sample space for the spinner.

George

I think the sample space is (shaded, unshaded).

Maria

I think the sample space is (shaded, shaded, shaded, unshaded).

Who determined the sample space correctly? Explain your reasoning.

Maria is correct. The sample space contains four items because there are four sections on the spinner. Each section represents a different outcome. If the sample space was (shaded, unshaded), that would mean the spinner had two sections.

9. What is the probability of landing on a shaded section? An unshaded section?

$P(\text{shaded}) = \frac{3}{4}$

$P(\text{unshaded}) = \frac{1}{4}$

There are 3 shaded sections and 4 total sections, so the probability of landing on a shaded section is $\frac{3}{4}$.

There is 1 unshaded section and 4 total sections, so the probability of landing on an unshaded section is $\frac{1}{4}$.

When the probabilities of the outcomes in a probability model are not all equal, the model is called a **non-uniform probability model**.

10. Construct a non-uniform probability model for spinning the spinner.

Outcome	Shaded (S)	Unshaded (U)
Probability	$\frac{3}{4}$	$\frac{1}{4}$

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Talk the Talk

Students describe the range of values for any probability and interpret the least and greatest possible values in the range.

Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

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11. Compare the probability models for spinning the spinner, rolling a number cube, and flipping a coin.

What do you notice about the probabilities in a uniform probability model and a non-uniform probability model?

In a uniform probability model, the probabilities of the outcomes are equal. But in a non-uniform probability model, the probabilities of the outcomes are not all equal.

The spinner's probability model is non-uniform. The number cube and coin probability models are uniform.

Talk the Talk

1. Refer to the probability models for the coin, number cube, and spinner in Problem 1. What can you conclude about the sum of the probabilities in each model?

The sum of the probabilities in each probability model is 1.

2. Describe the range of values for any probability. What do the least and greatest values in this range mean? Explain your reasoning.

Every probability is greater than or equal to 0 and less than or equal to 1. Probability cannot have a value less than 0 or greater than 1.

A probability of 0 represents an impossible event.

A probability of 1 represents an event that is certain to occur.

3. Bill, Larry, Earvin, and Kareem were playing a game with marbles. There are 8 blue marbles and 4 yellow marbles in a jar.

What is the probability of randomly selecting a blue marble from the jar?

Bill

The probability of selecting a blue marble is $\frac{8}{12}$.

Larry

The probability of selecting a blue marble is approximately 67%.

Kareem

The probability of selecting a blue marble is approximately 0.67.

Earvin

The probability of selecting a blue marble is $\frac{2}{3}$.

a. What are the similarities and differences of the probability values in each boy's statement?

Answers will vary.

Similarities:

All of the probability values for selecting a blue marble are equal or approximately equal.

$\frac{8}{12} = \frac{2}{3}$

$\frac{8}{12} \approx 67\%$

$\frac{8}{12} \approx 0.7$

Differences:

All of the statements express the probability of selecting a blue marble in a different form. Bill and Earvin used different but equivalent fractions. Kareem used a rounded decimal approximation and Larry used a rounded percent approximation.

b. What are the advantages and disadvantages of the probability values in each boy's statement?

Answers will vary.

Examples of Advantages:

- Bill's value, $\frac{8}{12}$, is the exact probability and also displays the exact number of blue marbles and total marbles. The 8 in the numerator represents the number of blue marbles and the 12 in the denominator represents the total number of marbles.
- Earvin's value, $\frac{2}{3}$, is the exact probability.
- Larry's value, 67%, is intuitively understandable. Because 67% is more than 50% that means there's a greater probability of selecting a blue marble compared to not selecting a blue marble.
- Kareem's value, 0.7, is intuitively understandable. The decimal 0.7 is closer to 1 than it is to 0 which means there's a greater probability of selecting a blue marble compared to not selecting a blue marble.

Examples of Disadvantages:

- The fractions used by Bill and Earvin may not be as intuitively understandable as the percent or decimal.
- The percent and decimal used by Larry and Kareem are not exact values. Both values are rounded.

c. Whose probability value do you prefer to use for this scenario? Explain your reasoning.

Answers will vary.

Example Response:

I prefer to use Bill's value, $\frac{8}{12}$, because it is the exact probability and also displays the exact number of blue marbles and total marbles. The 8 in the numerator represents the number of blue marbles and the 12 in the denominator represents the total number of marbles.

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Be prepared to share your solutions and methods.

Check for Students' Understanding

Consider the 12 months of a year. Suppose the name of each month is written on a separate piece of folded paper and placed into a bag. You reach into the bag and choose one piece of paper.

1. What is the probability of randomly choosing a month that has exactly 30 days?

   The probability of randomly choosing a month that has 30 days is $\frac{4}{12} = \frac{1}{3}$.

2. What is the probability of randomly choosing a month that begins with the letter *J*?

   The probability of randomly choosing a month that has 30 days is $\frac{3}{12} = \frac{1}{4}$.

3. Describe a possible outcome in this sample space that has a probability value of 0.

   Answers will vary.

   The probability of randomly choosing a month that has 6 Tuesdays is $\frac{0}{12} = 0$.

4. Describe a possible outcome in this sample space that has a probability value of 1.

   Answers will vary.

   The probability of randomly choosing a month that has 2 Tuesdays is $\frac{12}{12} = 1$.

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19.2

It's in the Cards

Compound Sample Spaces

LEARNING GOALS

In this lesson, you will:

- Develop a rule to determine the total number of outcomes in a sample space without listing each event.
- Classify events as independent or dependent.
- Use the Counting Principle to calculate the size of sample spaces.

KEY TERMS

- tree diagram
- organized list
- set
- element
- disjoint sets
- intersecting sets
- independent events
- dependent events
- Counting Principle

ESSENTIAL IDEAS

- Disjoint sets do not have common elements.
- Intersecting sets have at least one common element.
- Independent events are events for which the occurrence of one event has no impact on the occurrence of the other event.
- Dependent events are events for which the occurrence of one event has an impact on the occurrence of the following events. Some dependent events are selected by drawing from one set that does not allow repetitions.
- The Counting Principle states: If an action A can occur in m ways and for each of these m ways, an action B can occur in n ways, then Actions A and B can occur in $m \cdot n$ ways.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

S-CP Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Overview

Independent and dependent events are introduced in this lesson. Tree diagrams are used to list outcomes in a situation. Students are first given a worked example of tree diagrams and then in different situations, they asked to create their own tree diagrams and organized lists of the sample space. Students analyze the sample space in each situation by answering a series or related questions. Students conclude when the number of items in each set of the sample space is multiplied together, they can calculate the total number of possible outcomes. They distinguish between situations that involve independent events from disjoint sets and dependent events from intersecting sets. Students identify which situations are associated with dependent events that do not allow repetitions. The Counting Principle is stated and discussed as a shortcut for determining the size of a sample space.

Warm Up

A local pet store sells blue, red, black, green, and purple dog collars. They also sell engraved dog ID tags in silver, gold, and pink. Joan wants to buy a dog collar and engraved dog ID tag for her new puppy.

1. How many different dog collars does Joan have to choose from?

   Joan has 5 different dog collar choices. She can buy a blue, red, black, green, or purple dog collar.

2. How many different dog ID tags does Joan have to choose from?

   Joan has 3 different dog ID tag choices. She can buy a silver, gold, or pink dog ID tag.

3. How many different ways can Joan choose one dog collar and one dog ID tag?

   There are 15 different ways in which Joan can choose one dog collar and one dog ID tag.

4. How did you determine the answer to Question 3?

   Answers will vary.

   I made an organized list.

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It's in the Cards

Compound Sample Spaces

19.2

LEARNING GOALS

In this lesson, you will:

- Develop a rule to determine the total number of outcomes in a sample space without listing each event.
- Classify events as independent or dependent.
- Use the Counting Principle to calculate the size of sample spaces.

KEY TERMS

- tree diagram
- organized list
- set
- element
- disjoint sets
- intersecting sets
- independent events
- dependent events
- Counting Principle

You are due for a win! Your luck will run out soon!

Have you ever heard someone say something like this about a game of chance? People—especially people who don't know a lot about probability—are sometimes fooled into thinking that after a long series of losses or a long series of wins that the next turn will produce different results. This is often referred to as the Gambler's Fallacy.

Why is this thinking incorrect?

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Problem 1

Two different tree diagrams are included in a worked example illustrating possible combinations of round and small one topping pizzas. Students rewrite the sample space as an organized list. Next, they analyze the sample space by answering questions related to the situation. Students conclude that the sample space in each tree diagram is the same in spite of the different order of the outcomes.

Grouping

- Ask students to read the information. Discuss as a class.
- Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3

- What is a tree diagram?
- How do you read a tree diagram?
- What information does a tree diagram provide?
- Is there more than one way to construct a tree diagram?
- How many levels are in each of the two tree diagrams?
- What does each of the levels in the tree diagrams represent?

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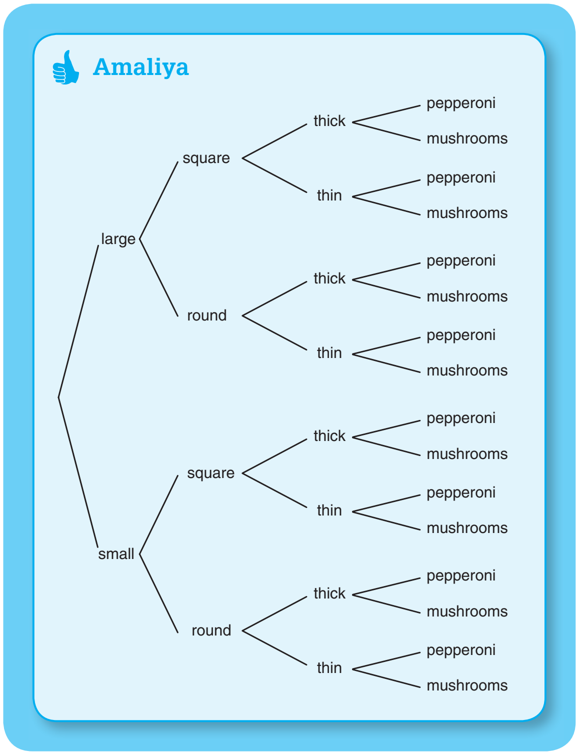
PROBLEM 1 Pizza Special

Mario's Pizzeria advertises special deals in the newspaper.

> Today's Special at Mario's Pizzeria
> Large one-topping pizza $9.00
> Small one-topping pizza $6.50
> Choose either a square or a round pizza with thick or thin crust.
> Available toppings: pepperoni or mushrooms
> Enjoy a fresh-baked pizza!!!

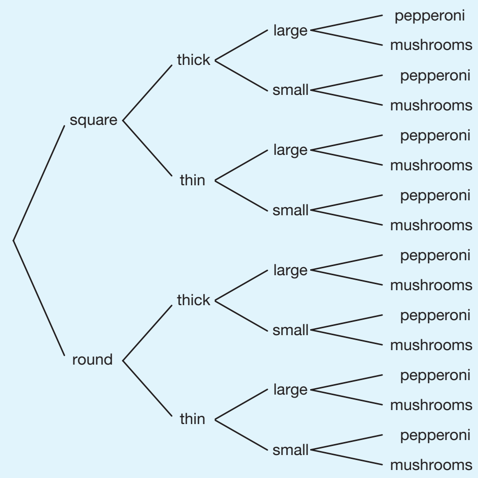
A **tree diagram** is a visual model for determining the sample space of multiple events.

Amaliya and Romeo sketched tree diagrams to show the possible pizza specials at Mario's Pizzeria.



- How many different ways could this tree diagram have been drawn?
- Do you notice a pattern emerging in your tree diagram with respect to the number of branches at each level?
- Does the sample space include the entire tree diagram?
- How did you determine the total number of outcomes in the tree diagram?

Romeo



Both tree diagrams show the same information, but each tree diagram is organized differently.

1. How many outcomes are included in the sample space of each tree diagram? Explain how you determined your answer.
   The sample space of each tree diagram contains 16 outcomes. I determined this by counting the number of branches in each tree diagram.

2. What does each outcome of the sample space represent?
   Each outcome in the sample space represents a type of one-topping pizza.

3. Compare the tree diagrams created by Amaliya and Romeo.

a. How are the tree diagrams similar and different?

Both tree diagrams show 16 total outcomes and both tree diagrams display the same varieties of pizzas. However, the tree diagrams display the same information using different types of organization.

b. Does the arrangement of the tree diagram affect the total number of possible outcomes? Explain why or why not.

No. The number of outcomes for each tree diagram is equal, but the organization of the tree diagrams is different. For example, if I order a large square pizza, this is the same as ordering a square pizza that is large.

The number of possible outcomes in the two tree diagrams doesn't change because I am choosing one item from each category: size, shape, thickness, and topping.

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Grouping

Have students complete Question 5 with a partner. Then have students share their responses as a class.

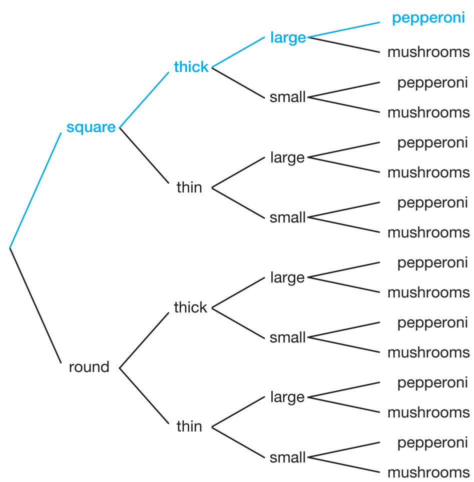
Guiding Questions for Share Phase, Question 5

- What is an organized list?
- How is an organized list constructed?
- Does an organized list give you the same information as the tree diagram?
- Is it easier to answer questions about the sample space using the organized list or the tree diagram? Explain.

An **organized list** is a visual model for determining the sample space of events.

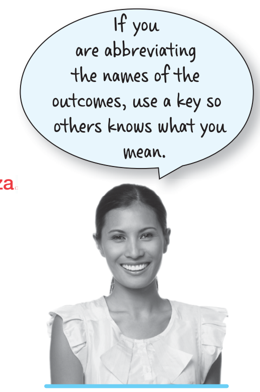
4. Use the tree diagram to write an organized list that displays all the possible pizza specials at Mario's Pizza. One entry is listed to help you get started.

Answers will vary.



Sq-Thick-L-P
Sq-Thick-L-M
Sq-Thick-S-P
Sq-Thick-S-M
Sq-Thin-L-P
Sq-Thin-L-M
Sq-Thin-S-P
Sq-Thin-S-M
R-Thick-L-P
R-Thick-L-M
R-Thick-S-P
R-Thick-S-M
R-Thin-L-P
R-Thin-L-M
R-Thin-S-P
R-Thin-S-M

Key:
S (Small), L (Large), Sq (Square), R (Round), P (pepperoni), M (mushrooms)



5. Analyze the sample space to answer each question.

a. How many possible round pizza specials can you order?

There are 8 possible outcomes that include a round pizza

R-Thick-L-P	R-Thick-S-P
R-Thick-L-M	R-Thick-S-M
R-Thin-L-P	R-Thin-S-P
R-Thin-L-M	R-Thin-S-M

b. How many possible thick and round pizza specials can you order?

There are 4 possible outcomes that include a round and thick pizza.

R-Thick-L-P	R-Thick-S-P
R-Thick-L-M	R-Thick-S-M

c. How many of the possible pizza choices are square but do not have mushrooms?

There are 4 possible outcomes that include a square pizza that does not have mushrooms.

Sq-Thick-L-P	Sq-Thick-S-P
Sq-Thin-L-P	Sq-Thin-S-P

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Problem 2

Five candidates are running for president and vice-president of student council. Students create a tree diagram listing the possible outcomes then they rewrite the sample space as an organized list. Students analyze the sample space by answering questions related to the situation.

Grouping

Have students complete Questions 1 through 7 with a partner. Then have students share their responses as a class.

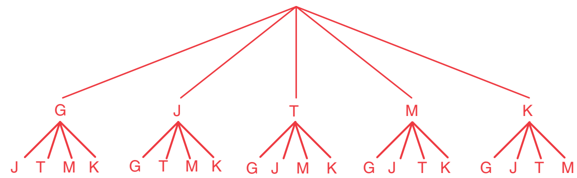
Guiding Questions for Share Phase, Questions 1 through 7

- Is there more than one way to construct the tree diagram?
- Do you notice a pattern emerging in your tree diagram with respect to the number of branches at each level?
- Was it easier to create the tree diagram or the organized list?
- Is the order important when determining the total number of outcomes in this situation?
- If Joshua is elected president, is he considered for the election of vice president as well?

PROBLEM 2 Student Council Election

Gwen, Joshua, Tamara, Miguel, and Khalil are running for student council offices. The student with the greatest number of votes is elected president, and the student receiving the next greatest number of votes is elected vice-president.

1. Create a tree diagram to represent the possible outcomes for the election.



Key:

G (Gwen), J (Joshua), T (Tamara), M (Miguel), K (Khalil)

2. What does each level in your diagram represent? Does order matter? Explain your reasoning.

The tree diagram has two rows because I am determining the sample space for electing a president and a vice-president. There are five student members listed, but only two will fill the presidential and vice-presidential positions.

- Can one person be both president and vice president in an organization at the same time?
- If Joshua receives the most votes, how many candidates are possible for the position of vice president?

3. Write the sample space as an organized list.

GJ, GT, GM, GK,

JG, JT, JM, JK,

TG, TJ, TM, TK,

MG, MJ, MT, MK,

KG, KJ, KT, KM

Key:

G (Gwen), J (Joshua), T (Tamara), M (Miguel), K (Khalil)

4. Analyze the sample space to answer each question.

   a. How many outcomes result in Gwen being elected president or vice-president?

   There are 8 possible outcomes in which Gwen is elected either president or vice-president.

   GJ, GT, GM, GK, JG, TG, MG, KG

   b. How many outcomes result in Tamara not being elected president?

   There are 16 possible outcomes in which Tamara is not elected president.

   GJ, GT, GM, GK,
   JG, JT, JM, JK,
   MG, MJ, MT, MK,
   KG, KJ, KT, KM

   c. How many outcomes result in Joshua or Khalil as president and Tamara or Gwen as vice-president?

   There are 4 possible outcomes for electing Joshua or Khalil president and Tamara or Gwen vice-president.

   JG, JT, KG, KT

5. The sample space includes both JM and MJ as outcomes. Explain the difference between these two outcomes.

JM means Joshua is president and Miguel is vice-president.

MJ means Miguel is president and Joshua is vice-president.

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6. Can you rearrange the tree diagram to produce a different number of total possible outcomes for the election? Explain why or why not.

No. Rearranging the tree diagram does not affect the number of possible election outcomes.

7. Consider your sample spaces from Problems 1 and 2.

   a. Does choosing a topping for a pizza at Mario's Pizzeria affect any of the other choices you have? Does choosing a size affect any of the other choices? Explain your reasoning.

   No. Choosing a particular topping or size does not affect choices from the other categories. For example, if I choose a large pizza, I can still choose thick or thin, square or round, and pepperoni or mushroom.

   b. Does electing the student council president affect the choices for vice-president? Explain your reasoning.

   Yes. Electing a president does affect the choices for vice-president because the person elected president cannot also be elected vice-president.

Problem 3

A cafeteria menu details possible entrées, side dishes, and desserts. Students answer a variety of questions which include creating a tree diagram for a specified entrée, and taking shortcuts to determine the total number of possible outcomes. Students conclude when the number of items in each set of the sample space is multiplied together, they can calculate the total number of possible outcomes.

Grouping

- Ask students to read the information. Discuss as a class.
- Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3

- How many entrées are listed?
- How many side dishes are listed?
- How many desserts are listed?
- How many levels are in your tree diagram?
- What do the levels represent in your tree diagram?
- Is there more than one way to construct the tree diagram?

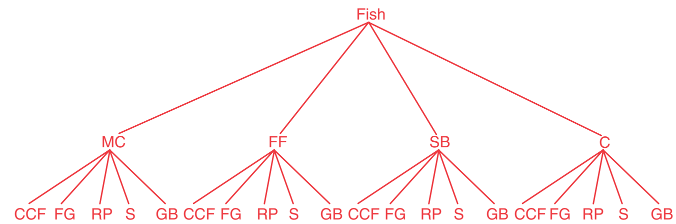
PROBLEM 3 Cafeteria Choices

The Friday lunch menu is available for Franklin Middle School.

Entrée	Side Dish	Dessert
fish sandwich	macaroni and cheese	cottage cheese and fruit
grilled cheese sandwich	french fries	fruit gelatin
hamburger	string beans	rice pudding
hot dog	carrots	strawberries
chicken nuggets		granola bar
pizza		

Each student chooses one entrée, one side dish, and one dessert to make their lunch.

1. How many lunches are possible with the fish sandwich as the entrée? Use a tree diagram to help you determine the answer.



