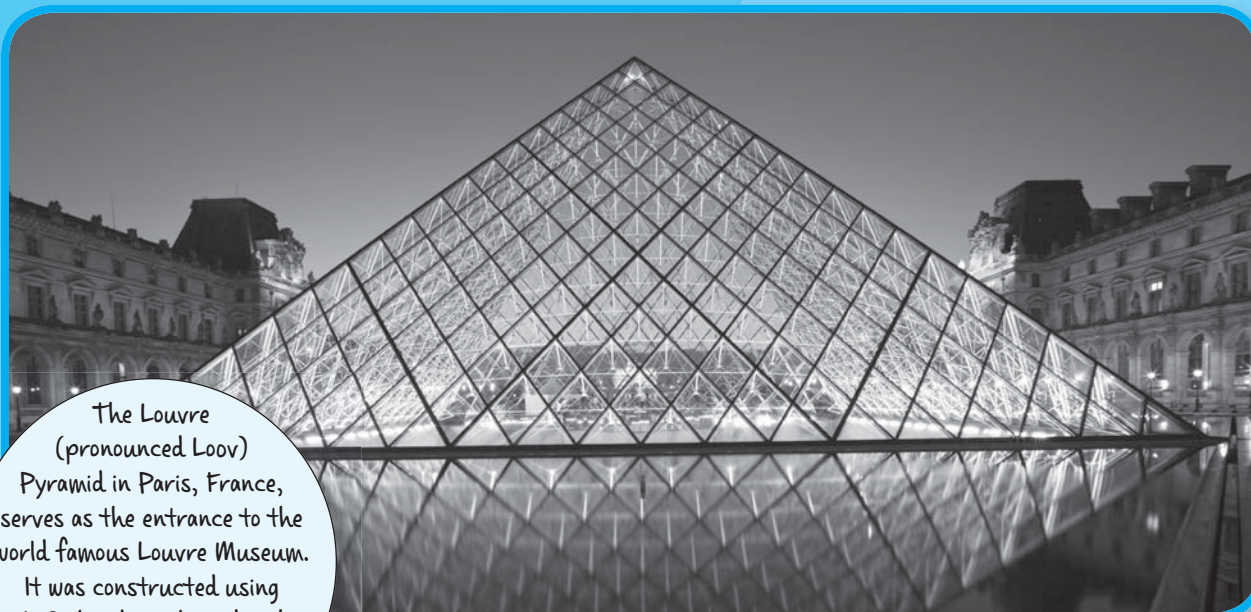


# Congruence Through Transformations

# 5



The Louvre (pronounced Loov) Pyramid in Paris, France, serves as the entrance to the world famous Louvre Museum. It was constructed using 673 rhombus-shaped and triangular glass segments.



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## Chapter 5 Overview

This chapter focuses on proving triangle congruence theorems and using the theorems to determine whether triangles are congruent.

Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
5.1	Translating, Rotating, and Reflecting Geometric Figures	G.CO.2 G.CO.3 G.CO.5	2	<p>This lesson explores rigid motions that preserve congruency.</p> <p>Students will translate, rotate, and reflect a single point and then a given trapezoid. They then determine the image coordinates without graphing.</p>	X	X	X	X	
5.2	Congruent Triangles	G.CO.6 G.CO.7 G.CO.8	1	<p>This lesson explores the relationship between corresponding sides and corresponding angles of congruent triangles on a coordinate plane.</p> <p>Questions then ask students to identify rigid motion transformations used to create new images, and write triangle congruence statements.</p>	X			X	
5.3	Side-Side-Side Congruence Theorem	G.CO.6 G.CO.7 G.CO.8 G.CO.10 G.CO.12	1	<p>This lesson begins by providing opportunities for student to explore the SSS Congruence Theorem using construction and triangles on the coordinate plane. Then, a formal two-column proof of the SSS Congruence Theorem is presented.</p> <p>Questions ask students to use the SSS Congruence Theorem to determine whether two triangles are congruent.</p>	X		X		
5.4	Side-Angle-Side Congruence Theorem	G.CO.6 G.CO.7 G.CO.8 G.CO.10 G.CO.12	1	<p>This lesson begins by providing opportunities for student to explore the SAS Congruence Theorem using construction and triangles on the coordinate plane. Then, a formal two-column proof of the SAS Congruence Theorem is presented.</p> <p>Questions ask students to determine whether two triangles are congruent by SSS or SAS.</p>	X	X	X		

Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
5.5	Angle-Side-Angle Congruence Theorem	G.CO.6 G.CO.7 G.CO.8 G.CO.10 G.CO.12	1	<p>This lesson begins by providing opportunities for student to explore the ASA Congruence Theorem using construction and triangles on the coordinate plane. Then, a formal two-column proof of the ASA Congruence Theorem is presented.</p> <p>Questions ask students to use the ASA Congruence Theorem to determine whether two triangles are congruent.</p>	X		X		
5.6	Angle-Angle-Side Congruence Theorem	G.CO.6 G.CO.7 G.CO.8 G.CO.10 G.CO.12	1	<p>This lesson begins by providing opportunities for student to explore the AAS Congruence Theorem using construction and triangles on the coordinate plane. Then, a formal two-column proof of the AAS Congruence Theorem is presented.</p> <p>Questions ask students to determine whether two triangles are congruent by ASA or AAS, and summarize the four triangle congruence theorems.</p>	X		X	X	
5.7	Using Congruent Triangles	G.CO.6 G.CO.7 G.CO.8 G.CO.9 G.CO.12	2	<p>This lesson provides student with opportunities to use all four theorems to determine triangle congruency.</p> <p>A proof about a perpendicular bisector of a line segment is included. Questions guide students to understand that AAA and SSA are not valid triangle congruence theorems.</p>	X				

## Skills Practice Correlation for Chapter 5

Lesson		Problem Set	Objectives
5.1	Translating, Rotating, and Reflecting Geometric Figures		Vocabulary
		1 – 10	Transform geometric figures on the coordinate plane
		11 – 16	Determine the coordinates of translated figures without graphing
		17 – 22	Determine the coordinates of rotated figures without graphing
		23 – 28	Determine the coordinates of reflected figures without graphing
5.2	Congruent Triangles	1 – 10	Identify the transformation used to create triangles on the coordinate plane and identify congruent sides and angles
		11 – 18	List corresponding sides and angles of triangles given congruence statements
5.3	Side-Side-Side Congruence Theorem		Vocabulary
		1 – 6	Use the Distance Formula to determine whether triangles are congruent by SSS
		7 – 12	Transform triangles on the coordinate plane and verify that the triangles are congruent by SSS
5.4	Side-Angle-Side Congruence Theorem		Vocabulary
		1 – 6	Use the Distance Formula to determine whether triangles are congruent by SAS
		7 – 12	Transform triangles on the coordinate plane and verify that the triangles are congruent by SAS
		13 – 20	Determine the angle measure or side measure needed to prove triangles congruent by SAS
		21 – 28	Determine whether there is enough information to prove triangles congruent by SSS or SAS
5.5	Angle-Side-Angle Congruence Theorem		Vocabulary
		1 – 6	Use the Distance Formula to determine whether triangles are congruent by ASA
		7 – 12	Transform triangles on the coordinate plane and verify that the triangles are congruent by ASA
		13 – 20	Determine the angle measure or side measure needed to prove triangles congruent by ASA

Lesson		Problem Set	Objectives
5.6	Angle-Angle-Side Congruence Theorem		Vocabulary
		1 – 6	Use the Distance Formula to determine whether triangles are congruent by AAS
		7 – 12	Transform triangles on the coordinate plane and verify that the triangles are congruent by AAS
		13 – 20	Determine the angle measure or side measure needed to prove triangles congruent by AAS
		21 – 28	Determine whether there is enough information to prove triangles congruent by ASA or AAS
5.7	Using Congruent Triangles	1 – 6	Construct the perpendicular bisectors of given line segments
		7 – 12	Use triangle congruence theorems to complete proofs
		13 – 18	Provide counterexamples to demonstrate that given theorems do not prove triangle congruence
		19 – 24	State congruence theorems that prove triangles congruent



# We Like to Move It!

## Translating, Rotating, and Reflecting Geometric Figures

### LEARNING GOALS

In this lesson, you will:

- Translate geometric figures on a coordinate plane.
- Rotate geometric figures on a coordinate plane.
- Reflect geometric figures on a coordinate plane.

### ESSENTIAL IDEAS

- When a horizontal translation occurs on a coordinate plane the  $x$ -coordinates of the pre-image change, but the  $y$ -coordinates remain the same.
- When a vertical translation occurs on a coordinate plane the  $y$ -coordinates of the pre-image change, but the  $x$ -coordinates remain the same.
- When a point or image on a coordinate plane is rotated  $90^\circ$  counterclockwise about the origin, its original coordinates  $(x, y)$  change to  $(-y, x)$ .
- When a point or image on a coordinate plane is rotated  $180^\circ$  counterclockwise about the origin, the original coordinates  $(x, y)$  change to  $(-x, -y)$ .
- When a point or image on a coordinate plane is rotated  $270^\circ$  counterclockwise about the origin, the original coordinates  $(x, y)$  change to  $(y, -x)$ .
- When a point or image on a coordinate plane is rotated  $360^\circ$  counterclockwise about the origin, the original coordinates  $(x, y)$  do not change.
- When a point or image on a coordinate plane is reflected over the  $x$ -axis, the original coordinates  $(x, y)$  change to  $(x, -y)$ .
- When a point or image on a coordinate plane is reflected over the  $y$ -axis, the original coordinates  $(x, y)$  change to  $(-x, y)$ .