There are 20 different lunches with the fish sandwich as the entrée.

Key:

MC (macaroni and cheese), FF (French fries), SB (string beans)

C (carrots), CCF (cottage cheese and fruit), FG (fruit gelatin)

RP (rice pudding), S (strawberries), GB (granola bar)

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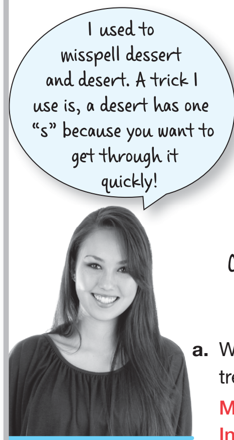
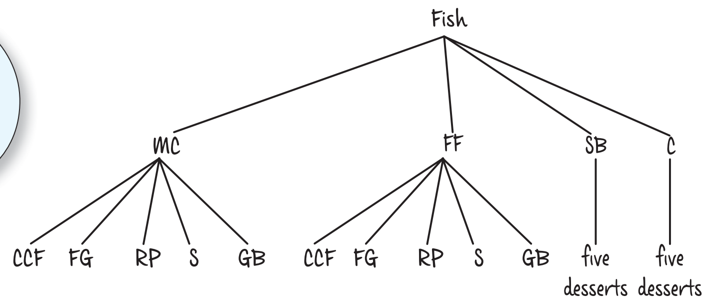
- Do you notice a pattern emerging in your tree diagram with respect to the number of branches at each level?
- How did you determine the total number of possible lunch meal combinations for the Friday menu?
- Why would it be cumbersome to create an organized list of the sample space?
- Does Paula's pattern use the number of branches in the tree diagram or the number of levels in the tree diagram?
- What pattern did Paula discover?

2. Maurice made the tree diagram shown for all the lunches with the fish sandwich as the entrée.





a. What differences do you notice between your tree diagram and Maurice's tree diagram?

Maurice didn't write all the different dessert items for string beans and carrots. Instead, Maurice wrote "five desserts" underneath string beans and "five desserts" underneath carrots.

b. Why do you think Maurice wrote "five desserts" instead of writing out the dessert combinations on his tree diagram?

Maurice may have seen a pattern in the tree diagram. Instead of writing out the 5 dessert options underneath SB and C, he may have written "five desserts" to save time.

c. Do you notice a pattern in your tree diagram? Explain your reasoning.

Yes. There are 5 dessert combinations for each of the side dishes. In my tree diagram, I noticed that there are 4 branches to represent the side dishes. Each of the side dishes split into 5 branches to represent the desserts. Maurice may have recognized that there was a pattern between each side dish and the number of desserts.

d. The fish sandwich has 4 possible side dishes and 5 possible desserts. Do you think this pattern will be the same for a grilled cheese sandwich? How many meal outcomes are possible when grilled cheese is the entrée? Explain your reasoning.

Yes. The pattern seems to work when the entrée is a grilled cheese sandwich. There are 20 possible meal outcomes with grilled cheese as the entrée because the side dish and dessert choices are the same as for the fish sandwich.

e. How many lunches are possible for the Friday menu? Explain your reasoning.

There are 120 possible meal outcomes for the Friday menu. I recognized that there is a pattern, and that for each entrée, there are 20 outcomes. Because there are 6 entrées, I multiplied the 20 outcomes by the 6 entrées to get an answer of 120 total meal outcomes.

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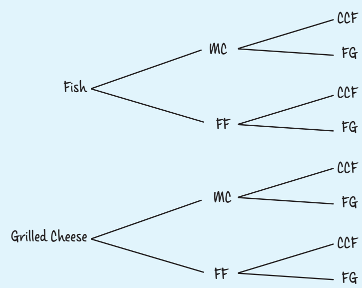
3. Paula created the tree diagram shown to determine all the possible meal outcomes.

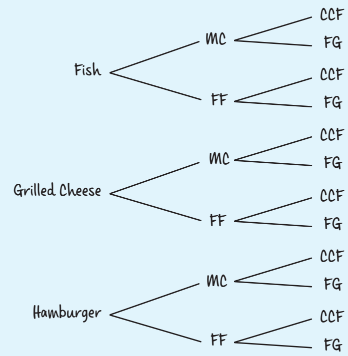
First, I pretended that there were just 2 choices for entrée, side dish, and dessert.



Entrée	Side Dish	Dessert	
2 choices	2 choices	2 choices	

2 × 2 × 2 = 8

Then, I pretended that there were 3 entrée choices, 2 side dish choices, and 2 dessert choices.



There are 12 outcomes.

Entrée	Side Dish	Dessert
3 choices	2 choices	2 choices

3 × 2 × 2 = 12

My pattern seems to work. So, for this problem,

Entrée	Side Dish	Dessert
6 choices	4 choices	5 choices

6 × 4 × 5 = 120

There are 120 different lunch possibilities.

The strategy I used was to make it a smaller problem to notice a pattern. Then I used the pattern to calculate all the possible meal outcomes!

Explain Paula's method for calculating the total number of possible meal outcomes. How did Paula's reasoning help her determine the number of meal outcomes?

Paula used a smaller number of choices for each category which made it more efficient to analyze different scenarios. She did not include all the food choices in the original problem. She tried two different problems and realized that if she multiplied the number of items in each set, she would be able to calculate the total number of possible combinations.

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Problem 4

Four flavors of frozen yogurt can be combined with two kinds of fruit and three additional toppings to create a frundae. Students create an organized list of the sample space to determine the total number of possible outcomes. Students continue to calculate the total number of possible outcomes, given certain parameters. In one situation repetition is allowed and this sample space is compared to the previously determined sample spaces.

Grouping

Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 5

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- How many flavors of yogurt are sold?
- How many choices of fruit are offered?
- How many toppings are available?
- If a tree diagram was created, how many levels would it have?
- What would each level represent?
- If a tree diagram was created, how many branches would it have?

PROBLEM 4 Stan's Frozen Yogurt

Stan's Frozen Yogurt Shop offers frundaes—frozen yogurt sundaes. The shop advertises different frundae options for customers.

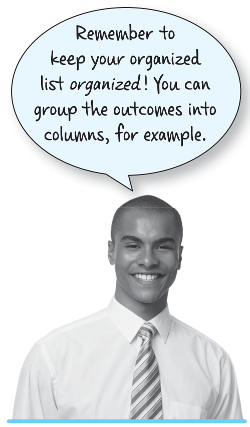
BUILD YOUR OWN FRUNDAE
Choose one yogurt flavor, fruit, and topping.

Frozen Yogurt Flavors	*Fruit*	*Toppings*
vanilla	bananas	nuts
chocolate	cherries	sprinkles
strawberry		granola
peach		

1. Write the sample space of different frundae options consisting of one yogurt flavor, one fruit, and one topping using an organized list.

V, B, N	Co, B, N	Sa, B, N	P, B, N
V, B, Sp	Co, B, Sp	Sa, B, Sp	P, B, Sp
V, B, G	Co, B, G	Sa, B, G	P, B, G
V, Ce, N	Co, Ce, N	Sa, Ce, N	P, Ce, N
V, Ce, Sp	Co, Ce, Sp	Sa, Ce, Sp	P, Ce, Sp
V, Ce, G	Co, Ce, G	Sa, Ce, G	P, Ce, G

Key:
V (vanilla), Co (chocolate), Sa (strawberry),
P (peach), B (bananas), Ce (cherries),
N (nuts), Sp (sprinkles), G (granola)



2. How many different frundaes can be created that contain one frozen yogurt flavor, one fruit, and one topping
There are 24 different frundaes in the sample space.

- What would each branch represent?
- Would it have been easier to construct a tree diagram rather than create an organized list? Explain.
- When Alec and Ella order a single-flavor frozen yogurt cone, do you see a pattern emerging in the organized list?

3. Alec and Ella each order a single-flavor frozen yogurt cone with no fruit or toppings.

a. Write an organized list for the sample space of the single-flavor frozen yogurt cones Alec and Ella could buy. Describe the sample space.

For each possible order, Alec's choice is listed first and Ella's choice is listed second.

V, V	Co, V	Sa, V	P, V
V, Co	Co, Co	Sa, Co	P, Co
V, Sa	Co, Sa	Sa, Sa	P, Sa
V, P	Co, P	Sa, P	P, P

Key:
V (vanilla), Co (chocolate), Sa (strawberry), P (peach)

b. How many outcomes are in the sample space? How did you determine the answer?

There are 16 outcomes in the sample space. I determined the answer by counting the outcomes in the organized list.

4. A Triple-Decker Froyanza is a frozen yogurt cone with three servings of yogurt. Tamara orders a Triple-Decker Froyanza with one serving of chocolate, one serving of vanilla, and one serving of strawberry frozen yogurt.

a. Write an organized list for the sample space that represents the order in which the server could stack the three servings of frozen yogurt on Tamara's cone.

Co, V, Sa	V, Co, Sa	Sa, Co, V
Co, Sa, V	V, Sa, Co	Sa, V, Co

Key:
V (vanilla), Co (chocolate), Sa (strawberry)

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b. How many different stacking orders are possible for the frozen yogurt flavors of Tamara's Triple-Decker Froyanza? Explain how you determined the answer.

There are 6 different ways to stack the frozen yogurt flavors. I determined the answer by counting the outcomes in the organized list.

5. Compare and contrast the sample spaces from Questions 1, 3, and 4.

a. How are the sample spaces alike?

All three sample spaces include outcomes of various frozen yogurt orders.

b. How is Question 1 and its sample space different from Question 3 and its sample space?

In Question 1, each outcome in the sample space consists of a yogurt flavor, a fruit, and a topping. In Question 3, each outcome in the sample space consists of two yogurt flavors. The first flavor is Alec's choice and the second flavor is Ella's choice.

c. How is Question 4 and its sample space different from Questions 1 and 3 and their sample spaces?

In Question 4, the sample space involves the same three frozen yogurt flavors represented by letters put in different orders. The question is about all the possible ways to stack the three yogurt flavors in a cone. In Question 1, the combinations of letters represent the three categories on the frundae menu. In Question 3, the combinations of letters in the sample space represent Alec's and Ella's single-flavor frozen yogurt choices.

Problem 5

Independent events, disjoint sets, and dependent events are defined. Students are referred to the previous problem to determine which situations could be considered as independent events from disjoint sets and independent events from one set. Students are also referred to the previous problem to site an example of dependent events from one set that cannot repeat.

Grouping

- Ask students to read the information. Discuss as a class.
- Have students complete Questions 1 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- What is the difference between disjoint sets and intersecting sets?
- What type of probability scenarios involve disjoint sets?
- What type of probability scenarios involve intersecting sets?

PROBLEM 5 Categorize Problems Involving Events

A **set** is a collection of items. If *x* is a member of set *B*, then *x* is an **element** of set *B*.

Two or more sets are **disjoint sets** if they do not have any common elements.

Two or more sets are **intersecting sets** if they do have common elements.

Consider the following examples of disjoint sets and intersecting sets.

Disjoint Sets:
Let N represent the set of 9th grade students. Let T represent the set of 10th grade students.

The set of 9th grade students and the set 10th grade students are disjoint because the two sets do not have any common elements. Any student can be in one grade only.

Intersecting Sets:
Let V represent the set of students who are on the girls' volleyball team. Let M represent the set of students who are in the math club. Julia is on the volleyball team and belongs to the math club.

The set of students who are on the girls' volleyball team and the set of students who are in the math club are intersecting because we know they have at least one common element, Julia.

1. Identify the sets in each scenario in problems 1 through 4. Then, determine whether the sets are disjoint or intersecting. Explain your reasoning.

   a. Problem 1: Pizza Special

   The Pizza Special scenario has 4 sets: pizza size, pizza shape, type of crust, and toppings.

   The sets are disjoint because they do not have any common elements.

   b. Problem 2: Student Council Election

   The Student Council Election scenario has 2 sets: presidential candidates and vice-presidential candidates.

   The sets are intersecting because they have common elements. The set of presidential candidates contains Gwen, Joshua, Tamara, Miguel, and Khalil. The set of vice-presidential candidates contains the 4 people who are not elected president.

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c. Problem 3: Cafeteria Choices

The Cafeteria Choices scenario has 3 sets: entrée, side dish, and desserts.

The sets are disjoint because they do not have any common elements.

d. Problem 4: Stan's Frozen Yogurt

Stan's Frozen Yogurt scenario has 3 sets: frozen yogurt flavors, fruit, and toppings.

The sets are disjoint because they do not have any common elements.

Independent events are events for which the occurrence of one event has no impact on the occurrence of the other event.

Dependent events are events for which the occurrence of one event has an impact on the occurrence of subsequent events.

Consider the following examples of independent events and dependent events.

A jar contains 1 blue marble, 1 green marble, and 2 yellow marbles.

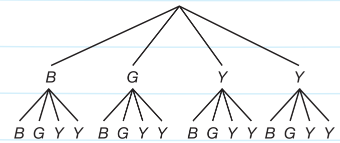
Independent Events:

You randomly choose a yellow marble, replace the marble in the jar, and then randomly choose a yellow marble again.

The event of choosing a yellow marble 1st does not affect the event of choosing a yellow marble 2nd because the yellow marble chosen 1st is replaced in the jar.

The events of randomly choosing a yellow marble first and randomly choosing a yellow marble 2nd are independent events because the 1st yellow marble was replaced in the jar.

You can see this visually using a tree diagram.



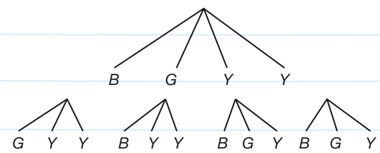
Dependent Events:

A jar contains 1 blue marble, 1 green marble, and 2 yellow marbles. You randomly choose a yellow marble without replacing the marble in the jar, and then randomly choose a yellow marble again.

The event of choosing a yellow marble 1st does affect the event of choosing a yellow marble 2nd because the yellow marble chosen 1st is not replaced in the jar.

The events of randomly choosing a yellow marble first and randomly choosing a yellow marble 2nd are dependent events because the 1st yellow marble was not replaced in the jar.

You can see this visually using a tree diagram.



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Grouping

Have students complete Questions 2 and 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 2 and 3

- What is the difference between independent events and dependent events?
- How can you determine whether or not two events are independent or dependent?

2. Sia made the following statement.

Sia

I think that choosing the first marble and choosing the second marble are dependent events.

Why is Sia's statement incorrect? Explain your reasoning.

Sia's statement is incorrect because choosing the first marble and choosing the second marble are actions, not events. Choosing a yellow marble first and choosing a yellow marble second are events. In general, events are more specific than actions.

One way to correct Sia's statement is to say, "The outcomes for choosing the second marble are affected by the outcomes of choosing the first marble."

3. Identify one outcome from each scenario in Problems 1 through 4. Then, state whether the events that result in that outcome are independent or dependent. Explain your reasoning.

a. Problem 1: Pizza Special

One possible outcome in the Pizza Special scenario is a large, square, thick crust pizza with mushrooms.

The 4 individual events of choosing a large size, choosing square shape, choosing a thick crust, and choosing mushroom toppings are independent because the occurrence of one event does not affect the occurrence of the other events.

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b. Problem 2: Student Council Election

One possible outcome in the Student Council Election scenario is electing Gwen president and Miguel vice-president.

The 2 individual events of electing Gwen president and electing Miguel vice-president are dependent because the event of electing a president affects the event of electing a vice-president. If Gwen is president, she cannot also be vice-president.

c. Problem 3: Cafeteria Choices

One possible outcome in the Cafeteria Choices scenario is a meal with chicken nuggets, carrots, and rice pudding.

The 3 individual events of choosing chicken nuggets, choosing carrots, and choosing rice pudding are independent because the occurrence of one event does not affect the occurrence of the other events.

d. Problem 4: Stan's Frozen Yogurt

One possible outcome in Stan's Frozen Yogurt scenario is a frundae with strawberry frozen yogurt, bananas, and granola.

The 3 individual events of choosing strawberry frozen yogurt, choosing bananas, and choosing granola are independent because the occurrence of one event does not affect the occurrence of the other events.

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Problem 6

The Counting Principle is stated. Students are given situations and they determine if the situation involves independent events from disjoint sets, independent events from the same set with repetitions allowed, or dependent events from the same set without repetitions. They use the Counting Principle to determine the size of the sample spaces for different types of events.

Grouping

- Ask students to read introduction. Discuss as a class.
- Have students complete Questions 1 through 7 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 7

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- How many rides are Devin's favorite rides?
- Can Devin ride the Bungee-Buggy two times in a row? Explain.
- Does this situation imply repetition or is it considered not repeating? Why?
- Does the second ride selection depend on the first ride selection?
- What information do you need to know to use the Counting Principle?

PROBLEM 6 The Counting Principle

To determine the total number of possible lunches in Problem 3, Paula used a mathematical principle called the *Counting Principle*. The principle is used to determine the number of outcomes in the sample space.

The **Counting Principle** states that if an action A can occur in m ways and for each of these m ways an action B can occur in n ways, then actions A and B can occur in $m \cdot n$ ways.

The Counting Principle can be generalized to more than two actions that happen in succession. If for each of the m and n ways A and B occur there is also an action C that can occur in s ways, then Actions A, B, and C can occur in $m \cdot n \cdot s$ ways.

1. Devin has an all-day pass for Scream amusement park. His favorite rides are the Bungee-Buggy, Head Rush roller coaster, Beep Beep go-karts, and Tsunami Slide water roller coaster. He never rides any other rides, and he can ride each of his favorite rides as many times as he wants.

   a. Describe the type of event for selecting ride order. Does this situation involve independent events from disjoint sets, independent events from the same set with repetitions allowed, or dependent events from the same set without repetitions? Explain your reasoning.

   The situation involves independent events from the same set. They are independent because one ride selection does not affect the next ride selection. Devin can keep riding the same ride, so repetition is allowed. Lastly, all choices are from the same set, Devin's 4 favorite rides.

   b. Use the Counting Principle to determine the number of possible ride orders for Devin's next two rides. Explain your reasoning.

   $4 \cdot 4 = 16$

   There are 16 different possible ride orders for Devin's next two rides. To calculate the answer, I multiplied 4 times 4 because he has 4 choices for the first ride and 4 choices for the second ride.

   c. How many ride order possibilities are there for Devin's next five rides?

   $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$

   There are 1024 different possible ride orders for Devin's next five rides. To calculate the answer, I multiplied $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ because he has 4 choices for the 1st ride, 4 choices for the 2nd ride, 4 choices for the 3rd ride, 4 choices for the 4th ride, and four choices for the 5th ride.

- How would you go about creating an organized list for this situation?
- How would you go about constructing a tree diagram for this situation?
- Does Sherry's second program selection depend on her first program selection?
- Are Sherry's choices from disjoint sets or intersecting sets?
- What information do you need to apply the Counting Principle?
- How many different classes does a student attend each day?

- When scheduling a student, does the subject chosen for the first period of the day affect the subject possibilities for the second period of the day?
- Are repetitions possible in this situation?
- Can the Counting Principle be used in this situation?
- What information is needed to use the Counting Principle?

2. Sherry stayed home from school Wednesday because she was ill. She watched a television program from 12:00 p.m. until 12:30 p.m., and another program from 12:30 p.m. until 1:00 p.m. From 12:00 p.m. until 12:30 p.m., her program choices were the news, cartoons, or a talk show. From 12:30 p.m. until 1:00 p.m., her program choices were a comedy, a soap opera, a game show, or a cooking show.

a. Describe the type of event for Sherry's television program selections. Explain your reasoning.

This situation involves independent events from disjoint sets. They are independent because Sherry's choice for the first television program does not affect her choice for the second program. The two choices are from different sets because the television programs offered at 12:00 p.m. are not the same as television programs offered at 12:30 p.m.

b. How many program selections can Sherry watch from 12:00 p.m. until 1:00 p.m?

$3 \cdot 4 = 12$

There are 12 different program selections Sherry could watch between 12:00 p.m. and 1:00 p.m. To calculate the answer, I multiplied 3 times 4 because Sherry has 3 choices for the first program and 4 choices for the second program.

3. A student's daily schedule includes math, English, science, social studies, foreign language, art, and physical education. Students are enrolled in each class for one period per day.

a. Describe the type of event for creating a student's daily schedule. Explain your reasoning.

This situation involves dependent events from the same set without repetitions. The situation is dependent because once a class is taken, a student cannot take the class a second time within the same day. The events are from the one set of 7 classes.

b. Determine how many different orders the classes can be arranged to fill a seven-period daily schedule.

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

There are 5040 different ways to arrange the schedule.

To calculate the answer I multiplied $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ because there are 7 choices for the 1st period class, 6 choices for the 2nd period class, 5 choices for the 3rd period class, etc. The number of choices decreases by one because each class is scheduled one time. After a class is scheduled the total number of class choices that still need to be scheduled decreases by one.

c. Lunch period is directly after fourth period. How many different class schedule arrangements are possible before lunch period? Explain your reasoning.

$7 \cdot 6 \cdot 5 \cdot 4 = 840$

There are 840 different ways to arrange the classes before lunch.

To calculate the answer I multiplied $7 \cdot 6 \cdot 5 \cdot 4$ because there are 7 choices for the 1st period class, 6 choices for the 2nd period class, 5 choices for the 3rd period class, and 4 choices for the 4th period class. I multiplied 4 numbers because there are 4 periods before lunch. The number of choices decreases by one because each class is scheduled one time. After a class is scheduled the total number of class choices that still need to be scheduled decreases by one.

4. The cell phone PIN to access voicemail is a 4-digit number. Each digit can be a number from 0 to 9, including 0 and 9. How many 4-digit numbers are possible? Repetition of numbers is allowed. Explain your calculation.

$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$

There are 10,000 possible 4-digit PINs.

To calculate the answer I multiplied $10 \cdot 10 \cdot 10 \cdot 10$ because there are 10 choices for each digit. There are 10 choices for each digit because the numbers in each digit of the PIN can be repeated.

5. If repeating digits is not permitted, how many different 4-digit PINs are possible?

$10 \cdot 9 \cdot 8 \cdot 7 = 5040$

There are 5040 possible 4-digit PINs. To calculate the answer I multiplied $10 \cdot 9 \cdot 8 \cdot 7$ because the number of choices for each successive digit decreases by one due to the no repetition restriction.

6. A typical license plate number for a car consists of three letters followed by four numbers ranging from 0 through 9, including 0 and 9. How many different license plates numbers are possible if letters and numbers can be repeated? Explain your calculation.

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 175{,}760{,}000$

There are 175,760,000 different license plate numbers.

To calculate the answer I multiplied $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ because there are 26 choices for each of the three letters and 10 choices for each of the the four digits.

7. How many different 3-letter, 4-digit license plate numbers are possible if letters and digits cannot be repeated?

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 78{,}624{,}000$

There are 78,624,000 different license plate numbers.

To calculate the answer I multiplied $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$ because each time a letter/number is selected, there is one less letter/number choice for the next letter/digit.

Talk the Talk

Students summarize the big ideas in the lesson by completing a chart and answering questions.

Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

Talk the Talk

The Counting Principle is used to determine the number of outcomes in a sample space. The calculations vary depending upon the type of situation.

1. For each given type of event and type of set, write a scenario and sample space calculation that represents the type of event and type of set.

   Answers will vary.

   Example responses are displayed.

Type of Events (independent or dependent)	Type of Sets (disjoint or intersecting)	Scenario	Sample Space Calculation
independent	three disjoint sets	How many different ways can Marianne accessorize her outfit using a belt, a bracelet, and a pair of shoes? She selects from a set of 3 different belts, a set of 3 different bracelets, and a set of 2 different pairs of shoes.	$3 \cdot 3 \cdot 2 = 18$
dependent	intersecting sets	In how many different orders can Timmy complete his homework for tonight? He has homework assignments from one set of 3 different classes, math, science, and history.	$3 \cdot 2 \cdot 1 = 6$
dependent	two disjoint sets	How many different styles of glasses can Tracy choose from? She selects from one set of 4 different frame designs, and from another set of 2 different frame materials.	$4 \cdot 2 = 8$

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2. Efi and Areti each describe a scenario with a sample space of 4 · 3 · 2 = 24.

Efi

The scenario has independent events and disjoint sets.

Areti

The scenario has dependent events and intersecting sets.

a. Based on Efi's description, write an example scenario involving 3 disjoint sets and 24 outcomes in the sample space.

Answers will vary.

Arnold is going to take 1 towel, 1 pair of sunglasses, and 1 pair of flip flops to the beach. He has 4 towels, 3 pairs of sunglasses, and 2 pairs of flip flops to choose from. How many outcomes are in the sample space?

b. Based on Areti's description, write an example scenario involving 3 intersecting sets and 24 outcomes in the sample space.

Answers will vary.

Four friends are racing to the end of the block. The friends' names are George, Vasilis, Vageli, and Chris. Nobody wants to finish last. How many outcomes are in the sample space for the top 3 finishers?

3. Describe how to calculate the number of outcomes in the sample space of situations involving either independent or dependent events.

For probability situations involving either independent or dependent events, I can use the Counting Principle to determine the number of outcomes in the sample space. The Counting Principle states that if an action A can occur in m ways and for each of these m ways, an action B can occur in n ways, then Actions A and B can occur in $m \cdot n$ ways. I multiply the number of ways one action can occur by the number of ways a second, third, etc., action can occur.

4. List advantages and disadvantages of using a tree diagram or an organized list to represent a sample space.

Answers will vary.

A tree diagram takes up more room, and it can be inefficient when there is a large number of actions.

An organized list can take up less space than a tree diagram. With an organized list, it's less obvious that an outcome or outcomes are missing.

Be prepared to share your solutions and methods.

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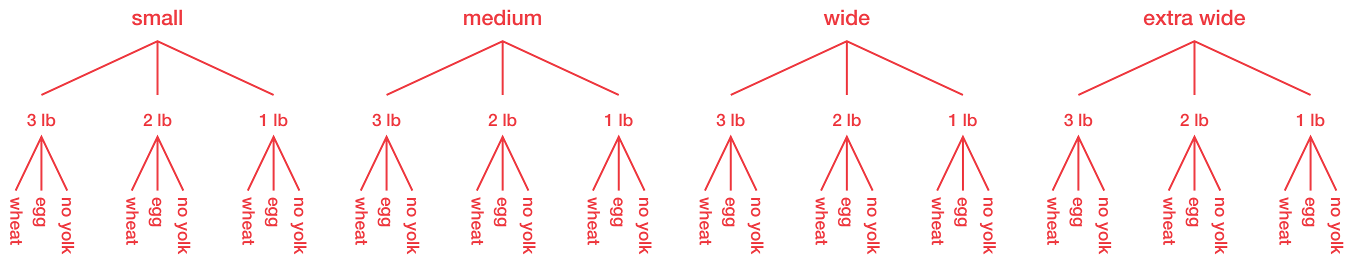
Check for Students' Understanding

The grocery store sells a variety of noodles. The possible sizes of the noodles are small, medium, wide, and extra-wide. The noodles are sold in 1 pound, 2 pound, or 3 pound bags. The noodles are also classified as no yolk, egg, and wheat.

1. Draw a tree diagram describing the variety of noodles available at the grocery store.



2. Create an organized list of the sample space.

S-1 lb – no yolk	S-1 lb – egg	S-1 lb – wheat
S-2 lb – no yolk	S-2 lb – egg	S-2 lb – wheat
S-3 lb – no yolk	S-3 lb – egg	S-3 lb – wheat
M-1 lb – no yolk	M-1 lb – egg	M-1 lb – wheat
M-2 lb – no yolk	M-2 lb – egg	M-2 lb – wheat
M-3 lb – no yolk	M-3 lb – egg	M-3 lb – wheat
W-1 lb – no yolk	W-1 lb – egg	W-1 lb – wheat
W-2 lb – no yolk	W-2 lb – egg	W-2 lb – wheat
W-3 lb – no yolk	W-3 lb – egg	W-3 lb – wheat
EW-1 lb – no yolk	EW-1 lb – egg	EW-1 lb – wheat
EW-2 lb – no yolk	EW-2 lb – egg	EW-2 lb – wheat
EW-3 lb – no yolk	EW-3 lb – egg	EW-3 lb – wheat

S-Small
M-Medium
W-Wide
EW-Extra Wide

3. Use the Counting Principle to calculate the size of the sample space.

$4 \times 3 \times 3 = 36$

There are 36 possible combinations.

4. Which method do you prefer to compute the size of the sample space?

Answers will vary.

I prefer to use the Counting Principle because it a shortcut.

5. What might be a disadvantage to using the Counting Principle?

A disadvantage of using the Counting Principle is the lack of visual representation for each individual outcome in the sample space. In order to see each individual outcome, I would need to create a tree diagram or an organized list.

19.3

And?

Compound Probability with "And"

LEARNING GOALS

In this lesson, you will:

- Determine the probability of two or more independent events.
- Determine the probability of two or more dependent events.

KEY TERMS

- compound event
- Rule of Compound Probability involving "and"

ESSENTIAL IDEAS

- A compound event is an event that consists of two or more events.
- The Rule of Compound Probability Involving "and" states: If Event *A* and Event *B* are independent, then the probability that Event *A* happens and Event *B* happens is the product of the probability that Event *A* happens and the probability that Event *B* happens, given that Event *A* has happened. Using probability notation, the Rule of Compound Probability involving "and" is $P(A \text{ and } B) = P(A) \cdot P(B)$.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

S-CP Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data

2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

8. Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

Overview

Students determine the probability of two or more independent events and two or more dependent events. The Rule of Compound Probability Involving "and" is stated and used to compute compound probabilities. Several situations present students with the opportunities to construct tree diagrams, create organized lists, and compute the probabilities of compound events.

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Warm Up

A box contains individually wrapped pieces of bubblegum. The flavors include grape, cherry, apple, raspberry, and lime. There are 20 pieces of each flavor.

1. Suppose you randomly choose one piece of bubblegum. What is the probability that the piece will be cherry flavored?

   The probability that the piece of bubblegum will be cherry flavored is $\frac{20}{100} = \frac{1}{5}$.

2. Suppose you randomly choose one piece of bubblegum. What is the probability that the piece will be not be cherry flavored?

   The probability that the piece of bubblegum will not be cherry flavored is $\frac{80}{100} = \frac{4}{5}$.

3. Suppose you randomly choose a first piece, replace the piece and choose a second piece. What is the probability that the first and second piece will both be cherry flavored?

   The probability that both pieces of bubblegum will be cherry flavored is $\frac{20}{100} \cdot \frac{20}{100} = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$.

4. Suppose you randomly choose a first piece, replace the piece and choose a second piece. What is the probability that the first piece is cherry flavored and second piece will not be cherry flavored?

   The probability that the first piece of bubblegum will be cherry flavored and the second piece will not be cherry flavored is $\frac{20}{100} \cdot \frac{80}{100} = \frac{1}{5} \cdot \frac{4}{5} = \frac{4}{25}$.

5. Suppose you randomly choose a first piece, replace the piece and choose a second piece. What is the probability that the neither piece is cherry flavored?

   The probability that neither piece of bubblegum will be cherry flavored is $\frac{80}{100} \cdot \frac{80}{100} = \frac{4}{5} \cdot \frac{4}{5} = \frac{16}{25}$.

6. Are the events described in the previous questions considered independent events or dependent events?

   They are considered independent events because there is replacement. The flavor of the piece that is selected first does not affect the flavor of the piece selected second.

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19.3

And?

Compound Probability with "And"

LEARNING GOALS

In this lesson, you will:

- Determine the probability of two or more independent events.
- Determine the probability of two or more dependent events.

KEY TERMS

- compound event
- Rule of Compound Probability involving "and"

There are three doors. Behind one door is a prize. Behind the other two doors are donkeys. You choose one door. The game show host opens one of the doors that you did not choose to reveal a donkey. Then, the host asks you if you would like to stay on the door you chose or switch to the other unopened door. Should you stay or switch? Or does it matter?

This famous probability problem is known as the Monty Hall problem—named after the host of the game show *Let's Make a Deal*, which featured this problem.

Surprisingly, the answer is that you should switch! When you switch you have a $\frac{2}{3}$ chance of getting the prize. If you stay, you have only a $\frac{1}{3}$ chance.

Can you figure out why you should switch every time? What if you had 100 doors to choose from and, after you made your choice, the game show host opened 98 doors to reveal 98 donkeys?

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Problem 1

A new juice is composed of 1 part fruit juice and 1 part vegetable juice. The two sets include 4 different fruit juices and 5 different vegetable juices. Students create an organized list and construct a tree diagram to represent the possible flavor blends. They determine the events of choosing a fruit juice and choosing a vegetable juice to create a blend is an independent event. Several questions focus on determining the probability under specified circumstances. A compound event is defined, and students describe the events in this situation that make up a compound event. They explore why multiplication is performed in compound probability problems using the word “and”. Flipping coins constitutes a second situation in which students continue to determine compound probabilities. The Rule of Compound Probability Involving “and” is stated.

Grouping

- Ask students to read the introduction. Discuss as a class.
- Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

PROBLEM 1 Juice It Up

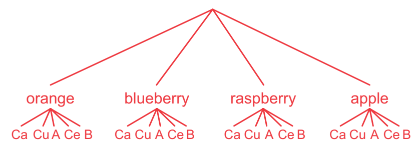
The NatureSwirl Juice Company plans to offer a new juice that features a fruit and vegetable blend. Customers will be able to select one of 4 fruits and one of 5 vegetables.

Fruit Choices: orange, blueberry, raspberry, apple

Vegetable Choices: carrot, cucumber, asparagus, celery, beet

1. Represent the possible flavor blends for this situation if one fruit and one vegetable are chosen.

Tree Diagram:



Organized List:

orange, Ca	blueberry, Ca	raspberry, Ca	apple, Ca
orange, Cu	blueberry, Cu	raspberry, Cu	apple, Cu
orange, A	blueberry, A	raspberry, A	apple, A
orange, Ce	blueberry, Ce	raspberry, Ce	apple, Ce
orange, B	blueberry, B	raspberry, B	apple, B

Key:
Ca = carrot
Cu = cucumber
A = asparagus
Ce = celery
B = beet

2. Does the action “choosing a fruit flavor” affect the outcomes of the action “choosing a vegetable flavor?” Explain your reasoning.

No. Choosing a fruit flavor does not affect the outcomes of choosing a vegetable flavor. No matter what fruit flavor I choose, I can still choose from all of the possible vegetable flavors.

Guiding Questions for Share Phase, Questions 1 through 3

- Did you use a tree diagram or an organized list to represent the possible flavor blends?
- How many berry flavors are there to choose from?
- What are the berry flavors?
- Do the berry flavors represent half of the flavor options?

3. If one fruit and one vegetable are chosen at random, what is the probability of choosing a juice that will include a berry flavor?

Matt and Kirsten used different methods to arrive at the correct answer. Their work is shown.

Kirsten

There are 20 possible outcomes in the sample space. Ten of those outcomes include a berry flavor—either blueberry or raspberry. So the probability of choosing a juice with a berry flavor is $\frac{2 \times 5}{4 \times 5} = \frac{10}{20} = \frac{1}{2}$.

orange, Ca	blueberry, Ca	raspberry, Ca	apple, Ca
orange, Cu	blueberry, Cu	raspberry, Cu	apple, Cu
orange, A	blueberry, A	raspberry, A	apple, A
orange, Ce	blueberry, Ce	raspberry, Ce	apple, Ce
orange, B	blueberry, B	raspberry, B	apple, B

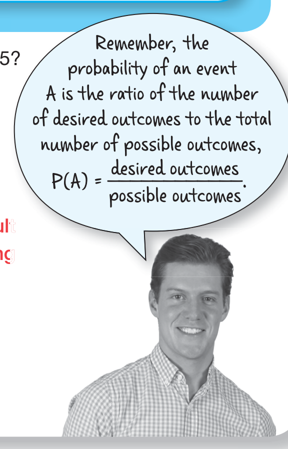
Key:
Ca = carrot   A = asparagus   Cu = cucumber   Ce = celery   B = beet

Matt

The probability of choosing a berry flavor is $\frac{1}{2}$, because there are 4 fruit flavors, and 2 of those fruit flavors (blueberry and raspberry) include a berry flavor: $\frac{2}{4} = \frac{1}{2}$. It doesn't matter what vegetable is chosen to go with the berry flavor, so the probability is $\frac{5}{5}$, or 1. The probability of choosing a juice with a berry flavor is $\frac{2}{4} \times \frac{5}{5} = \frac{1}{2} \times 1 = \frac{1}{2}$.

a. Why is the numerator shown in Kirsten's method 2×5?

The numerator in Kirsten's method represents the number of desired outcomes for choosing a juice with a berry flavor. She multiplied 2×5 because there are 2 berry flavors and 5 vegetable flavors. Kirsten used all 5 vegetable flavors because there is no restriction for which vegetable to use. The resul of her calculation is 10 desired outcomes of choosing a juice with a berry flavor.



b. Why is the denominator shown in Kirsten's method 4×5?

The denominator in Kirsten's method represents the total number of possible outcomes. She multiplied 4×5 because there are 4 fruit flavors and 5 vegetable flavors. The result is 20 possible outcomes for the fruit and vegetable blend.

c. Explain why Kirsten's expression $\frac{2 \times 5}{4 \times 5}$ and Matt's expression $\frac{2}{4} \times \frac{5}{5}$ both correctly represent the probability of choosing a juice with a berry flavor.

Matt used a different approach. He determined the probability of choosing a fruit that is a berry, the probability of choosing any vegetable, and then multiplied the two probabilities.

$P(\text{berry}) = \frac{2}{4}$ because 2 out of the 4 fruits are berries.

$P(\text{vegetable}) = \frac{5}{5}$ because any of the vegetables can be used.

$\frac{2}{4} \times \frac{5}{5} = \frac{1}{2} \times 1 = \frac{1}{2}$

d. Whose method do you prefer? Explain your reasoning.

Answers will vary.

I prefer Matt's method because it's cleaner and more efficient.

A **compound event** is an event that consists of two or more events.

4. Describe the events that make up the compound event in Question 3. Then list the outcomes for each event.

The first event is choosing a blueberry or raspberry fruit flavor. Both of these events are berry flavored outcomes.

The second event is choosing carrot, cucumber, asparagus, celery, or beet vegetable flavor. These are all of the possible vegetable flavor outcomes.

The compound event contains 10 outcomes.

blueberry-carrot	raspberry-carrot
blueberry-cucumber	raspberry-cucumber
blueberry-asparagus	raspberry-asparagus
blueberry-celery	raspberry-celery
blueberry-beet	raspberry-beet

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Grouping

Have students complete Questions 5 through 9 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 5 through 9

- What is the probability of choosing a vegetable that is not green?
- Can a compound event consist of two independent events?
- What is an example of a compound event that consists of two independent events?
- Can a compound event consist of two dependent events?
- What is an example of a compound event that consists of two dependent events?
- How is the Counting Principle related to the compound probability problem?
- Is the coin flipping situation considered a compound independent event or compound dependent event? Explain.
- What operation is associated with the Rule of Compound Probability Involving "and?"

5. What is the probability of choosing a juice blend that does not have a berry flavor and does have a green vegetable?

a. Let *A* represent the event of choosing a fruit that is not a berry flavor. What is *P*(*A*), the probability of choosing a fruit that is not a berry flavor?

$P(A) = \frac{2}{4} = \frac{1}{2}$ because 2 out of the 4 total fruits are not berry flavors: apple and orange.

b. Let *B* represent the event of choosing a green vegetable. What is *P*(*B*), the probability of choosing a green vegetable?

$P(B) = \frac{3}{5}$ because 3 out of the 5 total vegetables are green: cucumber, asparagus, and celery.

c. What is *P*(*A* and *B*), the probability of choosing a flavor that does not include a berry flavored fruit and does include a green vegetable? Explain how you determined your answer.

Answers will vary.

$P(A \text{ and } B) = \frac{3}{10}$

Using the Organized List:

By counting the outcomes in the organized list, I determined $P(A \text{ and } B) = \frac{6}{20} = \frac{3}{10}$.

Using Kirsten's Method:

I calculated the ratio of desired outcomes to total possible outcomes. The number of desired outcomes is 2 × 3 because 2 of the fruits are not berries and 3 of the vegetables are green. The number of total possible outcomes is 4 × 5 because there are 4 fruits and 5 vegetables to choose from.

$P(A \text{ and } B) = 2 \times \frac{3}{4} \times 5 = \frac{6}{20} = \frac{3}{10}$

Using Matt's Method:

I determined the probability of choosing a fruit that is not a berry, the probability of choosing a green vegetable, and then multiplied the two probabilities.

$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{2}{4} \cdot \frac{3}{5} = \frac{6}{20} = \frac{3}{10}$

6. What is the probability of choosing a juice blend that has a berry flavor and a vegetable that is not green?

   a. Let *A* represent the event of choosing a fruit that is a berry flavor. What is *P*(*A*), the probability of choosing a fruit that is a berry flavor?

      $P(A) = \frac{2}{4} = \frac{1}{2}$ because 2 out of the 4 total fruits are berry flavors: blueberry and raspberry.

   b. Let *B* represent the event of choosing a vegetable that is not green. What is *P*(*B*), the probability of choosing a vegetable that is not green?

      $P(B) = \frac{2}{5}$ because 2 out of the 5 total vegetables are not green: carrots and beets.

   c. What is *P*(*A* and *B*), the probability of choosing a juice blend that has a berry flavor and a vegetable that is not green?

      Answers will vary.

      $P(A \text{ and } B) = \frac{1}{5}$

      Using the Organized List:

      By counting the outcomes in the organized list, I determined $P(A \text{ and } B) = \frac{4}{20} = \frac{1}{5}$.