### COMMON CORE STATE STANDARDS FOR MATHEMATICS

#### G-CO Congruence

#### Experiment with transformations in the plane

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

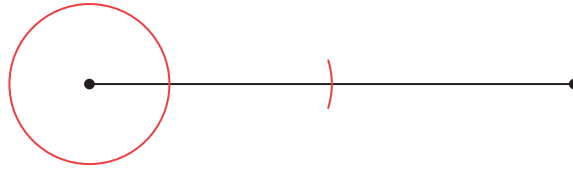
## Overview

This lesson begins with students cutting out a model of a trapezoid. Students will use this model on a coordinate plane to study rigid motion in this chapter. In the first problem, students will translate a single point and then a trapezoid. In the second problem, students rotate a single point, a trapezoid, and a parallelogram. In the third problem, students reflect a single point, a trapezoid.

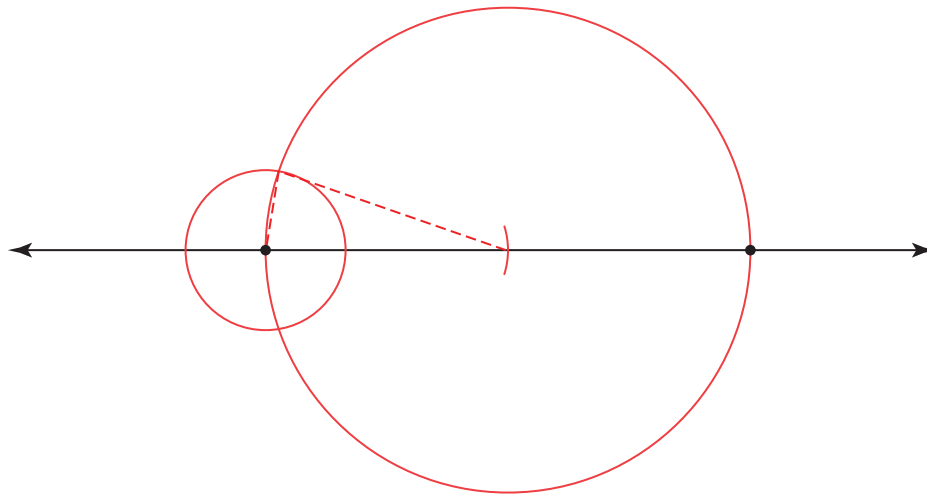


## Warm Up

The line segment shown represents the perimeter of a triangle.



1. Use the starter line to construct a triangle with this perimeter.



2. How many different triangles could be constructed from this given perimeter?  
**An infinite number of triangles could be constructed from this given perimeter.**
3. How did you begin the construction?  
**I began the construction by dividing the perimeter into three segments and then duplicated one of the segments on the starter line.**
4. How many arcs or circles were constructed, and how many line segments were duplicated for to complete this construction?  
**I constructed two circles and duplicated one line segment.**



# We Like to Move It!

## Translating, Rotating, and Reflecting Geometric Figures

### LEARNING GOALS

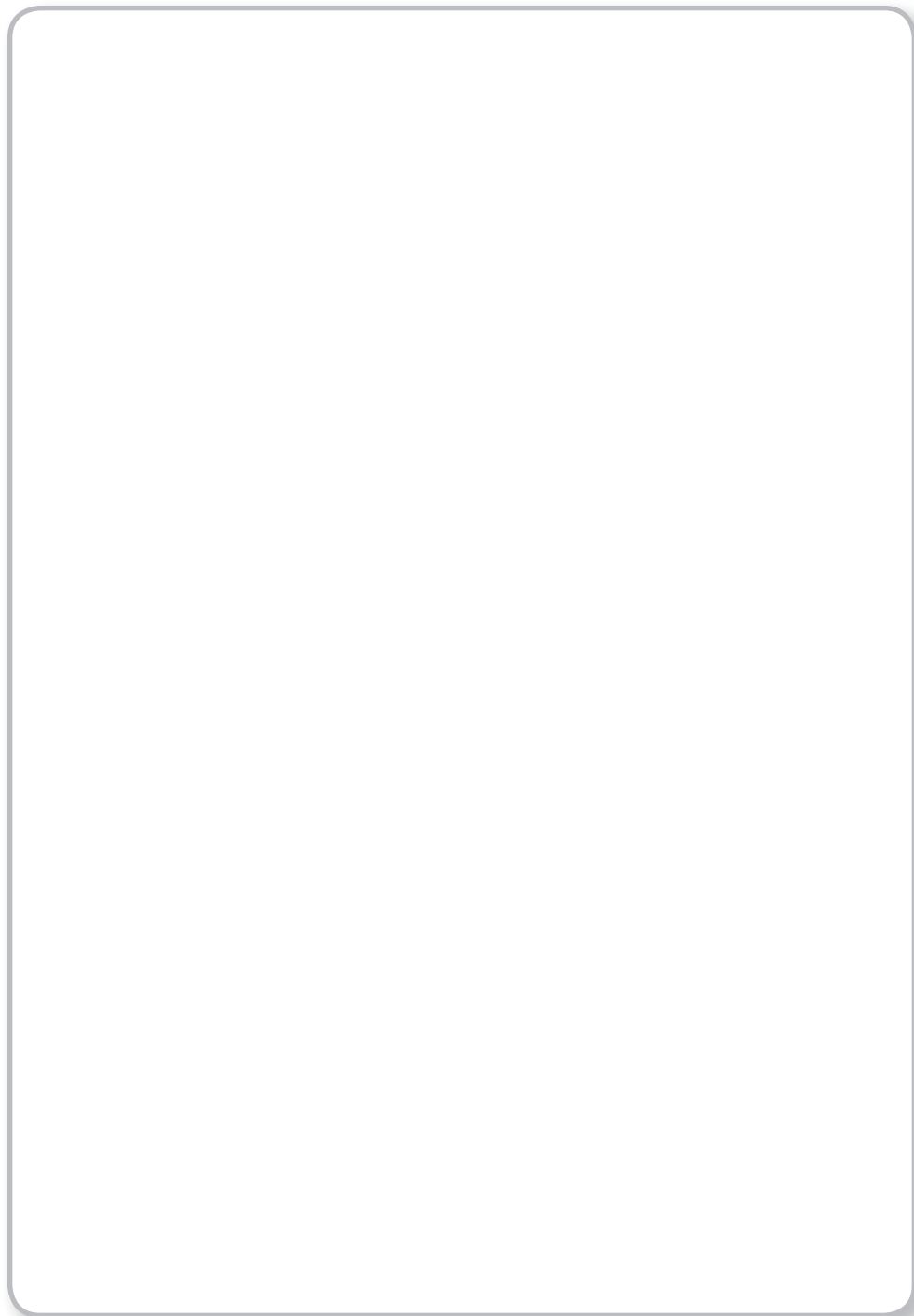
In this lesson, you will:

- Translate geometric figures on a coordinate plane.
- Rotate geometric figures on a coordinate plane.
- Reflect geometric figures on a coordinate plane.

**D**id you know that most textbooks are translated from English into at least one other language, usually Spanish? And in some school districts, general memos and letters to parents may be translated into up to five different languages! Of course, *translating* a language means something completely different from the word *translating* in geometry.

The same can be said for reflection. A “reflection pool” is a place where one can “reflect” on one’s thoughts, while also admiring reflections in the pool of still water.

How about rotation? What do you think the term *rotation* means in geometry? Is this different from its meaning in common language?



## Problem 1

Translation is described as a rigid motion that slides each point of a figure the same distance and direction. Students are first given four vertices in the first quadrant. They will use their model of a trapezoid to translate the trapezoid horizontally and vertically. Using translations, they form different trapezoids and record the coordinates of the vertices of the images. Finally, students are given the coordinates of the vertices of a parallelogram and without graphing they are able to determine the coordinates of images resulting from different translations.

## Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 and 2

- How did you determine the coordinates of the vertices of the trapezoid when it was translated left 15 units?
- How did you determine the coordinates of the vertices of the trapezoid when it was translated down 12 units?
- Could you have determined the coordinates of the vertices of the image without graphing? How?

## PROBLEM 1 Translating Geometric Figures on the Coordinate Plane

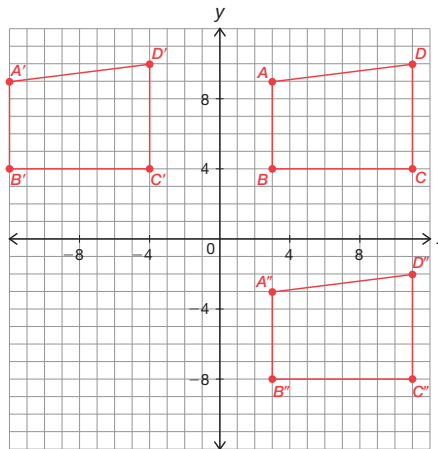
To begin this chapter, cut out a copy of the figure shown.



You will use this trapezoid throughout this lesson so don't lose it!