      Using Kirsten's Method:

      I calculated the ratio of desired outcomes to total possible outcomes. The number of desired outcomes is 2×2 because 2 of the fruits are berries and 2 of the vegetables are not green. The number of total possible outcomes is 4×5 because there are 4 fruits and 5 vegetables to choose from.

      $P(A \text{ and } B) = \frac{2 \times 2}{4 \times 5} = \frac{4}{20} = \frac{1}{5}$

      Using Matt's Method:

      I determined the probability of choosing a fruit that is a berry, the probability of choosing a vegetable that is not green, and then multiplied the two probabilities.

      $P(A \text{ and } B) = P(A) \cdot P(B) = \frac{2}{4} \cdot \frac{2}{5} = \frac{4}{20} = \frac{1}{5}$

7. Compare the previous problems. What relationship exists between *P*(*A*), *P*(*B*), and *P*(*A* and *B*)?

   The product of *P*(*A*) and *P*(*B*) is equal to *P*(*A* and *B*).

8. Why do you think that the probability of both events occurring is less than the probability of either event occurring by itself?

   The probability of both events occurring is less than the probability of either event occurring because the calculation for the probability of both events involves multiplying values that are both between 0 and 1, not including 0 and 1. When multiplying two probabilities with values between 0 and 1, not including 0 and 1, the result will always be less than the individual probabilities.

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Problem 2

A coin flipping scenario is presented. Questions guide students to discover that the the probability of compound event involving "and" with repeated trials can be determined by performing calculations using the specified events only. In such cases, it is not necessary to include the probability of each trial in the probability calculation.

Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3

- Explain how this scenario involves compound probability with "and."
- Part (c) asks to determine the probability of the 2nd coin landing tails up. What does this mean for the outcomes of the 1st and 3rd flips?
- In part (c), explain why it is not necessary to perform multiplication to answer the question.

9. Why do you think multiplication is performed in compound probability problems using the word "and"?

Using multiplication in compound probability problems using the word "and" is similar to using the Counting Principle for calculating outcomes. To calculate the probability of one event *and* another event, I can multiply the probability of one event by the probability of the other event.

PROBLEM 2 Flipping Coins

1. Suppose you flip 3 coins and record the result of each flip.

   a. What is the probability that all 3 coins land heads up?

   The probability that all 3 coins land heads up is $\frac{1}{8}$.

   $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

   b. What is the probability that the first 2 coins land heads up and the third coin lands tails up?

   The probability that the first 2 coins land heads up and the third coin lands tails up is $\frac{1}{8}$.

   $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

   c. What is the probability of the second coin landing tails up? Explain how you determined your answer.

   The probability of the second coin landing tails up is $\frac{1}{2}$.

   $\frac{2}{2} \times \frac{1}{2} \times \frac{2}{2} = 1 \times \frac{1}{2} \times 1 = \frac{1}{2}$

   Because the outcomes for the first and third coins are not specified, their probabilities are $\frac{2}{2}$, or 1.

   d. What is the probability of flipping heads on the first coin and tails on the third coin? Explain how you determined your answer.

   Only 2 of the coins affect the probability. The probability of flipping heads on the first coin and tails on the third coin is $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$.

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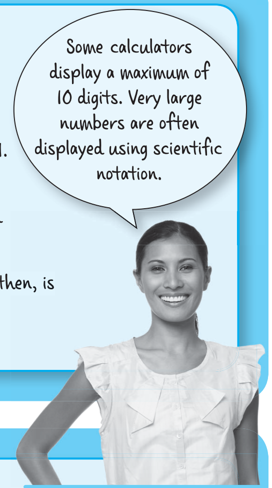
2. Suppose you flip 50 coins. What is the probability of flipping heads on the first and second coins? Derek and Lillian answered this question differently:

Derek

To calculate the number of total possible outcomes, I multiply 2 by itself 50 times, which is 2^{50}, or 1,125,899,906,842,624. There is 1 desired outcome for each of the first 2 coins, or 1×1. For the other 48 coins, there are 2 desired outcomes, because they can be heads or tails. This is 2^{48}. So, the number of desired outcomes is $1 \times 1 \times 2^{48}$, or 281,474,976,710,656.

The probability of flipping heads on the first and second coins, then, is $\frac{281,474,976,710,656}{1,125,899,906,842,624} = \frac{2^{48}}{2^{50}} = \frac{1}{2^2} = \frac{1}{4}$.



Lillian

I only need to do calculations with the probabilities for the first 2 coins because the probabilities for the other 48 coins are each $\frac{2}{2}$, or 1.

The probability of flipping heads on the first two of 50 coins is $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$.

3. Which strategy is more efficient? Why?

It is more efficient to use Lillian's strategy and only consider those actions which are specified to determine the probability. This makes the calculations simpler. Derek's strategy is correct but less efficient, especially because the numbers are very large.

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The **Rule of Compound Probability involving "and"** states: "If Event *A* and Event *B* are independent events, then the probability that Event *A* happens *and* Event *B* happens is the product of the probability that Event *A* happens and the probability that Event *B* happens, given that Event *A* has happened."

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Problem 3

A drawer contains 2 red socks, 1 blue sock and 3 green socks. At first, two socks are randomly chosen, one at a time, with replacement. Students create an organized list, and construct a tree diagram to fit the situation. Compound probabilities are computed using specified circumstances. The situation changes to no replacement and students continue to compute compound probabilities. The Counting Principle is used with independent events and dependent events.

Grouping

Have students complete Question 1 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- How many total socks are in the drawer?
- Are choosing the socks independent actions or dependent actions? Explain.
- What is the total number of choices for the first sock?
- What is the total number of choices for the second sock?

PROBLEM 3 One Sock, Two Sock, Red Sock, Blue Sock

You have 2 red, 1 blue, and 3 green socks in a drawer.

Suppose you reach into the drawer without looking and choose a sock, replace it, and then choose another sock. You choose a total of 2 socks.

1. Use this information to answer each question.

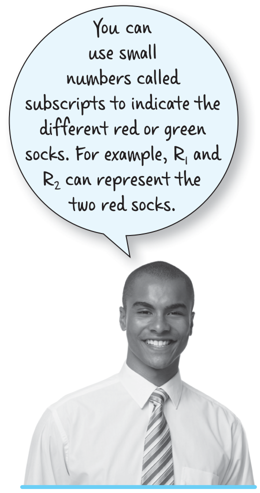
   a. Does the action "choosing the first sock" affect the outcomes of "choosing the second sock"? If so, how? Explain your reasoning.

   No. After I choose the first sock and replace it, there are still the same number of possible outcomes for the second sock.

   b. Use a tree diagram or organized list to represent the sample space for this situation.

R_1R_1	R_2R_1	BR_1	G_1R_1	G_2R_1	G_3R_1
R_1R_2	R_2R_2	BR_2	G_1R_2	G_2R_2	G_3R_2
R_1B	R_2B	BB	G_1B	G_2B	G_3B
R_1G_1	R_2G_1	BG_1	G_1G_1	G_2G_1	G_3G_1
R_1G_2	R_2G_2	BG_2	G_1G_2	G_2G_2	G_3G_2
R_1G_3	R_2G_3	BG_3	G_1G_3	G_2G_3	G_3G_3

   Key: R = red, B = blue, G = green



   c. How can you use the Counting Principle to determine the total number of possible outcomes? Explain your reasoning.

   I can multiply the number of outcomes for each sock choice. There are 6 possible outcomes for choosing the first sock and 6 possible outcomes for choosing the second sock. So, there are 6 × 6, or 36, total possible outcomes for choosing both socks.

d. Calculate the probability of choosing:

- a blue sock and then a red sock

$P(\text{blue 1st}) = \frac{1}{6}$

$P(\text{red 2nd}) = \frac{2}{6}$, or $\frac{1}{3}$

$P(\text{blue 1st and red 2nd}) = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$

- a red sock and then a sock that is not blue

$P(\text{red 1st}) = \frac{2}{6}$, or $\frac{1}{3}$

$P(\text{not blue 2nd}) = 1 - \frac{1}{6} = \frac{5}{6}$

$P(\text{red 1st and not blue 2nd}) = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$

- two socks with the 1st sock being green

$P(\text{green 1st}) = \frac{3}{6}$, or $\frac{1}{2}$

$P(\text{any color 2nd}) = \frac{6}{6}$, or 1

$P(\text{green 1st and any color 2nd}) = \frac{1}{2} \times 1 = \frac{1}{2}$

Suppose you reach into the drawer without looking and choose a sock, do NOT replace it, and then choose another sock. You choose a total of 2 socks. Remember, there are 2 red, 1 blue, and 3 green socks in a drawer.

2. Use this information to answer the questions.

a. Does the action "choosing the first sock" affect the outcomes of "choosing the second sock"? If so, how? Explain your reasoning.

Yes. After choosing the first sock, there is 1 less possible outcome for the second sock because the first sock is not replaced.

b. Use a diagram or organized list to represent the sample space for this situation.

Representations will vary.

R_1R_2	R_2R_1	BR_1	G_1R_1	G_2R_1	G_3R_1
R_1B	R_2B	BR_2	G_1R_2	G_2R_2	G_3R_2
R_1G_1	R_2G_1	BG_1	G_1B	G_2B	G_3B
R_1G_2	R_2G_2	BG_2	G_1G_2	G_2G_1	G_3G_1
R_1G_3	R_2G_3	BG_3	G_1G_3	G_2G_3	G_3G_2

Key: R = red, B = blue, G = green

Grouping

Have students complete Questions 2 and 3 with a partner. Then have students share their responses as a class.

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Guiding Questions for Share Phase, Questions 2 and 3

- What is the probability of choosing a blue sock and then a sock that is not blue?
- What is the probability of choosing a red sock and then a sock that is not red?
- How is this second situation different than the first situation?
- Will this difference affect the computation of the probabilities? Explain.

c. How can you use the Counting Principle to determine the total number of possible outcomes? Explain your reasoning.

I can multiply the number of outcomes for each sock choice. There are 6 possible outcomes for choosing the first sock and 5 possible outcomes for choosing the second sock. So, there are 6×5, or 30, total possible outcomes for choosing both socks.

d. Calculate the probability of choosing:

- a blue sock and then a red sock

$P(\text{blue 1st}) = \frac{1}{6}$

$P(\text{red 2nd}) = \frac{2}{5}$

$P(\text{blue 1st and red 2nd}) = \frac{1}{6} \times \frac{2}{5} = \frac{2}{30} = \frac{1}{15}$

- a red sock and then a sock that is not blue

$P(\text{red 1st}) = \frac{2}{6}$, or $\frac{1}{3}$

$P(\text{not blue 2nd}) = 1 - \frac{1}{5} = \frac{4}{5}$

$P(\text{red 1st and not blue 2nd}) = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

- two socks with the 1st sock being green

$P(\text{green 1st}) = \frac{3}{6}$, or $\frac{1}{2}$

$P(\text{any color 2nd}) = \frac{5}{5}$, or 1

$P(\text{green 1st and any color 2nd}) = \frac{1}{2} \times 1 = \frac{1}{2}$

3. What's different about the probability calculations in Question 1, part (d) and Question 2, part (d)?

In Question 1, part (d), the denominator of the probability of both events is 6 because the first sock is replaced in the drawer after it is chosen. There is the same number of socks in the drawer for both picks.

In Question 2, part (d), the denominator of the probability of both events decreases by 1 because the first sock is not replaced in the drawer after it is chosen. There is 1 less sock in the drawer for the second pick.

If Event *A* and Event *B* are dependent, then the probability that Event *A* happens and Event *B* happens is the product of the probability that Event *A* happens and the probability that Event *B* happens, given that Event *A* has already occurred.

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Grouping

Have students complete Questions 4 through 6 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 4 through 6

- Is Question 4 a problem that involve compound probability with “and?” Explain.
- Why is addition used to determine the answer for Question 4?
- What are the advantages and disadvantages of John’s solution method?
- What are the advantages and disadvantages of Faith’s solution method?
- Do you prefer John’s solution method or Faith’s solution method? Explain.

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4. Two red socks, 1 blue sock, and 3 green socks are in a drawer. One sock is randomly chosen without replacing it in the drawer, then a second sock is randomly chosen.

What is the probability that the second sock is red?

John explained his solution method.

> **John**
>
> We want the second sock to be red, but the first sock can be red or not red. The desired outcomes are “red and red” or “not red and red.”

Calculate the probability of choosing a red sock second.

$P(R \text{ and } R) + P(\text{not } R \text{ and } R)$

$\frac{2}{6} \times \frac{1}{5} + \frac{4}{6} \times \frac{2}{5}$

$\frac{2}{30} + \frac{8}{30}$

$\frac{10}{30} = \frac{1}{3}$

Remember, there are two cases that you should consider, depending on whether the first sock was red or not.

5. Faith reasoned differently to answer Question 4.

> **Faith**
>
> I made an organized list and saw that the probability of choosing a red sock second is $\frac{10}{30}$, or $\frac{1}{3}$.

Is Faith's explanation correct? Explain your reasoning.

Answers will vary.

Faith's explanation is correct.

You can verify $\frac{1}{3}$ is accurate by creating an organized list.

R_1R_2	R_2R_1	BR_1	G_1R_1	G_2R_1	G_3R_1
R_1B	R_2B	BR_2	G_1R_2	G_2R_2	G_3R_2
R_1G_1	R_2G_1	BG_1	G_1B	G_2B	G_3B
R_1G_2	R_2G_2	BG_2	G_1G_2	G_2G_1	G_3G_1
R_1G_3	R_2G_3	BG_3	G_1G_3	G_2G_3	G_3G_2

Key: R = red, B = blue, G = green

The organized list shows that 10 out of the 30 outcomes result in a red sock for the 2nd pick. So the probability of picking a red sock 2nd is $\frac{10}{30}$, or $\frac{1}{3}$.

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6. Two red socks, 1 blue sock, and 3 green socks are in a drawer. One sock is randomly chosen without replacing it in the drawer, and then a second sock is randomly chosen.

Use John's or Faith's method to answer each question.

a. What is the probability that the second sock is blue?

Verify your solution by creating an organized list.

I used John's method to answer the question. The probability of randomly choosing two socks with the second sock being blue is $\frac{1}{6}$.

I determined the answer by calculating P(Blue 1st, and blue 2nd given blue 1st) + P(Not blue 1st, and blue 2nd given not blue 1st) because the color of the 1st sock is not specified, which means it could be blue or not blue.

P(Blue 1st, and blue 2nd given blue 1st) = P(Blue 1st) · P(Blue 2nd, given blue 1st) $= \frac{1}{6} \cdot \frac{0}{5} = 0$

P(Not blue 1st, and blue 2nd given not blue 1st) = P(Not blue 1st) · P(Blue 2nd, given not blue 1st) $= \frac{5}{6} \cdot \frac{1}{5} = \frac{5}{30} = \frac{1}{6}$

P(Blue 1st, and blue 2nd given blue 1st) + P(Not blue 1st, and blue 2nd given not blue 1st) $= 0 + \frac{1}{6} = \frac{1}{6}$

The probability of randomly choosing two socks without replacement, that results in a blue sock 2nd is $\frac{1}{6}$.

Using an organized list, I verified that the probability of randomly choosing two socks without replacement, that results in a green sock 2nd is $\frac{5}{30}$ or $\frac{1}{6}$.

R_1R_2	R_2R_1	BR_1	G_1R_1	G_2R_1	G_3R_1
R_1B	R_2B	BR_2	G_1R_2	G_2R_2	G_3R_2
R_1G_1	R_2G_1	BG_1	G_1B	G_2B	G_3B
R_1G_2	R_2G_2	BG_2	G_1G_2	G_2G_1	G_3G_1
R_1G_3	R_2G_3	BG_3	G_1G_3	G_2G_3	G_3G_2

Key: R = red, B = blue, G = green

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b. What is the probability that the second sock is green?

Verify your solution by creating an organized list.

I used Faith's method to answer the question. The probability of randomly choosing two socks with the second sock being green is $\frac{1}{2}$.

Using an organized list, I verified that the probability of choosing a green sock 2nd is $\frac{15}{30}$, or $\frac{1}{2}$.

R_1R_2	R_2R_1	BR_1	G_1R_1	G_2R_1	G_3R_1
R_1B	R_2B	BR_2	G_1R_2	G_2R_2	G_3R_2
R_1G_1	R_2G_1	BG_1	G_1B	G_2B	G_3B
R_1G_2	R_2G_2	BG_2	G_1G_2	G_2G_1	G_3G_1
R_1G_3	R_2G_3	BG_3	G_1G_3	G_2G_3	G_3G_2

Key: R = red, B = blue, G = green

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Talk the Talk

Students summarize the big ideas in the lesson by answering two questions.

Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

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Talk the Talk

1. Compare the methods you used to determine the compound probability of two independent events both occurring and the compound probability of two dependent events both occurring. Describe the similarities and differences between the methods.

   For both independent and dependent events, I multiplied probabilities to determine the compound probability of both events occurring together.

   To determine the compound probability of two independent events both occurring, I multiplied the probability of each event. To determine the compound probability of two dependent events both occurring, I multiplied the probability of the first event by the probability of the second event, given that the first event already occurred.

2. What rules could you write to determine the compound probability of three or more independent events? Three or more dependent events? Include examples to support your conclusions.

   Answers will vary.

   Example for 3 Independent Events:

   A jar contains 2 gold marbles, 3 silver marbles, and 4 purple marbles. The probability of randomly choosing a gold marble, then a silver marble, then a purple marble with replacement is $\frac{8}{243}$.

   $P(\text{Gold 1st and silver 2nd and purple 3rd}) = \frac{2}{9} \cdot \frac{3}{9} \cdot \frac{4}{9} = \frac{24}{729} = \frac{8}{243}$

   I can determine the compound probability of n independent events by multiplying

   $P(A) \times P(B) \times P(C) \ldots \times P(n)$.

   Example for 3 Dependent Events:

   A jar contains 2 gold marbles, 3 silver marbles, and 4 purple marbles. The probability of randomly choosing a gold marble, then a silver marble, then a purple marble without replacement is $\frac{1}{21}$.

   $P(\text{Gold 1st and silver 2nd and purple 3rd}) = \frac{2}{9} \cdot \frac{3}{8} \cdot \frac{4}{7} = \frac{24}{504} = \frac{1}{21}$

   I can determine the compound probability of n dependent events by multiplying $P(A) \times P(B, \text{given } A) \times P(C, \text{given } A \text{ and } B) \ldots \times P(n, \text{given } A \text{ and } B \text{ and } C \ldots)$.

Be prepared to share your solutions and methods.

Check for Students' Understanding

A bucket contains individually wrapped pieces of bubblegum. The flavors include grape, cherry, apple, raspberry, and lime. There are 20 pieces of each flavor.

Suppose you randomly choose a first piece, do not replace the piece and choose a second piece. What is the probability that the second piece will be lime flavored?

Use John's method to determine the probability.

We want the second piece to be lime, but the first piece can be lime or not lime. The desired outcomes are "lime and lime" or "not lime and lime."

$$P(\text{Lime and Lime}) + P(\text{Not Lime and Lime})$$

$$\frac{20}{100} \cdot \frac{19}{99} + \frac{80}{100} \cdot \frac{20}{99}$$

$$\frac{380}{9900} + \frac{1600}{9900}$$

$$\frac{1980}{9900}$$

$$\frac{1}{5}$$

The probability that the second piece will be lime flavored is $\frac{1}{5}$.

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19.4

Or?

Compound Probability with "Or"

LEARNING GOALS

In this lesson, you will:

- Determine the probability of one or another independent events.
- Determine the probability of one or another dependent events.

KEY TERM

- Addition Rule for Probability

ESSENTIAL IDEAS

- A compound event is an event that consists of two or more events.
- The Addition Rule for Probability states: "The probability that Event A occurs or Event B occurs is the the probability that Event A occurs plus the probability that Event B minus the probability that both A and B occur." Using probability notation, the Addition Rule for Probability is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

S-CP Conditional Probability and the Rules of Probability

Use the rules of probability to compute probabilities of compound events in a uniform probability model

7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Overview

Students determine the probability of one or another independent events and the probability of one or another dependent events. The Addition Rule for Probability is stated and used to compute probabilities. Several situations present students with the opportunities to construct tree diagrams, create organized lists, complete tables and compute $P(A)$, $P(B)$, $P(A \text{ and } B)$, and $P(A \text{ or } B)$ with respect to the problem situation. Students create a graphic organizer to record the different types of compound events they have studied; independent events $P(A \text{ and } B)$, independent events $P(A \text{ or } B)$, dependent events $P(A \text{ and } B)$, dependent events $P(A \text{ or } B)$.

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Warm Up

Three coins are flipped and the result of each flip is recorded.

What is the probability of a heads up result on the first flip, heads up on the second flip, and heads up on the third flip?

1. Create a model to represent the sample space.

   HHH

   HHT

   HTH

   THH

   HTT

   THT

   TTH

   TTT

2. How many outcomes are possible?

   There are 8 possible outcomes.

3. What are the events in this probability situation?

   The first event is a heads up result on the first flip.

   The second event is a heads up result on the second flip.

   The third event is a heads up result on the third flip.

4. What is the probability of a heads up result on the first flip, $P(A)$?

   The probability of a heads up result on the first flip is $\frac{1}{2}$.

5. What is the probability of a heads up result on the second flip, $P(B)$?

   The probability of a heads up result on the second flip is $\frac{1}{2}$.

6. What is the probability of a heads up result on the third flip, $P(C)$?

   The probability of a heads up result on the third flip is $\frac{1}{2}$.

7. What is the probability a heads up result on the first flip, heads up on the second flip, and heads up on the third flip?

   The probability of a heads up result for all three flips is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

8. Are the events described in the previous questions considered independent events or dependent events?

   The events are independent.

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19.4

Or?

Compound Probability with "Or"

LEARNING GOALS

In this lesson, you will:

- Determine the probability of one or another independent events.
- Determine the probability of one or another dependent events.

KEY TERM

- Addition Rule for Probability

Do you frequently use the construction "and/or," as in, "I would like a new pair of shoes and/or a computer"?

Many writing instructors caution students to avoid using this construction because it can be confusing. Instead, they would suggest writing "I would like a new pair of shoes or a computer or both."

In this probability lesson, the word "or" means "one or the other or both."

Problem 1

Independent events are used to explore probability. Students complete a table to represent the sample space of two events. They determine $P(A)$, $P(B)$, $P(A \text{ and } B)$, and $P(A \text{ or } B)$ with respect to the problem situation. Students create a formula that relates $P(A)$, $P(B)$, and $P(A \text{ or } B)$. A second scenario is given and students once again construct a model to represent the sample space and compute $P(A)$, $P(B)$, $P(A \text{ and } B)$, and $P(A \text{ or } B)$. The Addition Rule is stated.

Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3

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- Is flipping heads the first event or the second event?
- Is flipping tails the first event or the second event?
- How many outcomes match the first event?
- How many outcomes match the second event?
- How many outcomes match both the first event and second event?
- How many total outcomes are possible?

PROBLEM 1 Or . . .?

Suppose you flip two coins. What is the probability of the first coin landing heads up or the second coin landing tails up?

1. Describe the events in this probability situation.

   The first event is the first coin landing heads up.

   The second event is the second coin landing tails up.

2. Complete the table to construct the sample space for this situation.

		Second Coin	
		H	T
First Coin	H	HH	HT
	T	TH	TT

   a. Draw a circle around the outcomes that match the first event.

   b. Draw a rectangle around the outcomes that match the second event.

3. How can you describe the outcome that is circled and has a rectangle around it in the sample space?

   The outcome HT represents a result of heads up for flipping the first coin and a result of tails up for flipping the second coin.

4. Kirk and Damon described the probability of a heads up result for the first coin or a tails up result for the second coin.

Kirk

I circled 2 outcomes and drew a rectangle around 2 outcomes, and there are 4 possible outcomes. So, the probability of flipping a heads on the first coin or a tails on the second is

$\frac{2+2}{4} = \frac{4}{4} = 1.$

Damon

I marked 4 outcomes, and there are 4 possible outcomes. But I marked 1 of the outcomes twice. I can count each outcome only once. So, the probability of a heads up result for the first coin or a tails up result for the second coin is $\frac{3}{4}$.

Grouping

Have students complete Questions 4 through 6 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 4 through 6

- What does a probability of 1 imply in Kirk's situation?
- Did Kirk and Damon mark the outcomes the same way?
- What did Damon do differently?
- What does $P(A \text{ and } B)$ represent in this situation?
- What does $P(A \text{ or } B)$ represent in this situation?
- What is the difference between $P(A \text{ and } B)$ and $P(A \text{ or } B)$?
- What operation(s) is used when determining $P(A \text{ and } B)$?
- What operation(s) is used when determining $P(A \text{ or } B)$?

Explain in your own words why Damon is correct and Kirk is incorrect.

Kirk is incorrect because his answer of 1 means that when flipping two coins, you are certain to get heads up on the first coin or tails up on the second coin. But this can't be true because one of the possible outcomes is TT.

Damon is correct because counting an outcome twice results in an incorrect answer. The correct probability is $\frac{3}{4}$.

5. Use the sample space and what you know about probability to answer each question.

a. What is the probability of a heads up result for flipping the first coin, $P(A)$?

The probability of a heads up result for the first coin flip is $\frac{1}{2}$. There are 2 outcomes out of 4 total outcomes in the sample space that show a heads up result for the first coin flip.

b. What is the probability of a tails up result for flipping the second coin, $P(B)$?

The probability of a tails up result for the second coin flip is $\frac{1}{2}$. There are 2 outcomes out of 4 total outcomes in the sample space that show a tails up result for the second coin flip.

c. What is the probability of a heads up result for the first coin flip AND a tails up result for the second coin flip, $P(A \text{ and } B)$?

The probability of a heads up result for the first coin flip and a tails up result for the second coin flip is $\frac{1}{4}$. One out of the 4 total outcomes in the sample space results in heads up on the first coin flip and tails up on the second coin flip.

d. What is the probability of a heads up result for the first coin flip OR a tails up result for the second coin flip $P(A \text{ or } B)$?

The probability of a heads up result for the first coin flip or a tails up result for the second coin flip is $\frac{3}{4}$. Three out of the 4 total outcomes in the sample space result in heads up on the first coin flip or tails up on the second coin flip.

6. Create a formula you could use to relate the probability of each event by itself, $P(A)$, $P(B)$, and the probability of the first event OR the second event $P(A \text{ or } B)$. Explain why your formula works.

Answers will vary.

$P(\text{Heads 1st or tails 2nd}) = \frac{1}{4}$

I determined the answer by adding P(Heads 1st) and P(Tails 2nd), then subtracting P(Heads 1st and tails 2nd).

$P(\text{Heads 1st}) = \frac{2}{4} = \frac{1}{2}$

$P(\text{Tails 2nd}) = \frac{2}{4} = \frac{1}{2}$

$P(\text{Heads 1st and tails 2nd}) = P(\text{Heads 1st}) \cdot P(\text{Tails 2nd}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(\text{Heads 1st or tails 2nd}) = P(\text{Heads 1st}) + P(\text{Tails 2nd}) - P(\text{Heads 1st and tails 2nd})$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

In general, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

A new holiday—Probability Day—is going to be celebrated at your school. It may be celebrated on any of the first 3 days of any month. The problem now is to choose which day it will fall on. Of course, the day will be selected at random. First the month will be selected and then the day.

7. How many dates for Probability Day are in the sample space? Explain how you determined the answer.

There are 36 possible dates for Probability Day in the sample space.

I can use the Counting Principle. There are 12 months to choose from and 3 days in each month to choose from. So, there are 12 × 3, or 36, different possible dates.

8. Construct a model to represent the sample space for this situation.

Answers will vary.

Jan 1, Jan 2, Jan 3
Feb 1, Feb 2, Feb 3
Mar 1, Mar 2, Mar 3
Apr 1, Apr 2, Apr 3
May 1, May 2, May 3
Jun 1, Jun 2, Jun 3
Jul 1, Jul 2, Jul 3
Aug 1, Aug 2, Aug 3
Sep 1, Sep 2, Sep 3
Oct 1, Oct 2, Oct 3
Nov 1, Nov 2, Nov 3
Dec 1, Dec 2, Dec 3

Grouping

Have students complete Questions 7 through 11 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 7 through 11

- How many months are in a year?
- How many days in each month satisfy the requirement?
- Is your model of the situation a tree diagram, a table, or an organized list?
- Which months are considered summer months?
- What does $P(A \text{ and } B)$ represent in this situation?
- What does $P(A \text{ or } B)$ represent in this situation?
- How can the Counting Principle be used to determine $P(A \text{ and } B)$?
- What needs to be calculated first before determining $P(A \text{ or } B)$?
- How does the Addition Rule compare to the formula you crated in Question 6 for $P(A \text{ or } B)$?

9. What is the probability that:

a. January 3rd will be randomly chosen?

P(January 3rd) $= \frac{1}{36}$

The probability of randomly choosing January 3rd is $\frac{1}{36}$ because January 3rd is 1 of the 36 possible outcomes.

b. May 2nd will be randomly chosen?

P(May 2nd) $= \frac{1}{36}$

The probability of randomly choosing May 2nd is $\frac{1}{36}$ because May 2nd is 1 of the 36 possible outcomes.

c. any specific day and month will be randomly chosen?

Let *A* represent the event of randomly choosing any of the 1st three days in any month.

$P(A) = \frac{1}{36}$

The probability of randomly choosing any of the 1st three days in any of the months is $\frac{1}{36}$ because each outcome in the sample space is 1 out of the 36 possible days.

10. Study the sample space. Let *P*(*A*) represent the probability of randomly choosing a summer month, and let *P*(*B*) represent the probability of randomly choosing the first day of the month.

a. Calculate *P*(*A*).

$P(A) = \frac{1}{4}$

Three of the months are summer months and the first 3 days of those months could be the holiday. So, $3 \times 3 = 9$ out of the 36 outcomes in the sample space are in the summer months, which means $P(A) = \frac{9}{36} = \frac{1}{4}$.

b. Calculate *P*(*A*).

$P(B) = \frac{1}{3}$

Twelve out of the 36 outcomes in the sample space are on the first day of a month, which means $P(B) = \frac{12}{36} = \frac{1}{3}$.

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c. Calculate $P(A \text{ and } B)$.

$P(A \text{ and } B) = \frac{1}{12}$

Solution Method Using the Organized List:

Three out of the 36 date outcomes in the sample space are a summer month and the 1st day of the month, which means $P(A \text{ and } B) = \frac{3}{36} = \frac{1}{12}$

Solution Method Using the Rule of Compound Probability Involving "and":

I used my work from parts (a) and (b) and Rule of Compound Probability involving "and" to determine the answer.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$
$$= \frac{1}{4} \cdot \frac{1}{3}$$
$$= \frac{1}{12}$$

d. Calculate $P(A \text{ or } B)$. Use the organized list you created to explain your answer.

$P(A \text{ or } B) = \frac{1}{2}$

Eighteen out of the 36 date outcomes in the sample space are a summer month or the 1st day of the month.

January 1, February 1, March 1, April 1, May 1,

June 1, June 2, June 3,

July 1, July 2, July 3,

August 1, August 2, August 3,

September 1, October 1, November 1, December 1

So, $P(A \text{ or } B) = \frac{18}{36} = \frac{1}{2}$

11. Use your answers from Question 10. Describe how you can calculate the probability of randomly choosing a summer month or the first day of a month without using a tree diagram or organized list.

The probability of randomly choosing a summer month or the first day of a month is $\frac{1}{2}$.

To determine the answer, I used my work from parts (a) through (c) and the formula for $P(A \text{ or } B)$ that I derived previously.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{12}$$
$$= \frac{3}{12} + \frac{4}{12} - \frac{1}{12}$$
$$= \frac{6}{12}$$
$$= \frac{1}{2}$$

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Grouping

Have students complete Questions 12 and 13 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 12 and 13

- Which months are considered winter months?
- What are disjoint sets?
- What are intersecting sets?
- What is the difference between disjoint sets and intersecting sets?

The **Addition Rule for Probability** states: "The probability that Event A occurs or Event B occurs is the probability that Event A occurs plus the probability that Event B occurs minus the probability that both A and B occur."

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

12. What is the probability of randomly choosing a summer month or a winter month?

The probability of randomly choosing a summer month or a winter month is $\frac{1}{2}$.

I determined the answer by using the Addition Rule for Probability, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Let A represent the event of choosing a summer month.

Let B represent the event of choosing a winter month.

$P(A) = \frac{9}{36} = \frac{1}{4}$ because 9 of the 36 date outcomes in the sample space are summer months.

$P(B) = \frac{9}{36} = \frac{1}{4}$ because 9 of the 36 date outcomes in the sample space are winter months.

$P(A \text{ and } B) = 0$ because none of the 36 date outcomes in the sample space are summer months and winter months.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{1}{4} + \frac{1}{4} - 0 \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

13. Jereld says that because the summer month outcomes and the winter month outcomes are disjoint sets, the probability that one event or the other occurs is just the sum of the probabilities. It's not necessary to subtract the probability of the one event and the other event. Is Jereld correct? Explain why or why not.

Jereld is correct.

The set of summer months and the set of winter months are disjoint because they don't have any outcomes in common. That means that the probability of common outcomes in the two sets is 0.

Therefore, the Addition Rule for Probability for disjoint sets can be simplified from $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ to $P(A \text{ or } B) = P(A) + P(B)$ because $P(A \text{ and } B) = 0$.

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Problem 2

Dependent events are used to explore probability. Students create an organized list to represent the sample space of two events. They determine *P*(*A*), *P*(*B*), *P*(*A* and *B*), and *P*(*A* or *B*) with respect to the problem situation and use the Addition Rule. Students conclude the Addition Rule can be applied to independent events as well as dependent events. In the last question, students create a graphic organizer in which they record examples of the different types of compound events they have studied.

Grouping

- Ask students to read introduction. Discuss as a class.
- Have students complete Questions 1 through 8 with a partner. Then have students share their responses as a class.

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Guiding Questions for Share Phase, Questions 1 through 8

- Does this situation involve independent events or dependent events? Explain.
- Is "choosing a yellow glove" the first event or the second event?
- Is "choosing a blue glove" the first event or the second event?
- How many outcomes satisfy the first event?

PROBLEM 2 Oh, Good. Gloves Now.

You have seen that you can use the Addition Rule for Probability to determine the probability that one or another of two independent events occurs. Can you also use this rule to determine the probability of one or another of two dependent events occurring? Let's find out!

Suppose you have 3 yellow, 1 purple, and 2 blue gloves in a drawer. You reach into the drawer without looking and randomly choose two gloves, one at a time.

1. Does the action "choosing the first glove" affect the action of "choosing the second glove"? If so, how? Explain your reasoning.

   Yes. Once I choose the first glove, there is 1 less possible outcome for the second glove.

2. Write an organized list to represent the sample space for this situation.

Y_1Y_2	Y_2Y_1	Y_3Y_1	PY_1	B_1Y_1	B_2Y_1
Y_1Y_3	Y_2Y_3	Y_3Y_2	PY_2	B_1Y_2	B_2Y_2
Y_1P	Y_2P	Y_3P	PY_3	B_1Y_3	B_2Y_3
Y_1B_1	Y_2B_1	Y_3B_1	PB_1	B_1P	B_2P
Y_1B_2	Y_2B_2	Y_3B_2	PB_2	B_1B_2	B_2B_1

Key: Y = yellow, P = purple, B = blue

3. Use the sample space to determine the probability of randomly choosing a yellow glove first or a blue glove second.

   a. Describe the events in this probability situation.

   The first event is choosing a yellow glove first.

   The second event is choosing a blue glove second.

   b. Draw a circle around the outcomes that match the first event and a rectangle around the outcomes that match the second event. How can you describe the outcomes that are circled and have a rectangle around them in the sample space?

   The outcomes that are circled and have a rectangle around them represent the outcomes in which a yellow glove is randomly chosen first AND a blue glove is randomly chosen second.

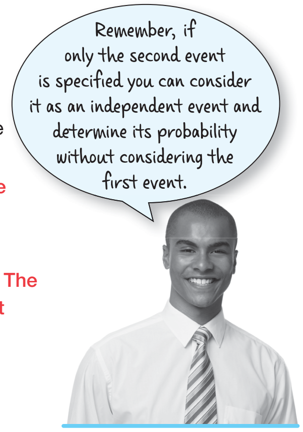
- How many outcomes satisfy the second event?
- How many outcomes satisfy both the first event and second event?
- How many total outcomes are possible?
- For Question 4, is Beth calculating *P*(*A* and *B*) or *P*(*B*)?
- What probability should Beth calculate?
- For Question 5, is Pedro treating the events as dependent events or independent events?

- What is the difference between calculating $P(A \text{ and } B)$ and calculating $P(A \text{ or } B)$?
- How do you know when use the calculation for $P(A \text{ and } B)$ versus the calculation for $P(A \text{ or } B)$?

c. What is the probability of randomly choosing a yellow glove first, $P(A)$?

The probability of randomly choosing a yellow glove first, $P(A)$, is $\frac{1}{2}$ because 2 out of the 6 gloves are blue.

I can also determine $P(A)$ from the sample space. The circled outcomes represent choosing a yellow glove first, $P(A) = \frac{15}{30} = \frac{1}{2}$.



d. What is the probability of randomly choosing a blue glove second, $P(B)$?

The probability of randomly choosing a blue glove second, $P(B)$, is $\frac{1}{3}$ because 2 out of the 6 gloves are blue.

I can also determine $P(B)$ from the sample space. The outcomes with a rectangle around them represent choosing a blue glove second, $P(B) = \frac{10}{30} = \frac{1}{3}$.

e. What is the probability of randomly choosing a yellow glove first AND a blue glove second, $P(A \text{ and } B)$?

The probability of randomly choosing a yellow glove first AND a blue glove second, $P(A \text{ and } B)$, is $\frac{1}{5}$. I used $P(B \text{ given } A)$ for the calculation because the events are dependent.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B \text{ given } A) \\ &= \frac{1}{2} \cdot \frac{2}{5} \\ &= \frac{2}{10} \\ &= \frac{1}{5} \end{aligned}$$

I can also determine $P(A \text{ and } B)$ from the sample space. The outcomes with a circle and rectangle drawn around them represent choosing a yellow glove first and a blue glove second, $P(A \text{ and } B) = \frac{6}{30} = \frac{1}{5}$.

f. What is the probability of randomly choosing a yellow glove first OR a blue glove second, $P(A \text{ or } B)$?

The probability of randomly choosing a yellow glove first OR a blue glove second, $P(A \text{ or } B)$, is $\frac{19}{30}$.

I determined $P(A \text{ or } B)$ from the sample space. The outcomes with a circle or rectangle drawn around them represent choosing a yellow glove first or a blue glove second, $P(A \text{ or } B) = \frac{19}{30}$.

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4. Beth wrote the following explanation for randomly choosing a blue glove second.

Beth

Because the actions are dependent in this problem, the probability of choosing a blue glove second is $\frac{2}{5}$. There are 5 gloves left after the first pick, and 2 of the gloves left are blue.

Explain why Beth's reasoning is incorrect.

Beth determined the probability of randomly choosing a blue glove second, after choosing a yellow glove first. But Question 3, part (d) only asks for the probability of choosing a blue glove second, *P*(Blue 2nd), regardless of what colored glove is chosen first.

One way to determine *P*(blue 2nd) is to consider "blue 2nd" as an independent event. Two out of the 6 gloves are blue which means *P*(blue 2nd) $= \frac{2}{6}$, or $\frac{1}{3}$.

Another way to determine the answer is to calculate the probability of choosing a blue glove 2nd, given a blue glove 1st or choosing a blue glove 2nd, given a non-blue glove 1st. In this case, I considered the events as dependent events.

Let *A* represent the event of randomly choosing a blue glove 1st.

Let *B* represent the event of randomly choosing a blue glove 2nd.

P(*A* and *B* given *A*) or

$$P(\text{not } A \text{ and } B \text{ given not } A) = P(A) \cdot P(B, \text{ given } A) + P(\text{not } A) \cdot P(B, \text{ given not } A)$$

$$= \left(\frac{2}{6}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{6}\right)\left(\frac{2}{5}\right)$$

$$= \frac{2}{30} + \frac{8}{30}$$

$$= \frac{10}{30}$$

$$= \frac{1}{3}$$

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5. Pedro wrote the following explanation for randomly choosing a yellow glove first and a blue glove second.

Pedro

In this problem, the probability of choosing a yellow glove first is $\frac{1}{2}$, and the probability of choosing a blue glove second is $\frac{1}{3}$. So, the probability of choosing both a yellow glove first and a blue glove second is $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Explain why Pedro's reasoning is incorrect.

Pedro's error is using $\frac{1}{3}$ for the calculation.

To determine the probability of choosing a yellow glove 1st and a blue glove 2nd, Pedro should multiply the probability of choosing a yellow glove 1st by the probability of choosing a blue glove 2nd, given the 1st glove is yellow. The events are dependent.

Let *A* represent the event of randomly choosing a yellow glove 1st.

Let *B* represent the event of randomly choosing a blue glove 2nd.

$$P(A \text{ and } B \text{ given } A) = P(A) \cdot P(B, \text{ given } A)$$
$$= \frac{3}{6} \cdot \frac{2}{5}$$
$$= \frac{6}{30}$$
$$= \frac{1}{5}$$

The probability of choosing a yellow glove 1st and a blue glove 2nd is $\frac{1}{5}$, not $\frac{1}{6}$.

6. Calculate the probability of randomly choosing a yellow glove first or a blue glove second, *P*(*A* or *B*), using the Addition Rule. Compare this to your answer in Question 3, part (f). What do you notice?