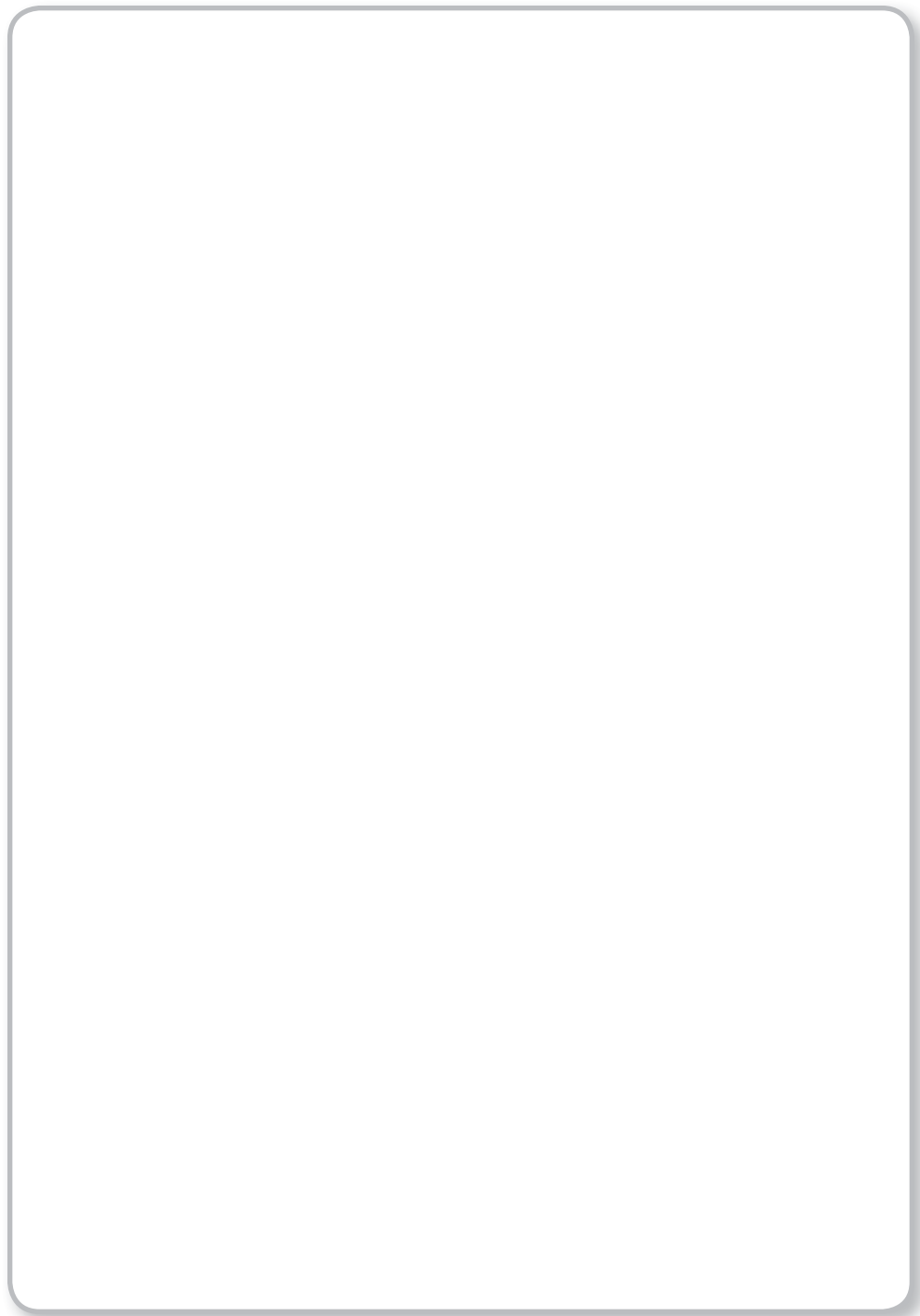


1. Graph trapezoid  $ABCD$  by plotting the points  $A(3, 9)$ ,  $B(3, 4)$ ,  $C(11, 4)$ , and  $D(11, 10)$ .



Use the model you cut out to help with the translations.





2. Translate trapezoid  $ABCD$  on the coordinate plane. Graph the image and record the vertex coordinates in the table.

a. Translate trapezoid  $ABCD$  15 units to the left to form trapezoid  $A'B'C'D'$ .

b. Translate trapezoid  $ABCD$  12 units down to form trapezoid  $A''B''C''D''$ .

Coordinates of Trapezoid $ABCD$	Coordinates of Trapezoid $A'B'C'D'$	Coordinates of Trapezoid $A''B''C''D''$
$A (3, 9)$	$A' (-12, 9)$	$A'' (3, -3)$
$B (3, 4)$	$B' (-12, 4)$	$B'' (3, -8)$
$C (11, 4)$	$C' (-4, 4)$	$C'' (11, -8)$
$D (11, 10)$	$D' (-4, 10)$	$D'' (11, -2)$



## Grouping

Have students complete Question 3 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Question 3

- Is translating a parallelogram different than translating a trapezoid? How?
- How do you know which coordinate changes during the transformation?
- How did you know which coordinate does not change during the transformation?

## Problem 2

Rotation is described as a rigid motion that turns a figure about a fixed point for a given angle. The problem begins with an example showing how to rotate a point located in the first quadrant  $90^\circ$  and  $180^\circ$  counterclockwise using the origin as the point of rotation. A table is given that shows the pattern of the coordinates from  $90^\circ$  and  $180^\circ$  counterclockwise rotation. Students will plot a point and use the table to determine the location of the point after a given rotation. Next, students explore rotations given a trapezoid

Let's consider translations without graphing.



3. The vertices of parallelogram  $DEFG$  are  $D(-9, 7)$ ,  $E(-12, 2)$ ,  $F(-3, 2)$ , and  $G(0, 7)$ .

- a. Determine the vertex coordinates of image  $D'E'F'G'$  if parallelogram  $DEFG$  is translated 14 units down.

The vertex coordinates of image  $D'E'F'G'$  are  $D'(-9, -7)$ ,  $E'(-12, -12)$ ,  $F'(-3, -12)$ , and  $G'(0, -7)$ .

- b. How did you determine the image coordinates without graphing?

I determined the image coordinates by adding  $-14$  to each of the  $y$ -coordinates. The  $x$ -coordinates stayed the same.

- c. Determine the vertex coordinates of image  $D''E''F''G''$  if parallelogram  $DEFG$  is translated 8 units to the right.

The vertex coordinates of image  $D''E''F''G''$  are  $D''(-1, 7)$ ,  $E''(-4, 2)$ ,  $F''(5, 2)$ , and  $G''(8, 7)$ .



- d. How did you determine the image coordinates without graphing?

I determined the image coordinates by adding 8 to each of the  $x$ -coordinates. The  $y$ -coordinates stayed the same.

## PROBLEM 2 Rotating Geometric Figures on the Coordinate Plane



Recall that a rotation is a rigid motion that turns a figure about a fixed point, called the point of rotation. The figure is rotated in a given direction for a given angle, called the angle of rotation. The angle of rotation is the measure of the amount the figure is rotated about the point of rotation. The direction of a rotation can either be clockwise or counterclockwise.

on the coordinate plane. Finally, students are given coordinates of vertices of a parallelogram and without graphing they determine the coordinates of images resulting from different rotations.

## Grouping

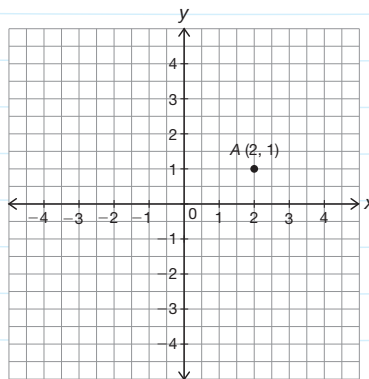
Ask students to read the introduction and worked example. Discuss as a class.



Let's rotate point  $A$  about the origin. The origin will be the point of rotation and you will rotate point  $A$   $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ .

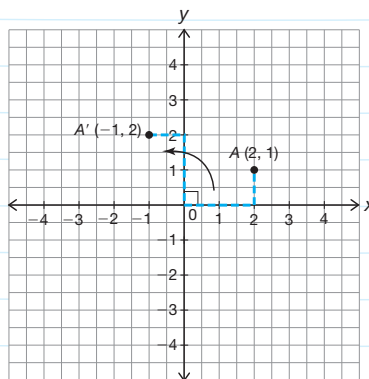
First, let's rotate point  $A$   $90^\circ$  counterclockwise about the origin.

**Step 1:** Plot a point anywhere in the first quadrant, but *not* at the origin.



Point  $A$  is plotted at  $(2, 1)$ .

**Step 2:** Next, draw a "hook" from the origin to point  $A$ , using the coordinates and horizontal and vertical line segments as shown.



**Step 3:** Rotate the "hook"  $90^\circ$  counterclockwise as shown.

Point  $A'$  is located at  $(-1, 2)$ . Point  $A$  has been rotated  $90^\circ$  counterclockwise about the origin.

## Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 4

- In which quadrant of the coordinate plane is point  $A$  located?
- If a point is rotated about the origin  $90^\circ$  counterclockwise, in which quadrant does the image appear?
- If a point is rotated about the origin  $180^\circ$  counterclockwise, in which quadrant does the image appear?
- How many degrees rotation about the origin are necessary for an image in the first quadrant to appear in the fourth quadrant?
- Is triangle  $ABC$  an enlargement of triangle  $DEF$  or is triangle  $DEF$  an enlargement of triangle  $ABC$ ?
- How would you describe the pattern represented in the table?
- Would this table change if a clockwise rotation occurred?
- What would be the coordinates if the point was rotated  $270^\circ$  counterclockwise about the origin?