The probability using the Addition Rule is $\frac{19}{30}$. I got the same answer in Question 3, part (f), using the sample space.

Addition Rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{5}$$
$$= \frac{15}{30} + \frac{10}{30} - \frac{6}{30}$$
$$= \frac{19}{30}$$

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Grouping

Have students complete Question 9 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 9

- Share an example of independent events.
- Share an example of dependent events.
- What is the difference between independent events and dependent events?
- For what type of scenario is multiplication used to calculate probability?
- For what type of scenario is addition and subtraction used to calculate probability?

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7. Determine the probability of choosing a yellow glove first or a yellow glove second using the Addition Rule. Verify your answer using the sample space in Question 2.

The probability of choosing a yellow glove first or a yellow glove second is $\frac{4}{5}$.

P(yellow 1st or yellow 2nd) = *P*(yellow 1st) + *P*(yellow 2nd) − *P*(yellow 1st and yellow 2nd)

$$= \frac{3}{6} + \frac{3}{6} - \left(\frac{3}{6}\right)\left(\frac{2}{5}\right)$$
$$= \frac{3}{6} + \frac{3}{6} - \frac{6}{30}$$
$$= \frac{15}{30} + \frac{15}{30} - \frac{6}{30}$$
$$= \frac{24}{30}$$
$$= \frac{4}{5}$$

8. Does the Addition Rule apply to independent events as well as dependent events? Explain your reasoning.

Explanations will vary.

Yes. The Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, applies to both independent and dependent events.

9. In this lesson and the previous lesson, you investigated the probabilities of different kinds of compound events. Complete the graphic organizer to record examples of the types of compound events you have studied. For each type of compound event, describe the methods you used to determine the probability.

Answers will vary.

Independent Events
$P(A \text{ and } B)$

Scenario: Flip 2 coins and record the result of each flip.

Question: What is the probability of flipping 2 heads?

Method: $P(A) \times P(B \text{ given } A)$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Independent Events
$P(A \text{ or } B)$

Scenario: Flip 2 coins and record the result of each flip.

Question: What is the probability of flipping heads on the first coin or heads on the second coin?

Method: $P(A) + P(B) - P(A \text{ and } B \text{ given } A)$

$\frac{1}{2} + \frac{1}{2} - \left(\frac{1}{2} \times \frac{1}{2}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

Compound Events

Scenario: Choose 2 socks from a drawer with 2 green, 2 blue, and 2 red socks, without replacement.

Question: What is the probability of choosing a blue sock and then a green sock?

Method: $P(A) \times P(B \text{ given } A)$

$\frac{2}{6} \times \frac{2}{5} = \frac{4}{30} = \frac{2}{15}$

Dependent Events
$P(A \text{ and } B)$

Scenario: Choose 2 socks from a drawer with 2 green, 2 blue, and 2 red socks, without replacement.

Question: What is the probability of choosing a green sock first or a green sock second?

Method: $P(A) + P(B) - P(A \text{ and } B \text{ given } A)$

$\frac{2}{6} + \frac{2}{6} - \left(\frac{2}{6} \times \frac{1}{5}\right) = \frac{4}{6} - \frac{2}{30} = \frac{18}{30} = \frac{3}{5}$

Dependent Events
$P(A \text{ or } B)$

Be prepared to share your solutions and methods.

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Check for Students' Understanding

A bucket contains individually wrapped pieces of bubblegum. The flavors include grape, cherry, apple, raspberry, and lime. There are 5 pieces of each flavor.

Jeff's favorite flavors are grape and lime. Jeff randomly chooses 2 pieces of bubblegum.

1. What is the probability of randomly choosing a piece of grape bubblegum first AND choosing a piece of lime bubblegum second, $P(A \text{ and } B)$?

The probability of choosing a piece of grape bubblegum first and choosing a piece of lime bubblegum second is $\frac{1}{24}$.

I determined the answer by using the Rule of Compound Probability involving "and."

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) \\ &= \frac{5}{25} \cdot \frac{5}{24} \\ &= \frac{1}{5} \cdot \frac{5}{24} \\ &= \frac{1}{24} \end{aligned}$$

2. What is the probability of randomly choosing a piece of grape bubblegum first OR a piece of lime bubblegum second, $P(A \text{ or } B)$?

The probability of choosing a grape piece first or a lime piece second is $\frac{43}{120}$.

I determined the answer by using the Addition Rule for Probability.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{5}{25} + \frac{5}{25} - \left(\frac{5}{25}\right)\left(\frac{5}{24}\right) \\ &= \frac{5}{25} + \frac{5}{25} - \frac{25}{600} \\ &= \frac{120}{600} + \frac{120}{600} - \frac{25}{600} \\ &= \frac{215}{600} \\ &= \frac{43}{120} \end{aligned}$$

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19.5

And, Or, and More!

Calculating Compound Probability

LEARNING GOAL

In this lesson, you will:

- Calculate compound probabilities with and without replacement.

ESSENTIAL IDEAS

- Situations "with replacement" generally involve independent events. Whether or not the first event happens has no effect on the second event.
- Situations "without replacement" generally involve dependent events. If the first event occurs, it has an impact on the probability of subsequent events.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

S-CP Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data

2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

8. Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

Overview

A standard deck of playing cards, choosing committee members, and a menu are used to determine compound probabilities. Students determine the probability of independent events $P(A \text{ and } B)$ with replacement, independent events $P(A \text{ or } B)$ with replacement, dependent events $P(A \text{ and } B)$ without replacement and dependent events $P(A \text{ or } B)$ without replacement. Some situations contain 3 or more events.

Warm Up

The local diner serves five different kinds of pie; apple, blueberry, lemon, rhubarb, and peach. They serve four flavors of ice cream; vanilla, strawberry, chocolate, and tangerine.

What is the probability that the next customer that orders pie with ice cream will order peach pie with vanilla ice cream?

1. Describe the events in the probability situation

   One event is a customer ordering peach pie.

   The other event is a customer ordering vanilla ice cream.

2. Complete the table to construct a model representing the sample space.

	Apple	Blueberry	Lemon	Rhubarb	Peach
Vanilla	V-A	V-B	V-L	V-R	V-P
Strawberry	S-A	S-B	S-L	S-R	S-P
Chocolate	C-A	C-B	C-L	C-R	C-P
Tangerine	T-A	T-B	T-L	T-R	T-P

3. What is the probability of a customer ordering peach pie, $P(A)$?

   The probability of a customer ordering peach pie is $\frac{4}{20}$, or $\frac{1}{5}$, because 4 out of the 20 outcomes result in choosing peach pie.

4. What is the probability of a customer ordering vanilla ice cream, $P(B)$?

   The probability of a customer ordering vanilla ice cream is $\frac{5}{20}$, or $\frac{1}{4}$, because 5 out of the 20 outcomes result in choosing vanilla ice cream.

5. What is the probability of a customer ordering peach pie AND ordering vanilla ice cream, $P(A \text{ and } B)$?

   The probability of a customer ordering peach pie and ordering vanilla ice cream is $\frac{1}{20}$.

   I determined the answer by using the Rule of Compound Probability involving "and."

   $$P(A \text{ and } B) = P(A) \cdot P(B)$$
   $$= \frac{1}{5} \cdot \frac{1}{4}$$
   $$= \frac{1}{20}$$

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6. What is the probability of a customer ordering peach pie OR ordering vanilla ice cream, $P(A \text{ or } B)$?

The probability of a customer ordering peach pie or ordering vanilla ice cream is $\frac{8}{20}$, or $\frac{2}{5}$. Eight out of the 20 outcomes in the sample space result in peach pie or vanilla ice cream.

7. Use the Addition Rule to calculate $P(A \text{ or } B)$ and verify your answer to Question 6.

The probability of a customer ordering peach pie or vanilla ice cream is $\frac{2}{5}$.

I used the Addition Rule for Probability to verify that the probability of a customer ordering peach pie or vanilla ice cream is $\frac{2}{5}$.

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
&= \frac{1}{5} + \frac{1}{4} - \left(\frac{1}{5}\right)\left(\frac{1}{4}\right) \\
&= \frac{1}{5} + \frac{1}{4} - \frac{1}{20} \\
&= \frac{4}{20} + \frac{5}{20} - \frac{1}{20} \\
&= \frac{8}{20} \\
&= \frac{2}{5}
\end{aligned}
$$

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And, Or, and More!

Calculating Compound Probability

19.5

LEARNING GOAL

In this lesson, you will:

- Calculate compound probabilities with and without replacement.

Are you an honest person? Do you have integrity? Do you do what is right even though there may not be laws telling you to do so? If yes, you may be described as having probity.

Talking about a person's probity can be a way to gauge the reliability of their testimony in court.

Did you know that the word "probability" comes from the word "probity"? How are these terms related? What are the similarities and differences in their meanings?

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Problem 1

Questions ask students to determine the probability of scenarios involving a standard deck of playing cards.

Grouping

- Ask students to read introduction. Discuss as a class.
- Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2

- How many cards are in a deck of standard playing cards?
- If the first card is replaced before drawing a second card, how does that affect your calculation?
- How many aces are in a deck of standard playing cards?
- How many tens are in a deck of standard playing cards?
- What does $P(A \text{ and } B)$ represent in this situation?
- What operation is associated with calculating $P(A \text{ and } B)$?
- What does $P(A \text{ or } B)$ represent in this situation?
- What operation is associated with calculating $P(A \text{ or } B)$?
- How is $P(A \text{ and } B)$ different than $P(A \text{ or } B)$?
- How many diamonds are in a deck of standard playing cards?

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PROBLEM 1 It's All in the Cards

You will often see the phrases "with replacement" and "without replacement" in probability problems. These phrases refer to whether or not the total sample space changes from the first to the last event.

For example, if you draw a marble from a bag, record the color, and then replace it, the sample space for the next event remains the same. If you do not replace it, then the sample space decreases.

A standard deck of playing cards contains 52 cards. There are 4 suits: spades, clubs, hearts, and diamonds. Each suit contains 13 cards: an ace, the numbers 2–10, and three "face cards," which are the Jack, Queen, and King.

A pair is 2 cards with the same number or face. Two aces is an example of a pair. A three of a kind is 3 cards with the same number or face. Three Jacks is an example of three of a kind.

1. Using a standard deck of playing cards, calculate each of the following probabilities for randomly drawing 2 cards, one at a time, *with* replacement.

   a. An ace first and a ten second

   The probability of randomly choosing an ace 1st and a ten 2nd is $\frac{1}{169}$.

   I calculated the answer by using the compound probability rule for "and."

   $$P(\text{ace 1st and ten 2nd}) = P(\text{ace 1st}) \cdot P(\text{ten 2nd})$$
   $$= \frac{4}{52} \cdot \frac{4}{52}$$
   $$= \frac{1}{13} \cdot \frac{1}{13}$$
   $$= \frac{1}{169}$$

- What is a pair?
- For determining the probability of any pair, can the first card chosen be any card?
- What is the probability of drawing any card? Is it a sure thing?
- What is the probability of a sure thing?
- What is three of a kind?
- How many kings are in a standard deck of playing cards?

b. An ace first or a ten second

The probability of randomly choosing an ace 1st or a ten 2nd is $\frac{103}{169}$.

I calculated the answer by using the compound probability rule for "or."

$$\begin{aligned} P(\text{ace 1st or ten 2nd}) &= P(\text{ace 1st}) + P(\text{ten 2nd}) - P(\text{ace 1st and ten 2nd}) \\ &= \frac{4}{13} + \frac{4}{13} - \frac{1}{169} \\ &= \frac{52}{169} + \frac{52}{169} - \frac{1}{169} \\ &= \frac{103}{169} \end{aligned}$$

c. two diamonds

The probability of randomly choosing two diamonds is $\frac{1}{16}$.

I calculated the answer by using the compound probability rule for "and" because choosing two diamonds means choosing a diamond 1st and a diamond 2nd.

$$\begin{aligned} P(\text{two diamonds}) &= P(\text{diamond 1st}) \cdot P(\text{diamond 2nd}) \\ &= \frac{13}{52} \cdot \frac{13}{52} \\ &= \frac{1}{4} \cdot \frac{1}{4} \\ &= \frac{1}{16} \end{aligned}$$

d. Any pair

The probability of randomly choosing any pair is $\frac{1}{13}$.

I calculated the answer by using the compound probability rule for "and" because choosing any pair means choosing any card 1st and the same number 2nd.

$$\begin{aligned} P(\text{any pair}) &= P(\text{any card 1st}) \cdot P(\text{same number as 1st card}) \\ &= \frac{52}{52} \cdot \frac{4}{52} \\ &= 1 \cdot \frac{1}{13} \\ &= \frac{1}{13} \end{aligned}$$

2. Calculate each of the following probabilities for drawing 3 cards, one at a time, *with* replacement.

a. Three of a kind

The probability of randomly choosing three of a kind is $\frac{1}{169}$.

I calculated the answer by using the compound probability rule for "and" because choosing three of a kind means choosing any card 1st and the same number 2nd and the same number 3rd.

Let A represent the event of randomly selecting any card 1st.

Let B represent the event of randomly selecting the same number as the first card 2nd.

Let C represent the event of randomly selecting the same number as the first card 3rd.

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$
$$= \frac{52}{52} \cdot \frac{4}{52} \cdot \frac{4}{52}$$
$$= 1 \cdot \frac{1}{13} \cdot \frac{1}{13}$$
$$= \frac{1}{169}$$

b. Three Kings

The probability of randomly choosing three Kings is $\frac{1}{2197}$.

I calculated the answer by using the compound probability rule for "and" because choosing three Kings means choosing a King 1st and a King 2nd and a King 3rd.

$$P(\text{three Kings}) = P(\text{King 1st}) \cdot P(\text{King 2nd}) \cdot P(\text{King 3rd})$$
$$= \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52}$$
$$= \frac{1}{13} \cdot \frac{1}{13} \cdot \frac{1}{13}$$
$$= \frac{1}{2197}$$

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Grouping

Have students complete Questions 3 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 3 through 5

- What does with replacement mean?
- What does without replacement mean?
- Are probability calculation for scenarios with replacement different than probability calculations for scenarios without replacement? Explain.
- Are independent events related to scenarios with replacement or scenarios without replacement? Explain.
- Are dependent events related to scenarios with replacement or scenarios without replacement? Explain.

3. Calculate each of the following probabilities for drawing 2 cards, one at a time, *without* replacement.

a. An ace first and a ten second

The probability of randomly choosing an ace 1st and a ten 2nd is $\frac{4}{663}$.

I calculated the answer by using the compound probability rule for "and."

$$P(\text{ace 1st and ten 2nd given ace 1st}) = P(\text{ace 1st}) \cdot P(\text{ten 2nd, given ace 1st})$$
$$= \frac{4}{52} \cdot \frac{4}{51}$$
$$= \frac{1}{13} \cdot \frac{4}{51}$$
$$= \frac{4}{663}$$

b. An ace first or a ten second

The probability of randomly choosing an ace 1st or a ten 2nd is $\frac{33}{221}$.

I calculated the answer by using the compound probability rule for "or."

$$P(\text{ace 1st or ten 2nd}) = P(\text{ace 1st}) + P(\text{ten 2nd, given ace 1st}) - P(\text{ace 1st and ten 2nd})$$
$$= \frac{4}{52} + \frac{4}{51} - \frac{4}{52} \cdot \frac{4}{51}$$
$$= \frac{1}{13} + \frac{4}{51} - \frac{16}{2652}$$
$$= \frac{51}{663} + \frac{52}{663} - \frac{4}{663}$$
$$= \frac{33}{221}$$

c. Two diamonds

The probability of randomly choosing two diamonds is $\frac{1}{17}$.

I calculated the answer by using the compound probability rule for "and" because choosing two diamonds means choosing a diamond 1st and a diamond 2nd.

$$P(\text{two diamonds}) = P(\text{diamond 1st}) \cdot P(\text{diamond 2nd, given diamond 1st})$$
$$= \frac{13}{52} \cdot \frac{12}{51}$$
$$= \frac{1}{4} \cdot \frac{4}{17}$$
$$= \frac{4}{68}$$
$$= \frac{1}{17}$$

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d. Any pair

The probability of randomly choosing any pair is $\frac{1}{17}$.

I calculated the answer by using the compound probability rule for "and" because choosing any pair means choosing any card 1st and the same number 2nd.

$$P(\text{any pair}) = P(\text{any card 1st}) \cdot P(\text{same number as 1st card})$$
$$= \frac{52}{52} \cdot \frac{3}{51}$$
$$= 1 \cdot \frac{1}{17}$$
$$= \frac{1}{17}$$

4. Calculate each of the following probabilities for drawing 3 cards, one at a time, *without* replacement.

a. Three of a kind

The probability of randomly choosing three of a kind is $\frac{1}{425}$.

I calculated the answer by using the compound probability rule for "and" because choosing three of a kind means choosing any card 1st and the same number 2nd and the same number 3rd.

Let *A* represent the event of randomly selecting any card 1st.

Let *B* represent the event of randomly selecting the same number as the first card 2nd.

Let *C* represent the event of randomly selecting the same number as the first card 3rd.

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$
$$= \frac{52}{52} \cdot \frac{3}{51} \cdot \frac{2}{50}$$
$$= 1 \cdot \frac{1}{17} \cdot \frac{1}{25}$$
$$= \frac{1}{425}$$

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b. Three Kings

The probability of randomly choosing three Kings is $\frac{1}{5525}$.

I calculated the answer by using the compound probability rule for "and" because choosing three Kings means choosing a King 1st and a King 2nd and a King 3rd.

Let A represent the event of randomly selecting a King 1st.

Let B represent the event of randomly selecting a King 2nd.

Let C represent the event of randomly selecting a King 3rd.

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B, \text{given } A) \cdot P(C, \text{given } A \text{ and } B)$$
$$= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50}$$
$$= \frac{1}{13} \cdot \frac{1}{17} \cdot \frac{1}{25}$$
$$= \frac{1}{5525}$$

5. What kinds of events generally correspond to situations "with replacement"? "Without replacement"? Why?

Situations "with replacement" generally involve independent events. In this type of situation, the first event does not affect subsequent events.

Situations "without replacement" generally involve dependent events. In this type of situation, the first event affects each subsequent event.

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Problem 2

Questions ask students to determine the probability of scenarios involving a three-member committee.

Grouping

Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 5

- Can the Rule of Probability involving "and" be used for more than 2 events?
- What is the maximum number of events that can be used with the Rule of Probability involving "and?"
- How can you determine whether to use the Rule of Probability involving "and" or the Addition Rule for Probability?

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PROBLEM 2 Let's Take It to the Committee

A homeowner's association has 25 members. The association needs to establish a three-member committee.

1. Calculate the probability that Bill, George, and Rio are randomly selected to serve on the committee.

   The probability that Bill, George, and Rio are selected for the committee is $\frac{1}{2300}$.

   I calculated the answer by using the compound probability rule for "and" because the question means choosing one person for the 1st spot and one person for the 2nd spot and one person for the 3rd spot.

   $$P(\text{Bill and George and Rio}) = P(\text{Bill}) \cdot P(\text{George}) \cdot P(\text{Rio})$$
   $$= \frac{3}{25} \cdot \frac{2}{24} \cdot \frac{1}{23}$$
   $$= \frac{3}{25} \cdot \frac{1}{12} \cdot \frac{1}{23}$$
   $$= \frac{3}{6900}$$
   $$= \frac{1}{2300}$$

2. Calculate the probability that Bill is randomly selected as president, George as vice-president, and Rio as treasurer.

   The probability that Bill is selected for president, George for vice-president, and Rio for treasurer is $\frac{1}{13{,}800}$.

   I calculated the answer by using the compound probability rule for "and" because the question means choosing Bill for president and George for vice-president and Rio for treasurer.

   Let A represent the event of randomly selecting Bill for president.

   Let B represent the event of randomly selecting George for vice-president.

   Let C represent the event of randomly selecting Rio for treasurer.

   $$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$
   $$= \frac{1}{25} \cdot \frac{1}{24} \cdot \frac{1}{23}$$
   $$= \frac{1}{13{,}800}$$

3. What is the difference between the situations in parts (a) and (b)?

   In part (a), the order in which Bill, George, and Rio are selected does not matter. But, in part (b) the order in which they are selected does matter.

4. Calculate the probability of five particular members being chosen to serve on a five-member committee.

The probability of 5 particular members being chosen for the committee is $\frac{1}{53,130}$.

I calculated the answer by using the compound probability rule for "and" because the question means choosing one person for the 1st spot and one person for the 2nd spot and one person for the 3rd spot and one person for the 4th spot and one person for the 5th spot.

Let *A* represent the event of randomly selecting the 1st member for the committee.

Let *B* represent the event of randomly selecting the 2nd member for the committee.

Let *C* represent the event of randomly selecting the 3rd member for the committee.

Let *D* represent the event of randomly selecting the 4th member for the committee.