1. What do you notice about the coordinates of point  $A$  and the coordinates of point  $A'$ ?

The  $x$ -coordinate of point  $A$  is 2, and the  $y$ -coordinate of point  $A'$  is 2.

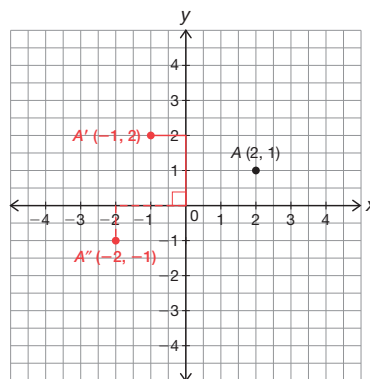
The  $y$ -coordinate of point  $A$  is 1, and the  $x$ -coordinate of point  $A'$  is  $-1$ .

So, the  $x$ -coordinate of point  $A$  is the  $y$ -coordinate of point  $A'$ , and the opposite of the  $y$ -coordinate of point  $A$  is the  $x$ -coordinate of point  $A'$ .

2. Predict what the coordinates of point  $A''$  will be if you rotate point  $A'$   $90^\circ$  counterclockwise about the origin.

If I follow the pattern, I believe point  $A''$  will have the coordinates  $(-2, -1)$ .

3. Rotate point  $A'$  about the origin  $90^\circ$  counterclockwise on the coordinate plane shown. Label the point  $A''$ .



- a. What are the coordinates of point  $A''$ ? Was your prediction for the coordinates of point  $A''$  correct?

The coordinates of point  $A''$  are  $(-2, -1)$ .

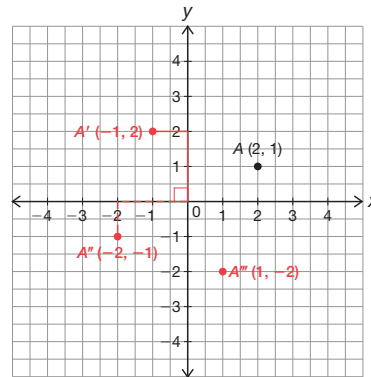
My prediction was correct.

- b. What do you notice about the coordinates of points  $A$  and  $A''$ ? How are the two points related?

The  $x$ -coordinate of point  $A''$  is the opposite of the  $x$ -coordinate of point  $A$ .

The  $y$ -coordinate of point  $A''$  is the opposite of the  $y$ -coordinate of point  $A$ .

4. Rotate point  $A'$  about the origin  $90^\circ$  counterclockwise on the coordinate plane shown. Label the point  $A'''$ .



- a. What are the coordinates of point  $A'''$ ?

The coordinates of point  $A'''$  are  $(1, -2)$ .



- b. What do you notice about the coordinates of point  $A$  and point  $A'''$ ? How are the two points related?

The  $x$ -coordinate of point  $A'''$  is the  $y$ -coordinate of point  $A$ .

The  $y$ -coordinate of point  $A'''$  is the opposite of the  $x$ -coordinate of point  $A$ .



You may have noticed that the values of the  $x$ - and  $y$ -coordinates seem to switch places for every  $90^\circ$  rotation about the origin. You may have also noticed that the rotation from point  $A$  to  $A''$  is a  $180^\circ$  counterclockwise rotation about the origin, and that the rotation from point  $A$  to  $A'''$  is a  $270^\circ$  counterclockwise rotation about the origin.

5. Determine the coordinates of point  $(x, y)$  after rotations of  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ , and  $360^\circ$ .

Original Point	Coordinates After a $90^\circ$ Counterclockwise Rotation About the Origin	Coordinates After a $180^\circ$ Counterclockwise Rotation About the Origin	Coordinates After a $270^\circ$ Counterclockwise Rotation About the Origin	Coordinates After a $360^\circ$ Counterclockwise Rotation About the Origin
$(x, y)$	$(-y, x)$	$(-x, -y)$	$(y, -x)$	$(x, y)$

Verify that the information in the table is correct by using a test point. Plot a point on a coordinate plane and rotate the point  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ , and  $360^\circ$  counterclockwise about the origin.

## Guiding Questions for Discuss Phase

- How can you determine the coordinates of point  $(2, -3)$  after a  $90^\circ$  degree counterclockwise rotation?
- How can you determine the coordinates of point  $(2, -3)$  after a  $90^\circ$  degree clockwise rotation?
- Compare rotating the point  $(2, -3)$   $180^\circ$  degree counterclockwise to rotating  $(2, -3)$   $180^\circ$  degrees clockwise rotation?
- In general, how can you use the table in Question 5 to determine the coordinates after a clockwise rotation?

## Grouping

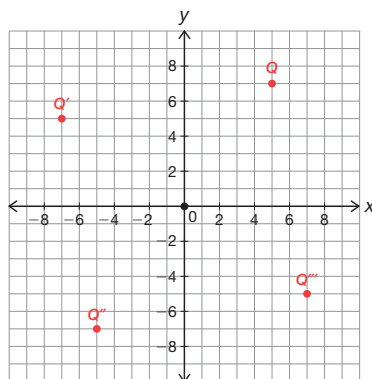
Have students complete Questions 6 and 7 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 6 and 7

- How did you determine the coordinates of the vertices of the image of the trapezoid when the trapezoid was rotated about the origin  $90^\circ$  counterclockwise?
- How did you determine the coordinates of the vertices of the image of the trapezoid when the trapezoid was rotated about the origin  $180^\circ$  counterclockwise?
- Could you have determined the coordinates of the vertices of the image without graphing?



6. Graph and label point  $Q$  at  $(5, 7)$  on the coordinate plane.



Remember that the table shows values for the coordinates, but coordinates for a plotted point are always in the form  $(x, y)$ !



7. Use the origin  $(0, 0)$  as the point of rotation.
- Rotate the pre-image  $Q$   $90^\circ$  counterclockwise about the origin. Label the image  $Q'$ . Determine the coordinates of image  $Q'$ , then describe how you determined the location of point  $Q'$ .

**Answers will vary.**  
I know that after a  $90^\circ$  counterclockwise rotation about the origin, the  $x$ -coordinate of the pre-image is the  $y$ -coordinate of the image. I also know that the opposite of the  $y$ -coordinate of the pre-image is the  $x$ -coordinate of the image.  
The location of image  $Q'$  is  $(-7, 5)$ .
  - Rotate the pre-image  $Q$   $180^\circ$  counterclockwise about the origin. Label the image  $Q''$ . Determine the coordinates of image  $Q''$ , then describe how you determined the location of image  $Q''$ .

**Answers will vary.**  
I determined the location of image  $Q''$  by using the information in the table.  
I know that after a  $180^\circ$  counterclockwise rotation about the origin the coordinates of the image are the opposite of the coordinates of the pre-image.  
The location of point  $Q''$  is  $(-5, -7)$ .
  - Rotate point  $Q$   $270^\circ$  counterclockwise about the origin. Label the image  $Q'''$ . Determine the coordinates of point  $Q'''$ , then describe how you determined the location of image  $Q'''$ .

**Answers will vary.**  
I know that after a  $270^\circ$  counterclockwise rotation about the origin, the  $x$ -coordinate of the image is the  $y$ -coordinate of the pre-image. I also know that the  $y$ -coordinate of the image is the opposite of the  $x$ -coordinate of the pre-image.  
The location of  $Q'''$  is  $(7, -5)$ .



- d. Rotate point  $Q$   $360^\circ$  counterclockwise about the origin. Label the image  $Q'''$ . Determine the coordinates of point  $Q'''$ , then describe how you determined the location of image  $Q'''$ .

Answers will vary.

I know that after a  $360^\circ$  counterclockwise rotation about the origin, the image and the pre-image have the same coordinates.

The location of  $Q'''$  is  $(5, 7)$ .

## Grouping

- Ask students to read the narrative. Discuss as a class.
- Have students complete Questions 8 through 10 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 8 through 10

- What is the difference between a clockwise rotation and a counterclockwise rotation?
- Is rotating the parallelogram different than rotating a trapezoid? If so, how?
- Is it possible to do a  $45^\circ$  rotation? How would you do it?
- Is it possible to rotate a figure about a point other than the origin? How would that be different?



You have been rotating points about the origin on a coordinate plane. However, do you think polygons can also be rotated on the coordinate plane?

You can use models to help show that you can rotate polygons on a coordinate plane. However, before we start modeling the rotation of a polygon on a coordinate plane, let's graph the trapezoid to establish the pre-image.



8. Graph trapezoid  $ABCD$  by plotting the points  $A(-12, 9)$ ,  $B(-12, 4)$ ,  $C(-4, 4)$ , and  $D(-4, 10)$ .

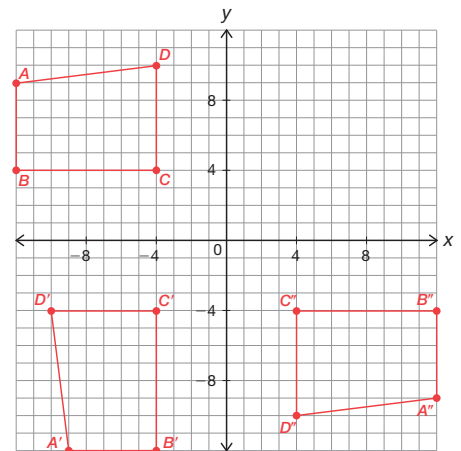
See coordinate plane.

Make sure you have the trapezoid that you cut out earlier.

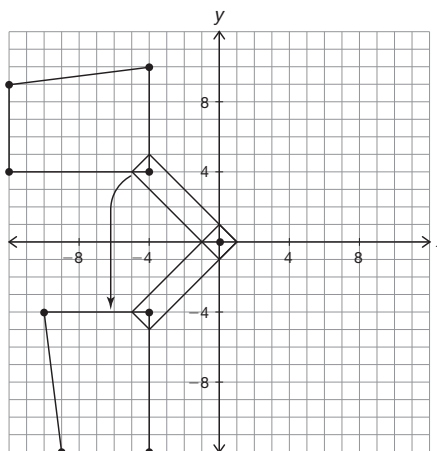


Now that you have graphed the pre-image, you are ready to model the rotation of the polygon on the coordinate plane.

- First, fold a piece of tape in half and tape it to both sides of the trapezoid you cut out previously.
- Then, take your trapezoid and set it on top of trapezoid  $ABCD$  on the coordinate plane, making sure that the tape covers the origin  $(0, 0)$ .
- Finally, put a pin or your pencil point through the tape at the origin and rotate your model counterclockwise.



The  $90^\circ$  counterclockwise rotation of trapezoid  $ABCD$  about the origin is shown.



9. Rotate trapezoid  $ABCD$  about the origin for each given angle of rotation. Graph and label each image on the coordinate plane and record the coordinates in the table.
- Rotate trapezoid  $ABCD$   $90^\circ$  counterclockwise about the origin to form trapezoid  $A'B'C'D'$ .  
See coordinate plane and table for answers.
  - Rotate trapezoid  $ABCD$   $180^\circ$  counterclockwise about the origin to form trapezoid  $A''B''C''D''$ .  
See coordinate plane and table for answers.

Coordinates of Trapezoid $ABCD$	Coordinates of Trapezoid $A'B'C'D'$	Coordinates of Trapezoid $A''B''C''D''$
$A (-12, 9)$	$A' (-9, -12)$	$A'' (12, -9)$
$B (-12, 4)$	$B' (-4, -12)$	$B'' (12, -4)$
$C (-4, 4)$	$C' (-4, -4)$	$C'' (4, -4)$
$D (-4, 10)$	$D' (-10, -4)$	$D'' (4, -10)$

## Grouping

Have students complete Questions 11 and 12 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 11 and 12

- Describe a clockwise rotation that produces the same result as a 90 degree counterclockwise rotation.
- Describe a clockwise rotation that produces the same result as a 180 degree counterclockwise rotation.
- Describe a clockwise rotation that produces the same result as a 270 degree counterclockwise rotation.
- Describe a clockwise rotation that produces the same result as a 450 degree counterclockwise rotation.



10. What similarities do you notice between rotating a single point about the origin and rotating a polygon about the origin?

Because a polygon is made of multiple points, the same methods and algebraic rule can be used when rotating the point(s) of the polygon about the origin.

Let's consider rotations without graphing.



11. The vertices of parallelogram  $DEFG$  are  $D(-9, 7)$ ,  $E(-12, 2)$ ,  $F(-3, 2)$ , and  $G(0, 7)$ .

- a. Determine the vertex coordinates of image  $D'E'F'G'$  if parallelogram  $DEFG$  is rotated  $90^\circ$  counterclockwise about the origin.

The vertex coordinates of image  $D'E'F'G'$  are  $D'(-7, -9)$ ,  $E'(-2, -12)$ ,  $F'(-2, -3)$ , and  $G'(-7, 0)$ .

- b. How did you determine the image coordinates without graphing?

I determined the image coordinates by following the pattern for rotating points  $90^\circ$  counterclockwise about the origin on a coordinate plane. Each  $x$ -coordinate of the pre-image became the  $y$ -coordinate of the image. The opposite of each  $y$ -coordinate of the pre-image became the  $x$ -coordinate of the image.

- c. Determine the vertex coordinates of image  $D''E''F''G''$  if parallelogram  $DEFG$  is rotated  $180^\circ$  counterclockwise about the origin.

The vertex coordinates of image  $D''E''F''G''$  are  $D''(9, -7)$ ,  $E''(12, -2)$ ,  $F''(3, -2)$ , and  $G''(0, -7)$ .

- d. How did you determine the image coordinates without graphing?

I determined the image coordinates by following the pattern for rotating points  $180^\circ$  about the origin on a coordinate plane. The image coordinates are the opposite values of the  $x$ - and  $y$ -coordinates of the pre-image.

- e. Determine the vertex coordinates of image  $D'''E'''F'''G'''$  if parallelogram  $DEFG$  is rotated  $270^\circ$  counterclockwise about the origin.

The vertex coordinates of image  $D'''E'''F'''G'''$  are  $D'''(7, 9)$ ,  $E'''(2, 12)$ ,  $F'''(2, 3)$ , and  $G'''(7, 0)$ .

- f. How did you determine the image coordinates without graphing?

I determined the image coordinates by following the pattern for rotating points  $270^\circ$  counterclockwise about the origin on a coordinate plane. Each  $y$ -coordinate of the pre-image became the  $x$ -coordinate of the image. The opposite of each  $x$ -coordinate of the pre-image became the  $y$ -coordinate of the image.



12. Dante claims that if he is trying to determine the coordinates of an image that is rotated  $180^\circ$  about the origin, it does not matter which direction the rotation occurred. Desmond claims that the direction is important to know when determining the image coordinates. Who is correct? Explain why the correct student's rationale is correct.

Dante is correct. Because the rotation is  $180^\circ$ , the rotation will always end up at the same location whether it is rotated clockwise or counterclockwise. The direction is not important for a  $180^\circ$  rotation.



### Problem 3

Reflection is described as a rigid motion that flips a figure over a reflection line. The problem begins with an example showing how to reflect a point over the  $y$ -axis. Students will perform a reflection of the point over the  $x$ -axis. A table is given that shows the pattern of the coordinates from a reflection over the  $x$ - and  $y$ -axis. Students then plot a point and use the table to determine the location of the point after a given reflection. Next, they are given the coordinates of four vertices of a trapezoid and graph the trapezoid. Using reflections over the  $x$ -axis and  $y$ -axis, they will form two different trapezoids and record the coordinates of the vertices of the images. Finally, students are given the coordinates of the vertices of a parallelogram and without graphing they are able to determine the coordinates of images resulting from different reflections.

### Grouping

Ask students to read introduction and worked examples. Discuss as a class.

### PROBLEM 3 Reflecting Geometric Figures on the Coordinate Plane

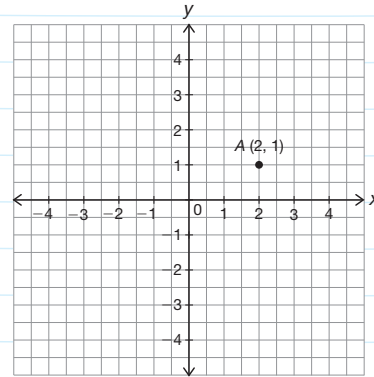


Recall that figures that are mirror images of each other are called reflections. A reflection is a rigid motion that reflects, or “flips,” a figure over a given line called a line of reflection. A line of reflection is a line over which a figure is reflected so that corresponding points are the same distance from the line.

Let’s reflect point  $A$  over the  $y$ -axis.

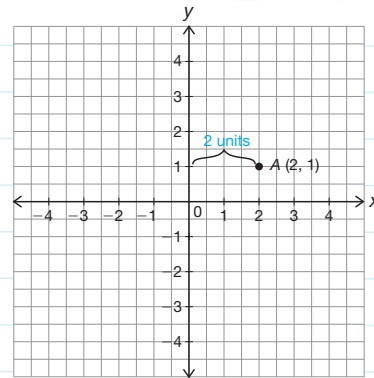


**Step 1:** Plot a point anywhere in the first quadrant, but *not* at the origin.



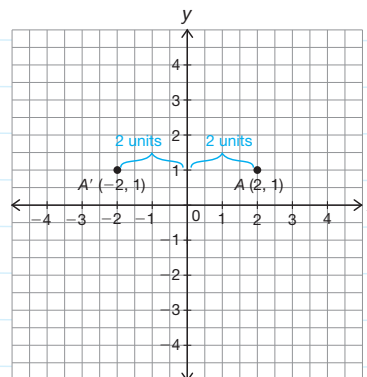
Point  $A$  is plotted at  $(2, 1)$ .

**Step 2:** Next, count the number of  $x$ -units from point  $A$  to the  $y$ -axis.



Point  $A$  is 2 units from the  $y$ -axis.

**Step 3:** Then, count the same number of  $x$ -units on the opposite side of the  $y$ -axis to locate the reflection of point  $A$ . Label the point  $A'$ .