Let *E* represent the event of randomly selecting the 5th member for the committee.

$$\begin{aligned} P(A \text{ and } B \text{ and } C \text{ and } D \text{ and } E) &= P(A) \cdot P(B) \cdot P(C) \cdot P(D) \cdot P(E) \\ &= \frac{5}{25} \cdot \frac{4}{24} \cdot \frac{3}{23} \cdot \frac{2}{22} \cdot \frac{1}{21} \\ &= \frac{1}{5} \cdot \frac{1}{6} \cdot \frac{3}{23} \cdot \frac{1}{11} \cdot \frac{1}{21} \\ &= \frac{3}{159,390} \\ &= \frac{1}{53,130} \end{aligned}$$

5. Calculate the probability of a particular member of the homeowner's association being chosen to serve as one of five officers.

The probability that a particular member is chosen as one of the five officers is $\frac{77,293}{354,200}$.

Let *A* represent the event of randomly selecting the 1st officer.

Let *B* represent the event of randomly selecting the 2nd officer.

Let *C* represent the event of randomly selecting the 3rd officer.

Let *D* represent the event of randomly selecting the 4th officer.

Let *E* represent the event of randomly selecting the 5th officer.

$$\begin{aligned} P(A \text{ or } B \text{ or } C \text{ or } D \text{ or } E) &= P(A) + P(B) + P(C) + P(D) + P(E) \\ &= \frac{1}{25} + \frac{1}{24} + \frac{1}{23} + \frac{1}{22} + \frac{1}{21} \\ &= \frac{42,504}{1,062,600} + \frac{44,275}{1,062,600} + \frac{46,200}{1,062,600} + \frac{48,300}{1,062,600} \\ &\quad + \frac{50,600}{1,062,600} \\ &= \frac{231,879}{1,062,600} \\ &= \frac{77,293}{354,200} \end{aligned}$$

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Problem 3

Questions ask students to determine the probability of scenarios involving a restaurant menu.

Grouping

Have students complete Questions 1 through 8 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 8

- Can the Rule of Compound Probability involving "and" be used for more than two events? Explain.
- Can the Rule of Compound Probability involving "and" be used for independent events and dependent events? Explain.
- Can the Addition Rule for Probability be used for more than two events? Explain.

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PROBLEM 3 What's on the Menu

The menu at a local restaurant offers the following items.

Appetizers: Shrimp, Veggies, Avocado Dip, Soup, Stuffed Mushrooms
Salads: Garden, Caesar, Pasta, House
Entrées: Steak, Pizza, Ravioli, Meatloaf, Chicken, Flounder, Spaghetti, Pork, Ham, Shrimp
Desserts: Ice Cream, Cookies, Fruit, Chocolate Cake, Pie, Cheese Cake, Sorbet

1. A dinner consists of one selection from each category. How many different dinners exist?

   There are 1400 possible dinners.

   $5 \cdot 4 \cdot 10 \cdot 7 = 1400$

2. What is the probability that a patron selects chicken?

   The probability that a patron selects chicken is $\frac{1}{10}$ because 1 out of the 10 entrées is chicken.

3. What is the probability that a patron selects meatloaf and chocolate cake?

   The probability that a patron selects meatloaf and chocolate cake is $\frac{1}{70}$.

   I calculated the answer by using the compound probability rule for "and."

   $$\begin{aligned} P(\text{meatloaf and chocolate cake}) &= P(\text{meatloaf}) \cdot P(\text{chocolate cake}) \\ &= \frac{1}{10} \cdot \frac{1}{7} \\ &= \frac{1}{70} \end{aligned}$$

4. What is the probability that one patron selects steak or another patron selects pie?

The probability that a patron selects steak or another patron selects pie is $\frac{8}{35}$.

I calculated the answer by using the compound probability rule for "and."

$$P(\text{steak or pie}) = P(\text{steak}) + P(\text{pie}) - P(\text{steak and pie})$$
$$= \frac{1}{10} + \frac{1}{7} - \left(\frac{1}{10}\right)\left(\frac{1}{7}\right)$$
$$= \frac{1}{10} + \frac{1}{7} - \frac{1}{70}$$
$$= \frac{7}{70} + \frac{10}{70} - \frac{1}{70}$$
$$= \frac{16}{70}$$
$$= \frac{8}{35}$$

5. What is the probability that a patron selects meatloaf? Any entrée except meatloaf?

The probability that a patron selects meatloaf is $\frac{1}{10}$ because 1 out of the 10 entrées is meatloaf.

The probability that a patron selects any entrée except meatloaf is $\frac{9}{10}$ because 9 out of the 10 entrees are not meatloaf.

The two events are complementary.

6. What is the probability that a patron selects flounder, Caesar salad, and fruit?

The probability that a patron selects flounder, Caesar salad, and fruit is $\frac{1}{280}$.

I calculated the answer by using the compound probability rule for "and."

$$P(\text{Flounder, Caesar salad, and fruit}) = P(\text{flounder}) \cdot P(\text{Caesar salad}) \cdot P(\text{fruit})$$
$$= \frac{1}{10} \cdot \frac{1}{4} \cdot \frac{1}{7}$$
$$= \frac{1}{280}$$

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7. What is the probability that a patron selects pizza, cookies, and any salad except a garden salad?

The probability that a patron selects pizza, cookies, and any salad except a garden salad is $\frac{3}{280}$.

I calculated the answer by using the compound probability rule for "and."

$$P(\text{pizza, cookies, and non-garden salad}) = P(\text{pizza}) \cdot P(\text{cookies}) \cdot P(\text{non-garden salad})$$
$$= \frac{1}{10} \cdot \frac{1}{7} \cdot \frac{3}{4}$$
$$= \frac{3}{280}$$

8. What is the probability that a patron selects soup, chicken, any salad, and any dessert?

The probability that a patron selects soup, chicken, any salad, and any dessert is $\frac{1}{50}$.

I calculated the answer by using the compound probability rule for "and."

Let A represent the event of a patron randomly selecting the soup appetizer.

Let B represent the event of a patron randomly selecting the chicken entrée.

Let C represent the event of a patron randomly selecting any salad.

Let D represent the event of a patron randomly selecting any dessert.

$$P(A \text{ and } B \text{ and } C \text{ and } D) = P(A) \cdot P(B) \cdot P(C) \cdot P(D)$$
$$= \frac{1}{5} \cdot \frac{1}{10} \cdot \frac{4}{4} \cdot \frac{7}{7}$$
$$= \frac{1}{5} \cdot \frac{1}{10} \cdot 1 \cdot 1$$
$$= \frac{1}{50}$$

Be prepared to share your solutions and methods.

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Check for Students' Understanding

A pack of 200 trivia cards is divided evenly into five categories; sports, music, movies, books, and occupations.

1. Suppose you randomly choose one card. What is the probability of selecting a card in the sports category?

   The probability of drawing a card in the sports category is $\frac{40}{200}$, or $\frac{1}{5}$, because 40 out of the 200 cards are in the sports category.

2. Suppose you randomly choose two cards without replacement. What is the probability of choosing two cards in the sports category?

   The probability of choosing two cards in the sports category is $\frac{39}{995}$.

   I determined the answer by using the Rule of Compound Probability involving "and."

   $$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) \\ &= \frac{40}{200} \cdot \frac{39}{199} \\ &= \frac{1}{5} \cdot \frac{39}{199} \\ &= \frac{39}{995} \end{aligned}$$

3. Suppose you randomly choose three cards without replacement. What is the probability of you drawing three cards in the sports category?

   The probability of choosing three cards in the sports category is $\frac{247}{32{,}835}$.

   I determined the answer by using the Rule of Compound Probability involving "and."

   $$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) \\ &= \frac{40}{200} \cdot \frac{39}{199} \cdot \frac{38}{198} \\ &= \frac{1}{5} \cdot \frac{39}{199} \cdot \frac{19}{99} \\ &= \frac{741}{98{,}505} \\ &= \frac{247}{32{,}835} \end{aligned}$$

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Do You Have a Better Chance of Winning the Lottery or Getting Struck By Lightning?

19.6

Investigate Magnitude through Theoretical Probability and Experimental Probability

LEARNING GOALS

In this lesson, you will:

- Simulate events using the random number generator on a graphing calculator.
- Compare experimental and theoretical probability.

KEY TERMS

- theoretical probability
- simulation
- experimental probability

ESSENTIAL IDEAS

- The graphing calculator is used to generate random numbers.
- A random number generator is used to simulate events.
- Experimental and theoretical probabilities are compared.
- A simulation is an experiment that models a real-life situation.
- Experimental probability is defined as $\frac{\textit{number of times an outcome occurs}}{\textit{total number of trials performed}}$

COMMON CORE STATE STANDARDS FOR MATHEMATICS

S-IC Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments

2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Overview

Lottery, lightning, and raffle scenarios are used to determine probabilities for compound independent events. Students determine the probability of compound independent events using the Rule for Compound Probability involving "and." The term simulation is defined and the formula for experimental probability is given. The graphing calculator is used to generate random numbers. Steps for using the random number generator are provided. Students simulate a raffle drawing using the random number generator, performing 100 trials and compare the experimental probability of winning the raffle to the theoretical probability of winning the raffle. Students notice that as the number of trials increase, the experimental probability approaches the theoretical probability.

Warm Up

The Millionaire Raffle sells a limited number of tickets. The advertisement states only 500,000 tickets are sold. Four winning raffle numbers are randomly selected and each winner will receive 1 million dollars.

What is the probability of your raffle ticket winning the lottery?

The probability of winning the lottery is $\frac{4}{500,000}$, or $\frac{1}{125,000}$, because 4 out of the 500,000 tickets will win the million dollar prize.

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- Compare experimental and theoretical probability.

KEY TERMS

- theoretical probability
- simulation
- experimental probability

State lotteries have received mixed reviews ever since the first one was established in New Hampshire in 1964. Some people feel that state lotteries promote gambling and irresponsible game playing. Others feel that state lotteries are beneficial to schools and other public services. Can you think of some ideas that would benefit public services, but not be associated with gambling? Discuss your ideas with your partner.

Problem 1

Students explore the probabilities associated with winning a 3-digit lottery.

Grouping

- Ask students to read information. Discuss as a class.
- Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Discuss Phase, Questions 1 through 5

- What is 001 the smallest 3-digit number?
- What is the largest 3-digit number?
- Is 000 a possible 3-digit number?
- Is 1000 considered a 3-digit number?
- Would Ms. Mason win if the order of her three digits were different?
- Is the order of the digits important in this lottery?
- Would this be considered a situation with or without replacement? Explain.
- How many chances does Ms. Mason's book club have to win the lottery?
- Can one member choose 3-4-5 while a second member chooses 5-4-3?

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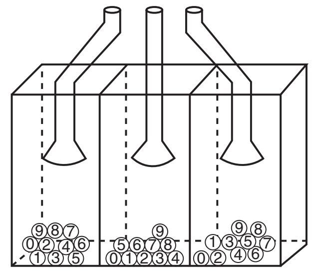
PROBLEM 1 Select a 3-digit Number and Win a Prize!

In many states, a common lottery game requires a player to purchase a ticket containing a 3-digit number. Each digit in the number ranges from 0 through 9, and repetition of numbers is allowed. Each day, a number is randomly chosen from three bins, each bin containing the numbers 0 through 9. If a player's 3-digit number matches the number randomly selected from the three bins, the player wins a cash lottery prize.



1. How many 3-digit lottery numbers are possible? Explain your reasoning.

   One-thousand different lottery numbers are possible.

   I used the Counting Principle to determine the answer.

   $10 \times 10 \times 10 = 1000$

2. Ms. Mason chooses the number 2-7-9. What is the probability that she will win the lottery?

   The probability Ms. Mason will win is $\frac{1}{1000}$.

   I calculated the answer by using the compound probability rule for "and."

   $P(\text{winning number}) = P(\text{winning 1st digit}) \cdot P(\text{winning 2nd digit}) \cdot P(\text{winning 3rd digit})$

   $= \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   $= \frac{1}{1000}$

- Would the book club members have a better chance of winning the lottery if they each placed their bets separately? Explain.
- If all of the members of the book club played the 4-digit lottery instead of the 3-digit lottery, what would be their chances of winning?
- Is one in a thousand a good chance of winning?
- What do you think is a good chance of winning?

- Does everyone have the same chance of winning the lottery? Explain.
- Do you have a better chance of winning the lottery if you play with a group of people or if you play separately? Explain.

3. Ms. Mason's book club occasionally plays the lottery as a group. There are 12 members in Ms. Mason's group. If each book club member chooses a different 3-digit number, what is the chance the book club will win a lottery cash prize?

The probability of the book club winning the lottery is $\frac{3}{250}$.

To determine the answer, I multiplied the probability of Ms. Mason selecting the winning number by 12.

$$P(\text{winning number}) = 12 \cdot [P(\text{winning 1st digit}) \cdot P(\text{winning 2nd digit}) \cdot P(\text{winning 3rd digit})]$$
$$= 12\left[\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right]$$
$$= 12 \cdot \left[\frac{1}{1000}\right]$$
$$= \frac{12}{1000}$$
$$= \frac{3}{250}$$

4. At one book club meeting, Mr. Kaplan said, "If I play the lottery game that involves choosing a 4-digit number by myself, the chance of winning will decrease." Is Mr. Kaplan correct? Explain your reasoning.

Mr. Kaplan is correct. The probability would decrease from $\frac{1}{1000}$ to $\frac{1}{10{,}000}$.

I calculated the answer by using the compound probability rule for "and."

Let *A* represent the event of randomly selecting the winning 1st digit.

Let *B* represent the event of randomly selecting the winning 2nd digit.

Let *C* represent the event of randomly selecting the winning 3rd digit.

Let *D* represent the event of randomly selecting the winning 4th digit.

$$P(A \text{ and } B \text{ and } C \text{ and } D) = P(A) \cdot P(B) \cdot P(C) \cdot P(D)$$
$$= \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$$
$$= \frac{1}{10{,}000}$$

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5. A similar lottery game claims that there is a "one in a million" chance to win. Assuming that the slogan is true, how many digits are in this lottery game? Explain your reasoning.

If there is a one in a million chance to win, then the lottery game has 6 digits because $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$.

I verified my answer by using the compound probability rule for "and."

Let A represent the event of randomly selecting the winning 1st digit.

Let B represent the event of randomly selecting the winning 2nd digit.

Let C represent the event of randomly selecting the winning 3rd digit.

Let D represent the event of randomly selecting the winning 4th digit.

Let C represent the event of randomly selecting the winning 5th digit.

Let D represent the event of randomly selecting the winning 6th digit.

$$P(A \text{ and } B \text{ and } C \text{ and } D \text{ and } E \text{ and } F) = P(A) \cdot P(B) \cdot P(C) \cdot P(D) \cdot P(E) \cdot P(F)$$
$$= \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$$
$$= \frac{1}{1{,}000{,}000}$$

Problem 2

Students compare the probability of winning a 3-digit lottery with the probability of winning a 6-digit lottery.

Grouping

- Ask students to read the information. Discuss as a class
- Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 3

- How did you determine the number of independent events for the Wishes Come True Lottery?
- How did you determine the number of outcomes in each event for the Supersonic Lottery?

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PROBLEM 2 A Good Chance of Winning? Think Again!

Sometimes, lottery companies encourage players to play for greater prizes, but fail to mention that the chances of winning are extremely low. Many times, lottery companies create different games where the chances of winning are not as obvious.

To participate in the Supersonic Lottery, players choose 3 numbers, each number ranges from 1 through 50 and repetition of numbers is allowed. Then, the winning numbers are randomly selected from 3 bins, each bin containing the numbers 1 through 50. The player's three numbers need to match the randomly selected three numbers in the same order to win the cash lottery prize.

In a similar fashion, players participate in the Wishes Come True Lottery by choosing 6 numbers, each number ranging from 1 through 25. Then, the winning numbers are randomly selected from six bins, each bin containing the numbers 1 through 25.

1. Use the information given to complete the table.

Lottery Name	Number of Independent Events	Number of Outcomes in Each Event	Probability of Winning
Supersonic Lottery	3	50	$\frac{1}{125{,}000}$
Wishes Come True Lottery	6	25	$\frac{1}{244{,}140{,}625}$

- What is 50 raised to the third power?
- What is 25 raised to the sixth power?
- Which do you think would be of greater value, 50 to the third power or 25 to the sixth power? Explain.
- Which has more impact on the answer in terms of largeness, the exponent or the base number?

2. Explain how you calculated the probability of winning each lottery.

I calculated the probability of winning the Supersonic Lottery by using the compound probability rule for "and."

Let *A* represent the event of randomly selecting the winning 1st digit.

Let *B* represent the event of randomly selecting the winning 2nd digit.

Let *C* represent the event of randomly selecting the winning 3rd digit.

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$
$$= \frac{1}{50} \cdot \frac{1}{50} \cdot \frac{1}{50}$$
$$= \frac{1}{125{,}000}$$

I also used the compound probability rule for "and" to calculate the probability of winning the Wishes Come True Lottery.

Let *A* represent the event of randomly selecting the winning 1st digit.

Let *B* represent the event of randomly selecting the winning 2nd digit.

Let *C* represent the event of randomly selecting the winning 3rd digit.

Let *D* represent the event of randomly selecting the winning 4th digit.

Let *E* represent the event of randomly selecting the winning 5th digit.

Let *F* represent the event of randomly selecting the winning 6th digit.

$$P(A, B, C, D, E, \text{ and } F) = P(A) \cdot P(B) \cdot P(C) \cdot P(D) \cdot P(E) \cdot P(F)$$
$$= \frac{1}{25} \cdot \frac{1}{25} \cdot \frac{1}{25} \cdot \frac{1}{25} \cdot \frac{1}{25} \cdot \frac{1}{25}$$
$$= \frac{1}{244{,}140{,}625}$$

3. Does the number of actions or the number of outcomes in each action have a greater impact on the probability? How can you tell?

The number of independent events has a greater impact on the probability.

The Wishes Come True Lottery has a much smaller probability. Compared to the Supersonic Lottery, the Wishes Come True Lottery has twice as many events and half as many outcomes in each event. From this example, it seems that the number of events has a greater impact on probability.

Problem 3

Students compare probabilities of winning the lottery to being struck by lightning. They conclude that a person is approximately 5 times more likely to be struck by lightning.

Grouping

- Ask students to read the information. Discuss as a class.
- Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2

- How many outcomes are possible in the Special 5 Lottery?
- What is 19 raised to the fifth power?
- Is $\frac{600}{300{,}000{,}000}$ greater than or less than $\frac{1}{2{,}476{,}099}$?
- What is $\frac{600}{300{,}000{,}000}$ reduced to lowest terms?
- Approximately how many times does 500,000 go into 2,476,099?

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PROBLEM 3 Lottery vs. Lightning

Use the information provided to answer Questions 1 and 2.

- To play the Special 5 Lottery, players choose 5 numbers. Each number ranges from 1 through 19, and repetition of numbers is allowed. The winning numbers are randomly selected from 5 bins, each bin containing the numbers 1 through 19. The player's five numbers need to match the randomly selected five numbers in the same order to win the cash lottery prize.
- In 2008, the National Oceanic and Atmospheric Administration (NOAA) estimated that there were 600 victims of lightning strikes in the United States. In 2008, the approximate population of the United States was 300,000,000.

1. Do you have a better chance of winning the Special 5 Lottery or getting struck by lightning? Explain your reasoning.

   It is more likely to be struck by lightning. I answered the question by calculating each probability and comparing their values.

   I calculated the probability of winning the Special Lottery by using the compound probability rule for "and."

   Let *A* represent the event of randomly selecting the winning 1st digit.

   Let *B* represent the event of randomly selecting the winning 2nd digit.

   Let *C* represent the event of randomly selecting the winning 3rd digit.

   Let *D* represent the event of randomly selecting the winning 4th digit.

   Let *E* represent the event of randomly selecting the winning 5th digit.

   $$P(A, B, C, D, \text{ and } E) = P(A) \cdot P(B) \cdot P(C) \cdot P(D) \cdot P(E)$$
   $$= \frac{1}{19} \cdot \frac{1}{19} \cdot \frac{1}{19} \cdot \frac{1}{19} \cdot \frac{1}{19}$$
   $$= \frac{1}{2{,}476{,}099}$$

   To determine the probability of getting hit by lightning, I wrote a ratio using the given number of people who were hit by lightning and the total population.

   $$P(\text{getting hit by lightning}) = \frac{\text{desired outcomes}}{\text{possible outcomes}}$$
   $$= \frac{600}{300{,}000{,}000}$$
   $$= \frac{1}{500{,}000}$$

   By comparing the probabilities $\frac{1}{2{,}476{,}099}$ and $\frac{1}{500{,}000}$, I can see that $\frac{1}{500{,}000}$ is greater than $\frac{1}{2{,}476{,}099}$. This means it is more likely to be struck by lightning than it is to win the Special 5 Lottery.