**Step 4:** Point  $A'$  is located at  $(-2, 1)$ . Point  $A$  has been reflected over the  $y$ -axis.

## Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 4

- What are the coordinates of the point  $(-4, -2)$  after reflecting over the  $x$ -axis?
- In general, how do the coordinates of a point change when reflected over the  $x$ -axis?
- What are the coordinates of the point  $(-4, -2)$  after reflecting over the  $y$ -axis?
- In general, how do the coordinates of a point change when reflected over the  $y$ -axis?



1. What do you notice about the coordinates of point  $A$  and the coordinates of image  $A'$ ?

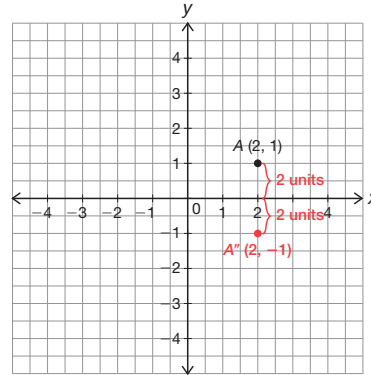
The  $x$ -coordinates are opposites while the  $y$ -coordinates remained the same.

2. Predict the coordinates of  $A''$  if point  $A$  is reflected over the  $x$ -axis. Explain your reasoning.

The coordinates of  $A''$  will be  $(2, -1)$ .

To reflect point  $A$  over the  $x$ -axis, I would count the number of  $y$ -units from point  $A$  to the  $x$ -axis. Then, I would count the same number of  $y$ -units on the opposite side of the  $x$ -axis to determine the location of the reflected point.

3. Reflect point A over the x-axis on the coordinate plane shown. Verify whether your prediction for the location of the image was correct. Graph the image and label it A''.



The location of A'' after the reflection is (2, -1).

Yes. My prediction was correct.



4. What do you notice about the coordinates of A and A''?

The x-coordinates are the same, but the y-coordinates are opposite.



The coordinates of a pre-image reflected over either the x-axis or the y-axis can be used to determine the coordinates of the image.

5. Determine the coordinates of point (x, y) after reflections about the x-axis or y-axis.

Original Point	Coordinates of Image After a Reflection Over the x-axis	Coordinates of Image After a Reflection Over the y-axis
(x, y)	(x, -y)	(-x, y)

Does this table still make sense if the line of reflection is not the x- or y-axis?



## Grouping

Ask students to complete Question 5. Discuss as a class.

## Grouping

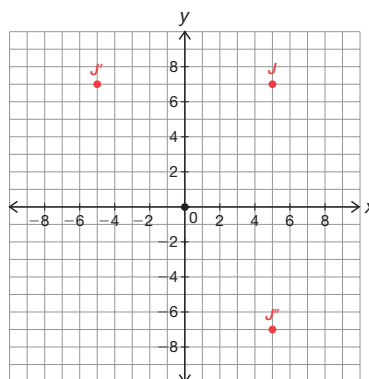
Have students complete Questions 6 through 8 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 6 through 8

- What are the coordinates of the point  $(5, 7)$  after reflecting over the line  $x = 1$ ?
- What are the coordinates of the point  $(5, 7)$  after reflecting over the line  $x = -1$ ?
- What are the coordinates of the point  $(5, 7)$  after reflecting over the line  $y = 2$ ?
- What are the coordinates of the point  $(5, 7)$  after reflecting over the line  $y = -2$ ?



6. Graph point  $J$  at  $(5, 7)$  on the coordinate plane shown.  
See coordinate plane.



7. Reflect point  $J$  over the  $y$ -axis on the coordinate plane. Label the image  $J'$ . Determine the coordinates of  $J'$ . Then, describe how you determined the location of image  $J'$ .  
The coordinates of  $J'$  are  $(-5, 7)$ . Because I am reflecting  $J$  over the  $y$ -axis, I determined the location of point  $J'$  using the information in the table. I knew that by reflecting a point over the  $y$ -axis, the  $x$ -coordinate of the image would be the opposite of the pre-image and the  $y$ -coordinate would stay the same.



8. Reflect point  $J$  over the  $x$ -axis on the coordinate plane. Label the image  $J''$ . Determine the coordinates of  $J''$ . Then, describe how you determined the location of image  $J''$ .  
The coordinates of  $J''$  are  $(5, -7)$ . Because I am reflecting  $J$  over the  $x$ -axis, I determined the location of point  $J''$  using the information in the table. I knew that by reflecting a point over the  $x$ -axis, the  $x$ -coordinate of the image would be the same as the pre-image and the  $y$ -coordinate would be the opposite.

## Grouping

Have students complete Questions 9 through 11 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 9 through 11

- What are the coordinates of trapezoid  $ABCD$  after reflecting over the line  $x = 2$ ?
- What are the coordinates of trapezoid  $ABCD$  after reflecting over the line  $x = -3$ ?
- What are the coordinates of trapezoid  $ABCD$  after reflecting over the line  $y = 4$ ?
- What are the coordinates of trapezoid  $ABCD$  after reflecting over the line  $y = -3$ ?

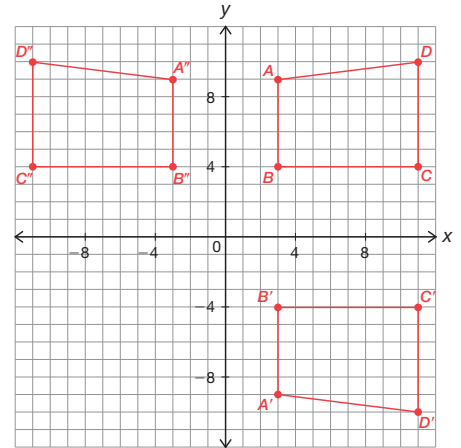
You can also reflect polygons on the coordinate plane. You can model the reflection of a polygon across a line of reflection. Just as with rotating a polygon on a coordinate plane, you will first need to establish a pre-image.



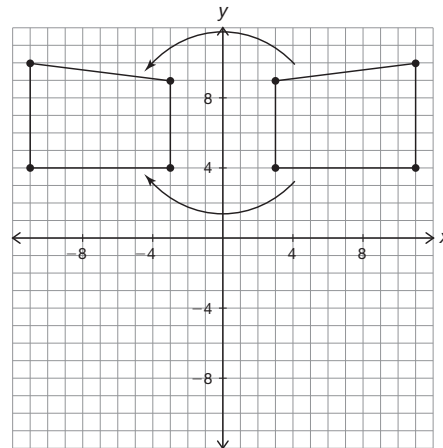
9. Graph trapezoid  $ABCD$  by plotting the points  $A(3, 9)$ ,  $B(3, 4)$ ,  $C(11, 4)$ , and  $D(11, 10)$ .  
See coordinate plane.

Now that you have graphed the pre-image, you are ready to model the reflection of the polygon on the coordinate plane. For this modeling, you will reflect the polygon over the  $y$ -axis.

- First, take your trapezoid that you cut out previously and set it on top of trapezoid  $ABCD$  on the coordinate plane.
- Next, determine the number of units point  $A$  is from the  $y$ -axis.
- Then, count the same number of units on the opposite side of the  $y$ -axis to determine where to place the image in Quadrant II.
- Finally, physically flip the trapezoid over the  $y$ -axis like you are flipping a page in a book.



The reflection of trapezoid  $ABCD$  over the  $y$ -axis is shown.



10. Reflect trapezoid  $ABCD$  over each given line of reflection. Graph and label each image on the coordinate plane and record each image's coordinates in the table.
- Reflect trapezoid  $ABCD$  over the  $x$ -axis to form trapezoid  $A'B'C'D'$ .
  - Reflect trapezoid  $ABCD$  over the  $y$ -axis to form trapezoid  $A''B''C''D''$ .

Coordinates of Trapezoid $ABCD$	Coordinates of Trapezoid $A'B'C'D'$	Coordinates of Trapezoid $A''B''C''D''$
$A (3, 9)$	$A' (3, -9)$	$A'' (-3, 9)$
$B (3, 4)$	$B' (3, -4)$	$B'' (-3, 4)$
$C (11, 4)$	$C' (11, -4)$	$C'' (-11, 4)$
$D (11, 10)$	$D' (11, -10)$	$D'' (-11, 10)$



11. What similarities do you notice between reflecting a single point over the  $x$ - or  $y$ -axis and reflecting a polygon over the  $x$ - or  $y$ -axis?

The similarities I notice are that even though a polygon consists of multiple points, the same methods and algebraic rule can be used when reflecting the point(s) over the  $x$ - or  $y$ -axis.

Let's consider reflections without graphing.



12. The vertices of parallelogram  $DEFG$  are  $D (-9, 7)$ ,  $E (-12, 2)$ ,  $F (-3, 2)$ , and  $G (0, 7)$ .

- Determine the vertex coordinates of image  $D'E'F'G'$  if parallelogram  $DEFG$  is reflected over the  $x$ -axis.

The vertex coordinates of the image  $D'E'F'G'$  are  $D' (-9, -7)$ ,  $E' (-12, -2)$ ,  $F' (-3, -2)$ , and  $G' (0, -7)$ .

- How did you determine the image coordinates without graphing?

I determined the image coordinates by following the pattern for reflecting over the  $x$ -axis. The  $x$ -coordinate of each point remained the same. The opposite of the  $y$ -coordinate of the pre-image became the  $y$ -coordinate of the image.

## Grouping

Have students complete Question 12 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Question 12

- How does the  $x$ -coordinate change when reflecting a point over the  $x$ -axis?
- How does the  $x$ -coordinate change when reflecting a point over the  $y$ -axis?
- How does the  $y$ -coordinate change when reflecting a point over the  $x$ -axis?
- How does the  $y$ -coordinate change when reflecting a point over the  $y$ -axis?

- c. Determine the vertex coordinates of image  $D'E'F'G'$  if parallelogram  $DEFG$  is reflected over the  $y$ -axis.

The vertex coordinates of image  $D'E'F'G'$  are  $D'$  (9, 7),  $E'$  (12, 2),  $F'$  (3, 2), and  $G'$  (0, -7).



- d. How did you determine the image coordinates without graphing?

I determined the image coordinates by following the pattern for reflecting over the  $y$ -axis. The opposite of the  $x$ -coordinate of the pre-image became the  $x$ -coordinate of the image. The  $y$ -coordinate of each point remained the same.

## Talk the Talk

Students rewrite the slope ratio to describe the slope of a line that has been rotated 90 degrees and conclude that it is the negative reciprocal of the ratio for the original line. In other words, the image and pre-image have a perpendicular relationship.

## Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

### Talk the Talk

You know that a line is determined by two points. The slope of any line represented on a coordinate plane can be given by  $\frac{y_2 - y_1}{x_2 - x_1}$ .

You also now know that when rotating a point  $(x, y)$  90° counterclockwise about the origin, the  $x$ -coordinate of the original point maps to the  $y$ -coordinate of the transformed point and the  $y$ -coordinate of the original point maps to the opposite of the  $x$ -coordinate of the transformed point.



1. Rewrite the slope ratio above to describe the slope of a line that has been rotated 90° counterclockwise. What do you notice? Explain your reasoning.

$$\frac{y_2 - y_1}{x_2 - x_1}$$

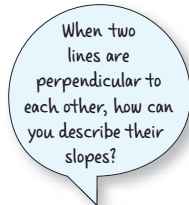
Replace  $x$  with  $y$  and replace  $y$  with  $-x$ .

$$\frac{-x_2 - (-x_1)}{y_2 - y_1}$$

$$\frac{-(x_2 - x_1)}{(y_2 - y_1)}$$

The slope ratio for a line that is rotated 90° counterclockwise is the negative reciprocal of the ratio for the original line.

When a line is rotated 90° counterclockwise it is perpendicular to the original line. A line that is perpendicular to another line has a slope that is the negative reciprocal of the slope of the original line.



2. Complete the sentence using *always*, *sometimes*, or *never*.

Images that result from a translation, rotation, or reflection are always congruent to the original figure.



Be prepared to share your solutions and methods.



## Check for Students' Understanding

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The coordinates of a pre-image are as follows.

$A(0, 0)$        $B(13, 0)$        $C(13, 4)$        $D(4, 4)$

Consider the coordinates of each image listed below and describe the transformation.

1.  $A'(0, -7)$        $B'(13, -7)$        $C'(13, -3)$        $D'(4, -3)$

a translation down 7 units

2.  $A'(0, 0)$        $B'(13, 0)$        $C'(13, -4)$        $D'(4, -4)$

a reflection over the  $y$ -axis

3.  $A'(0, 0)$        $B'(-13, 0)$        $C'(-13, -4)$        $D'(-4, -4)$

a 180 degree clockwise or counterclockwise rotation about the origin

4. Did you have to graph the pre-image and image to describe the transformation?

Explain your reasoning.

Answers will vary.

No, I did not have to graph the pre-image and image to describe the transformation. I was able to recognize the change in the coordinates of the pre-image to the image and knew what transformation the change in values implied.



# Hey, Haven't I Seen You Before?

## Congruent Triangles

### LEARNING GOALS

In this lesson, you will:

- Identify corresponding sides and corresponding angles of congruent triangles.
- Explore the relationship between the corresponding sides of congruent triangles.
- Explore the relationship between the corresponding angles of congruent triangles.
- Write congruence statements for congruent triangles.
- Identify and use rigid motion to create new images.

### ESSENTIAL IDEAS

- Corresponding side lengths of congruent triangles are congruent.
- Corresponding angle measures of congruent triangles are congruent.
- Listing statements of triangle congruence requires accurate identification of the corresponding vertices, segments, and angles.
- Translations, rotations, and reflections are rigid motions that preserve the size and shape of figures.

### COMMON CORE STATE STANDARDS FOR MATHEMATICS

#### G-CO Congruence

#### Understand congruence in terms of rigid motions

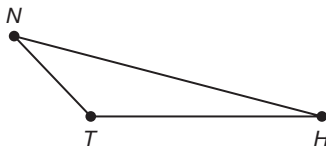
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

## Overview

This lesson begins with definitions and symbols associated with congruent line segments and congruent angles. Students will explore the properties of congruent triangles on a coordinate plane. A translation and the Pythagorean Theorem are used to determine corresponding sides of congruent triangles are congruent. A protractor is used to determine the corresponding angles of congruent triangles are congruent. Students then write triangle congruence statements and use the statements to list congruent corresponding sides and congruent corresponding angles.

## Warm Up

Use triangle  $NTH$  to answer each question.



1. Determine the length of each side of triangle  $NTH$ .

$$NT = 3.6 \text{ cm}$$

$$TH = 10 \text{ cm}$$

$$NH = 5.12 \text{ cm}$$

2. Determine the measure of each angle in triangle  $NTH$ .

$$m\angle N = 47^\circ$$

$$m\angle T = 117^\circ$$

$$m\angle H = 16^\circ$$

3. If triangle  $NTH$  was copied on to a coordinate plane, then translated down 9 units, how would the length of the sides of the image compare to the length of the sides of the pre-image?

**They would be the same.**

4. If triangle  $NTH$  was copied on to a coordinate plane, then rotated  $90^\circ$  counterclockwise about the origin, how would the length of the sides of the image compare to the length of the sides of the pre-image?

**They would be the same.**

5. If triangle  $NTH$  was copied on to a coordinate plane, then reflected over the  $x$ -axis, how would the length of the sides of the image compare to the length of the sides of the pre-image?

**They would be the same.**

6. If triangle  $NTH$  was copied on to a coordinate plane, then translated up 3 units, how would the measures of the angles of the image compare to the measures of the angles of the pre-image?

**They would be the same.**

7. If triangle  $NTH$  was copied on to a coordinate plane, then rotated  $180^\circ$  counterclockwise about the origin, how would the measures of the angles of the image compare to the measures of the angles of the pre-image?

**They would be the same.**

8. If triangle  $NTH$  was copied on to a coordinate plane, then reflected over the  $y$ -axis, how would the measures of the angles of the image compare to the measures of the angles of the pre-image?

**They would be the same.**



# Hey, Haven't I Seen You Before?

## Congruent Triangles

### LEARNING GOALS

In this lesson, you will:

- Identify corresponding sides and corresponding angles of congruent triangles.
- Explore the relationship between the corresponding sides of congruent triangles.
- Explore the relationship between the corresponding angles of congruent triangles.
- Write congruence statements for congruent triangles.
- Identify and use rigid motion to create new images.

**I**n mathematics, when a geometric figure is transformed, the size and shape of the figure do not change. However, in physics, things are a little different. An idea known as length contraction explains that when an object is in motion, its length appears to be slightly less than it really is. This cannot be seen with everyday objects because they do not move fast enough. To truly see this phenomenon you would have to view an object moving close to the speed of light. In fact, if an object was moving past you at the speed of light, the length of the object would seem to be practically zero!

This theory is very difficult to prove and yet scientists came up with the idea in the late 1800s. How do you think scientists test and prove length contraction? Do you think geometry is used in these verifications?

## Problem 1

Congruent line segments and congruent angles are defined. Symbols are used to represent side and angle relationships. A distinction is made between the symbols used to represent the actual length of a line segment and the measure of an angle, and the symbols used to represent the geometric models of a line segment and an angle. The definition of corresponding sides is provided. Students are given the coordinates of the vertices of a right triangle and graph the triangle. After determining the length of each side, using the Pythagorean Theorem and the distance between two horizontal and vertical points, students will translate the triangle. Similarly, students then determine the lengths of the sides of the image and conclude that corresponding sides are congruent. Then triangles  $ABC$  and  $DEF$  are used in conjunction with a protractor to determine the corresponding angles of congruent triangles are congruent.

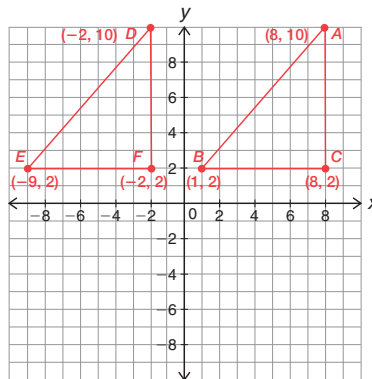
## Grouping

Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

### PROBLEM 1 Understanding Congruence



1. Graph triangle  $ABC$  by plotting the points  $A(8, 10)$ ,  $B(1, 2)$ , and  $C(8, 2)$ .



Let's explore the properties of congruent triangles.



- a. Classify triangle  $ABC$ . Explain your reasoning.

Triangle  $ABC$  is a right triangle because line segment  $AC$  is a vertical line segment. I know this because the  $x$ -coordinates of the points are the same, but the  $y$ -coordinates are different. I also know that line segment  $BC$  is a horizontal line segment because the  $y$ -coordinates are the same but the  $x$ -coordinates are different. The intersection of a vertical line segment and a horizontal line segment is a right angle.