Problem 4

As a class, students select a 2-digit raffle number and calculate the probability of winning the raffle. The term simulation is defined, and the formula for experimental probability is given. Steps for using a graphing calculator to generate random numbers are provided. Students perform a total of 100 trials using the random number generator to determine the experimental probability of the classes' 2-digit raffle number winning the raffle. This experimental probability is compared to the theoretical probability.

Grouping

- Ask students to read the information. Complete Questions 1 and 2 as a class.
- Have students complete Questions 3 through 6 with a partner. Then have students share their responses as a class.

2. Based on your answer to Question 1, how many times greater is the probability of the more likely event to occur than the less likely event?

It is about 5 times more likely for a person to be struck by lightning than it is to win the Special 5 Lottery.

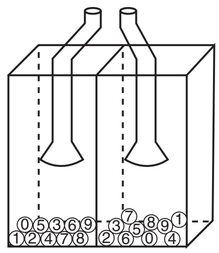
I determined the answer by dividing the denominators of the probabilities.

$\frac{2,476,099}{500,000} \approx 4.95$

PROBLEM 4 A Simulation of a Raffle

Some raffles operate like lotteries.

In this raffle, players choose a two-digit number between 00 and 99. A number is randomly chosen from 2 bins, each bin containing the numbers 0 through 9.



1. As a class, select a 2-digit raffle number.

Answers will vary.

For example, the class may choose the number 32.

2. Calculate the probability of winning the raffle. Show your work.

The probability of winning the raffle is $\frac{1}{100}$.

I calculated the probability of winning the raffle by using the compound probability rule for "and."

Let A represent the event of randomly selecting the winning 1st digit.

Let B represent the event of randomly selecting the winning 2nd digit.

$P(A \text{ and } B) = P(A) \cdot P(B)$

$= \frac{1}{10} \cdot \frac{1}{10}$

$= \frac{1}{100}$

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Guiding Questions for Discuss Phase, Questions 1 and 2

- What 2-digit raffle number did the class select?
- Is the number 00 or 99 a possible raffle number?
- How many total outcomes are possible?
- Did you calculate theoretical probability?
- What is theoretical probability?
- What is an example of a simulation?

- Have you ever used a simulation? Explain.
- What is the difference between experimental probability and theoretical probability?
- Which do you think is more accurate, experimental probability or theoretical probability? Explain.

The **theoretical probability** of an event is the likelihood that the event will occur. It is determined from a sample space of known outcomes.

$$\text{Theoretical Probability} = \frac{\text{number of desired outcomes in known sample space}}{\text{total number of possible outcomes in known sample space}}$$

You can *simulate* the selection of raffle numbers by using the random number generator on a graphing calculator. A **simulation** is an experiment that models a real-life situation.

The data from a simulation is used to determine **experimental probability**.

$$\text{Experimental Probability} = \frac{\text{number of times an outcome occurs}}{\text{total number of trials performed}}$$

Use your graphing calculator to generate random numbers with the following key strokes.

Step 1: Press **MATH**.

```
MATH NUM CPX PRB
1:▶Frac
2:▶Dec
3:³
4:³√(
5:ˣ√
6:fMin(
7↓fMax(
```

Step 2: Scroll across to **PRB** using the arrows.

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

Step 3: Scroll down to **randInt(** using the arrows. This command generates random integers.

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

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Step 4: Press **ENTER** and then type the minimum number in the range, the maximum number in the range, and number of times you would like to generate numbers.

This command will generate 5 random integers between 0 and 99, including 0 and 99.

```
randInt(0,99,5)█
```

Step 5: Press **ENTER**.
Your screen should show the random numbers.

```
randInt(0,99,5)
{11 10 68 70 50}
█
```

Generating 5 numbers at a time is efficient because your calculator can display five 2-digit numbers on one line. So, you can display 100 numbers with twenty rows of 5 numbers.

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Guiding Questions for Share Phase, Questions 3 through 6

- How does the appearance of the classes' raffle number compare to other classmates results?
- How many trials did you and your partner conduct?
- How many trials did your entire class conduct collectively?
- What is the experimental probability resulting from the entire classes' trials?
- Are all of the experimental probabilities approximately the same?
- As the number of trials increase, what relationship occurs between the experimental probability and the theoretical probability?

3. Work with a partner. Complete this activity twice, once for each of you.

- Generate twenty sets of five random numbers using your calculator. This will provide 100 trials.
- Record the number of times that the class's raffle number matches one of the random numbers.

The class's raffle number appeared ___2___ times out of 100 trials.

4. Determine the experimental probability for the raffle simulation.

Answers will vary.

Experimental Probability

$= \frac{\text{number of times an outcome occurs}}{\text{total amount of trials performed}}$

$= \frac{2}{100}$

$= \frac{1}{50}$

5. Are the values for the theoretical probability and the experimental probability equivalent?

The probabilities are very close to equivalent.

6. Collect the data for the entire class.

Determine the experimental probability using the results from the entire class.

Write the experimental probability as a fraction with a denominator of 100, and a numerator that may be a decimal number rounded to one decimal place.

The probabilities should be approximately $\frac{1}{100}$.

In mathematical simulations, the value of the experimental probability approaches the value of the theoretical probability when the number of trials increases.

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Talk the Talk

Questions compare theoretical and experimental probabilities.

Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

Talk the Talk

1. How are experimental probability and theoretical probability alike?

   Both probabilities measure the likelihood of an event occurring. If there are a large number of trials when determining experimental probability, the result should be very close to the theoretical probability.

2. How are experimental probability and theoretical probability different?

   The experimental probability can vary because it is based on the number of times an outcome occurs and the total number of trials can vary. However, theoretical probability does not vary because it is based on an established number of desired outcomes and an established number of possible outcomes.

Be prepared to show your solutions and methods.

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Check for Students' Understanding

The Daily Number Lottery is a 3-digit number raffle with repetition, each digit ranging from 0 – 9.

1. PLAY IT STRAIGHT:

   What is the probability of your lottery ticket having an exact match to the winning lottery number? Order **is** important.

   The probability of an exact match is $\frac{1}{1000}$.

   I determined the answer by using the Rule of Compound Probability involving "and."

   Let *A* represent selecting the winning 1st digit.

   Let *B* represent selecting the winning 2nd digit.

   Let *C* represent selecting the winning 3rd digit.

   $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$

   $= \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   $= \frac{1}{1000}$

2. PLAY FRONT PAIR:

   What is the probability of choosing only the front two digits of the three digit winning lottery number? Order **is** important

   The probability of having the front two digits of the winning lottery number in order is $\frac{9}{1000}$.

   I determined the answer by using the Rule of Compound Probability involving "and."

   Let *A* represent selecting the winning 1st digit.

   Let *B* represent selecting the winning 2nd digit.

   Let *C* represent selecting a non-winning 3rd digit.

   $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$

   $= \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{9}{10}$

   $= \frac{9}{1000}$

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3. PLAY BACK PAIR:

What is the probability of choosing only the back two digits of the three digit winning lottery number? Order **is** important

The probability of having the back two digits of the winning lottery number in order is $\frac{9}{1000}$.

I determined the answer by using the Rule of Compound Probability involving "and."

Let *A* represent selecting a non-winning 1st digit.

Let *B* represent selecting the winning 2nd digit.

Let *C* represent selecting the winning 3rd digit.

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$
$$= \frac{9}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$$
$$= \frac{9}{1000}$$

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Chapter 19 Summary

KEY TERMS

- outcome (19.1)
- sample space (19.1)
- event (19.1)
- probability (19.1)
- probability model (19.1)
- uniform probability model (19.1)
- complement of an event (19.1)
- non-uniform probability model (19.1)
- tree diagram (19.2)
- organized list (19.2)
- set (19.2)
- element (19.2)
- disjoint sets (19.2)
- intersecting sets (19.2)
- independent events (19.2)
- dependent events (19.2)
- Counting Principle (19.2)
- compound event (19.3)
- Rule of Compound Probability involving "and" (19.3)
- Addition Rule for Probability (19.4)
- theoretical probability (19.6)
- simulation (19.6)
- experimental probability (19.6)

19.1 Listing a Sample Space for a Situation Involving Probability

All of the possible outcomes in a probability experiment or situation are called the sample space.

The probability of an event *A* is the ratio of the number of desired outcomes to the total number of possible outcomes.

$$P(A) = \frac{\text{desired outcomes}}{\text{possible outcomes}}$$

Example

For breakfast, Jake can have pancakes, oatmeal, or cereal. He can choose either orange juice or apple juice to drink.

The sample space of breakfast choices is:

pancakes and orange juice	oatmeal and orange juice	cereal and orange juice
pancakes and apple juice	oatmeal and apple juice	cereal and apple juice

The probability Jake randomly chooses pancakes and apple juice is $\frac{1}{6}$.

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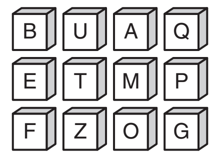
19.1 Describing the Probability of an Event and Its Complement

An event (E) is an outcome or set of outcomes in a sample space. The complement of an event (E^c) is an event that contains all the outcomes in the sample space that are not outcomes in the event. The probability of an event and the probability of its complement must equal 1 since together they make up the entire sample space: $P(E) + P(E^c) = 1$.

Example



If one of the blocks is chosen at random, what is the probability that it will have a vowel on it?

The event is choosing a block with a vowel on it.

The complement of the event is choosing a block that does not have a vowel on it.

$P(E) = \frac{1}{3}$

$P(E^c) = \frac{2}{3}$

$P(E) + P(E^c) = \frac{1}{3} + \frac{2}{3} = 1$

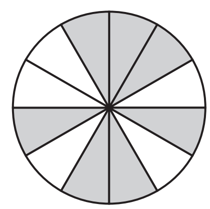
19.1 Constructing a Probability Model for a Situation

A probability model lists all of the possible outcomes and each outcome's probability. In a probability model, the sum of the probabilities must equal 1.

Example

The probability model for the spinner is:

Outcomes	Shaded	Unshaded
Probability	$\frac{7}{12}$, or 0.5833	$\frac{5}{12}$, or 0.4167

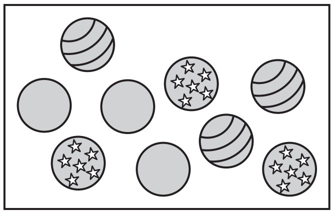


19.1 Differentiating Between Uniform and Non-uniform Probability Models

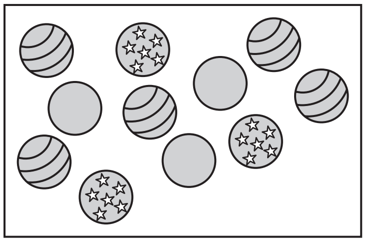
If the outcomes in a probability model are all equal, the model is called a uniform probability model. If the outcomes are not all equal, the model is called a non-uniform probability model.

Example

Uniform probability model:



Non-uniform probability model:



Outcomes	Plain Ball	Ball with Stripes	Ball with Stars
Probability	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Outcomes	Plain Ball	Ball with Stripes	Ball with Stars
Probability	$\frac{3}{11}$	$\frac{5}{11}$	$\frac{3}{11}$

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19.2 Categorizing Problems Involving Events

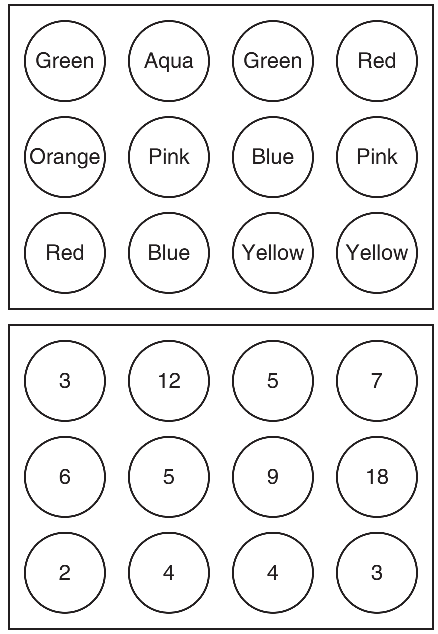
Independent events are events for which the occurrence of one event has no impact on the occurrence of the other event. Independent events may be selected from either intersecting sets or from disjoint sets. Intersecting sets have at least one common element. Disjoint sets do not have any common elements. Dependent events are events for which the occurrence of one event has an impact on the occurrence of the following events.

Example

Independent event from disjoint sets:
 Choose 1 ball from the color set and, choose 1 ball from the number set.

Independent event from intersecting sets:
 Choose 1 ball from the color set. Replace it. Then choose another ball from the color set.

Dependent event from intersecting sets:
 Choose 1 ball from the color set. Do not replace it. Then choose another ball from the color set.



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19.2 Using the Counting Principle

You can use the Counting Principle to calculate the size of a sample space for a situation. According to the Counting Principle, if an action A can occur in m ways, and for each of these m ways, an action B can occur in n ways, then actions A and B can occur in $m \cdot n$ ways. This principle can be extended to include more actions. If, for example, after A and B occur, then another action C can occur in s ways, then actions A, B, and C can occur in $m \cdot n \cdot s$ ways.

Example

Eight students are on the school tennis team. A photographer wants to arrange the students in a line for a picture. How many arrangements are possible?

There are 8 choices for the first student, leaving 7 students for the remaining spots. Then, there are 7 choices for the second student, leaving 6 students for the remaining spots, and so on. Each choice is therefore a dependent event from intersecting sets. We can use the Counting Principle to determine the number of arrangements:

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$$

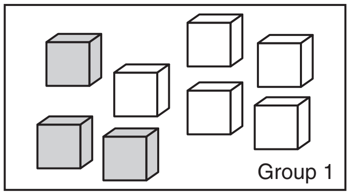
19.3 Determining the Probability of Two or More Independent Events Involving "and"

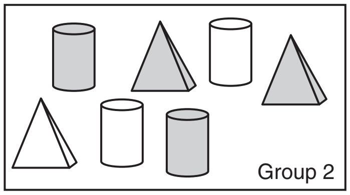
A compound event consists of two or more events. When calculating the probability of a compound event involving "and," you multiply the probabilities of each individual event. For independent events, the Rule of Compound Probability involving "and" is:

> The probability that Event A happens and Event B happens is the product of the probability that Event A happens and the probability that Event B happens:

$$P(A \text{ and } B) = P(A) \times P(B)$$

Example





What is the probability of randomly choosing a shaded block from Group 1 and then choosing a pyramid from Group 2?

Event A is choosing a shaded block from Group 1.
Event B is choosing a pyramid from Group 2.
These are independent events.

$$P(A \text{ and } B) = P(A) \times P(B) = \frac{3}{8} \times \frac{3}{7} = \frac{9}{56}$$

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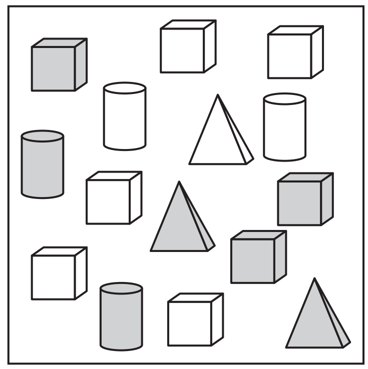
19.3 Determining the Probability of Two or More Dependent Events Involving "and"

If compound events are dependent, the probability of an event is altered by previous events. For dependent events, the Rule of Compound Probability involving "and" is:

> The probability that Event A happens and Event B happens is the product of the probability that Event A happens and the probability that Event B happens given that Event A has happened.

$$P(A \text{ and } B) = P(A) \times P(B, \text{ given } A)$$

Example



What is the probability of randomly choosing a shaded block, and then randomly choosing a block that is not shaded, and then randomly choosing another shaded block without replacing any of the objects?

Event A is choosing a shaded block from the group without replacing it.

Event B is choosing a block that is not shaded from the group after a shaded block is chosen.

Event C is choosing a block that is shaded from the group after a shaded block and a not-shaded block are chosen.

These are dependent events.

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B, \text{ given } A) \times P(C, \text{ given } A \text{ and } B) = \frac{7}{15} \times \frac{8}{14} \times \frac{6}{13} = \frac{8}{65}$$

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19.4 Determining the Probability of Independent Events Involving "or"

To determine the probability that either of two compound events will occur, use the Addition Rule for Probability:

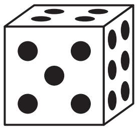
> The probability that Event A occurs or Event B occurs is the probability that Event A occurs plus the probability that Event B occurs minus the probability that both A and B occur.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

For independent events A and B, $P(A \text{ and } B) = P(A) \times P(B)$.

Example

A number cube has a different number on each of 6 sides. What is the probability of rolling a number cube twice such that you roll a 5 the first time and a 6 the second time?



$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{1}{6} + \frac{1}{6} - \left(\frac{1}{6} \times \frac{1}{6}\right) \\ &= \frac{2}{6} - \frac{1}{36} \\ &= \frac{12}{36} - \frac{1}{36} \\ &= \frac{11}{36} \end{aligned}$$

19.4 Determining the Probability of Dependent Events Involving "or"

You can also apply the Addition Rule for Probability to determine the probability that either of two compound events will occur when the events are dependent. In this case, however, the probability of Event B occurring is influenced by Event A.

$$P(A \text{ or } B) = P(A) + P(B\text{, given } A) - P(A \text{ and } B \text{ given A})$$

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Example

Terrence has different colors of tee-shirts in his drawer: 2 red t-shirts, 2 green t-shirts, and 3 blue t-shirts. If he chooses a t-shirt at random from the drawer without replacing it, and then randomly chooses another, what is the probability of choosing a blue shirt first or a green shirt second?

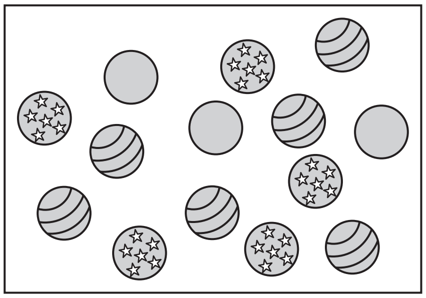
The events are dependent.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B\text{, given } A) - P(A \text{ and } B \text{ given } A) \\ &= \frac{3}{7} + \frac{1}{3} - \left(\frac{3}{7} \times \frac{1}{3}\right) \\ &= \frac{3}{7} + \frac{1}{3} - \frac{1}{7} \\ &= \frac{7}{21} \end{aligned}$$

19.5 Calculating Compound Probabilities With and Without Replacement

You often see the phrases "with replacement" and "without replacement" in probability problems. These phrases refer to whether the sample space changes from the first to the last event. Situations "with replacement" generally involve independent events. The first event has no effect on the occurrence of the following events. Situations "without replacement" generally involve dependent events. The first event has an impact on the occurrence of later events.

Example



What is the probability of randomly choosing a ball from the set that has stars on it and then randomly choosing a ball that is not plain?

Event A is choosing a ball that has stars on it.

Event B is choosing a ball that is not plain.

With replacement, the events are independent:

$$P(A \text{ and } B) = P(A) \times P(B) = \frac{5}{14} \times \frac{11}{14} = \frac{55}{196}$$

Without replacement, the events are dependent:

$$P(A \text{ and } B) = P(A) \times P(B, \text{ given } A) = \frac{5}{14} \times \frac{11}{13} = \frac{55}{182}$$

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19.6 Comparing Experimental and Theoretical Probabilities

A simulation is an experiment that models a real-life situation. You can use random numbers for a simulation. The following keystrokes on a graphing calculator generate random numbers:

- Press MATH.
- Scroll across to PRB using the arrows.
- Scroll down to randInt(using the arrows.
- Press ENTER.
- Enter 0,99,5) to generate random numbers from 0 to 99, generated 5 at a time.
- Press ENTER to view the random numbers.

Because you are completing an experiment, you use the experimental probability formula:

$$\text{experimental probability} = \frac{\text{number of times an outcome occurs}}{\text{total number of trials performed}}$$

The expected outcome of an experiment is called the theoretical probability:

$$\text{theoretical probability} = \frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}}$$

In mathematical simulations, the value of the experimental probability approaches the value of the theoretical probability when the number of trials increases.

Example

Suppose you buy three raffle tickets. Only 100 tickets are sold, and each has a number from 0 to 99 on it. What is the probability that you will win the raffle?

The theoretical probability is always the same:

$$\text{theoretical probability} = \frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}} = \frac{3}{100}$$

The experimental probability can vary based on the number of trials. Using the random number generator on a calculator, you can simulate 100 trials. Suppose one of your three raffle numbers appears 11 times:

$$\text{experimental probability} = \frac{\text{number of times an outcome occurs}}{\text{total number of trials performed}} = \frac{11}{100}$$

You run another experiment, simulating 300 trials. Suppose one of your three raffle numbers appears 12 times:

$$\text{experimental probability} = \frac{\text{number of times an outcome occurs}}{\text{total number of trials performed}} = \frac{12}{300} = \frac{1}{25}$$

Notice the experimental probability for 100 trials is different from the experimental probability for 300 trials. As the number of trials increases, the value of the experimental probability will get closer and closer to the value of the theoretical probability.

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