- b. Calculate the length of side  $\overline{AB}$ .

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 7^2 + 8^2 &= c^2 \\ 49 + 64 &= c^2 \\ c^2 &= 113 \\ c &= \sqrt{113} \approx 10.6 \end{aligned}$$

The length of side  $\overline{AB}$  is approximately 10.6 units.

I would use the Pythagorean Theorem to calculate the length.



2. Translate triangle  $ABC$  10 units to the left to form triangle  $DEF$ . Graph triangle  $DEF$  and list the coordinates of points  $D$ ,  $E$ , and  $F$ .

The coordinates of triangle  $DEF$  are  $D(-2, 10)$ ,  $E(-9, 2)$ , and  $F(-2, 2)$ .

## Guiding Questions for Share Phase, Questions 1 and 2

- Triangle  $ABC$  lies in which quadrant?
- Is triangle  $ABC$  a right triangle? How do you know?
- How did you determine the length of the legs of the right triangle?
- What do you know about the hypotenuse of a right triangle?
- What do you need to know to determine the length of the hypotenuse?
- How did you determine the coordinates of the vertices of the image of triangle  $ABC$ ?



## Guiding Questions for Share Phase, Questions 3 through 5

- Which side of triangle  $ABC$  is in the same relative position as side  $DE$  in triangle  $DEF$ ?
- Which side of triangle  $ABC$  is in the same relative position as side  $EF$  in triangle  $DEF$ ?
- Which side of triangle  $ABC$  is in the same relative position as side  $DF$  in triangle  $DEF$ ?
- Did you have to use the Pythagorean Theorem to determine the length of side  $DE$ ? Why or why not?

Triangle  $ABC$  and triangle  $DEF$  in Question 1 are the same size and the same shape. Each side of triangle  $ABC$  matches, or corresponds to, a specific side of triangle  $DEF$ .

3. Given what you know about corresponding sides of congruent triangles, predict the side lengths of triangle  $DEF$ .

$$DE \approx 10.6, EF = 7, DF = 8$$

4. Verify your prediction.

- a. Identify the pairs of corresponding sides of triangle  $ABC$  and triangle  $DEF$ .

Side  $\overline{AC}$  in triangle  $ABC$  corresponds to side  $\overline{DF}$  in triangle  $DEF$ .

Side  $\overline{BC}$  in triangle  $ABC$  corresponds to side  $\overline{EF}$  in triangle  $DEF$ .

Side  $\overline{AB}$  in triangle  $ABC$  corresponds to side  $\overline{DE}$  in triangle  $DEF$ .

- b. Determine the side lengths of triangle  $DEF$ .

The length of side  $\overline{DF}$  is 8 units.

The length of side  $\overline{EF}$  is 7 units.

The length of side  $\overline{DE}$  is approximately 10.6 units.

- c. Compare the lengths of the sides of triangle  $ABC$  to the lengths of the corresponding sides of triangle  $DEF$ . What do you notice?

The length of side  $\overline{AC}$  is equal to the length of side  $\overline{DF}$ .

The length of side  $\overline{BC}$  is equal to the length of side  $\overline{EF}$ .

The length of side  $\overline{AB}$  is equal to the length of side  $\overline{DE}$ .



5. In general, what can you conclude about the relationship between the corresponding sides of congruent triangles?

Corresponding sides of congruent triangles are congruent.

Corresponding sides are sides that have the same relative positions in corresponding geometric figures.



Would there ever be a time when corresponding sides of figures would not be congruent?



## Grouping

Have students complete Questions 6 through 9 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 6 through 9

- How did you know which scale to use on the protractor when measuring the angles of triangle  $ABC$ ?
- Which angle of triangle  $ABC$  is in the same relative position as  $\angle D$  in triangle  $DEF$ ?
- Which angle of triangle  $ABC$  is in the same relative position as  $\angle E$  in triangle  $DEF$ ?
- Which angle of triangle  $ABC$  is in the same relative position as  $\angle F$  in triangle  $DEF$ ?



Use triangle  $ABC$  and triangle  $DEF$  from Question 1 to answer each question.

6. Use a protractor to determine the measures of  $\angle A$ ,  $\angle B$ , and  $\angle C$ .

The measure of  $\angle C$  is equal to  $90^\circ$ .

The measure of  $\angle A$  is equal to  $40^\circ$ .

The measure of  $\angle B$  is equal to  $50^\circ$ .

7. What would you predict to be true about the measures of corresponding angles of congruent triangles?

The measures of corresponding angles of congruent triangles are equal.

8. Verify your prediction.

- a. Identify the corresponding angles of triangle  $ABC$  and triangle  $DEF$ .

Angle  $A$  in triangle  $ABC$  corresponds to angle  $D$  in triangle  $DEF$ .

Angle  $B$  in triangle  $ABC$  corresponds to angle  $E$  in triangle  $DEF$ .

Angle  $C$  in triangle  $ABC$  corresponds to angle  $F$  in triangle  $DEF$ .

- b. Use a protractor to determine the measures of angles  $D$ ,  $E$ , and  $F$ .

The measure of angle  $F$  is equal to  $90^\circ$ .

The measure of angle  $D$  is equal to  $40^\circ$ .

The measure of angle  $E$  is equal to  $50^\circ$ .

- c. Compare the measures of the angles of triangle  $ABC$  to the measures of the corresponding angles of triangle  $DEF$ .

The measure of angle  $A$  is equal to the measure of angle  $D$ .

The measure of angle  $B$  is equal to the measure of angle  $E$ .

The measure of angle  $C$  is equal to the measure of angle  $F$ .



9. In general, what can you conclude about the relationship between the corresponding angles of congruent triangles?

Corresponding angles of congruent triangles are congruent.

Each angle in triangle  $ABC$  corresponds to a specific angle in triangle  $DEF$ . Corresponding angles are angles that have the same relative positions in corresponding geometric figures.



So, what can you say about corresponding sides and corresponding angles of congruent triangles?



## Problem 2

Students will practice listing congruent corresponding sides and angles of two triangles given only a congruence statement. An image and pre-image of a triangle is provided and students then identify the transformation used, acknowledge the transformation preserves both size and shape, write a triangle congruence statement, and use the statement to list the congruent sides and congruent angles.

## Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 3

- Do you need a diagram of the triangles to determine the congruent corresponding sides? Why or why not?
- Do you need a diagram of the triangles to determine the congruent corresponding angles? Why or why not?
- How is the triangle congruency statement used to determine the congruent corresponding sides?
- How is the triangle congruency statement used to determine the congruent corresponding angles?
- How could you tell the transformation was not a translation?

## PROBLEM 2 Statements of Triangle Congruence



1. Consider the congruence statement  $\triangle JRB \cong \triangle MNS$ .

a. Identify the congruent angles.

$$\angle J \cong \angle M$$

$$\angle R \cong \angle N$$

$$\angle B \cong \angle S$$

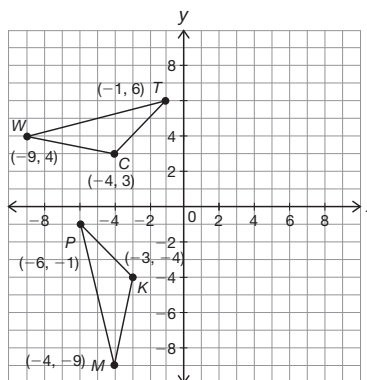
b. Identify the congruent sides.

$$\overline{JR} \cong \overline{MN}$$

$$\overline{RB} \cong \overline{NS}$$

$$\overline{JB} \cong \overline{MS}$$

2. Analyze the two triangles shown.



Remember, the  $\cong$  means "is congruent to."



a. Determine the transformation used to create triangle  $PMK$ .

**Triangle  $TWC$  was rotated  $90^\circ$  counterclockwise to create triangle  $PMK$ .**

b. Does the transformation preserve the size and shape of the triangle in this problem situation? Why or why not?

**Yes. Rotations preserve both the size and shape of figures.**

c. Write a triangle congruence statement for the triangles.

$$\triangle TWC \cong \triangle PMK$$

d. Identify the congruent angles and congruent sides.

$$\angle T \cong \angle P$$

$$\angle W \cong \angle M$$

$$\angle C \cong \angle K$$

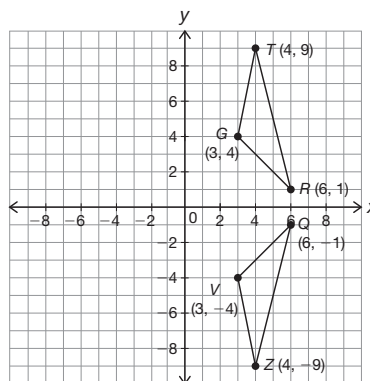
$$\overline{TW} \cong \overline{PM}$$

$$\overline{WC} \cong \overline{MK}$$

$$\overline{TC} \cong \overline{PK}$$

- How could you tell the transformation was not a reflection?
- Do all of the transformations you have studied preserve both size and shape? Explain.
- Is there more than one way to write the triangle congruence statement? Explain.
- What is another way to write the triangle congruence statement?
- If the triangle congruence statement is written differently, will that change the congruent parts of the triangle? Explain.

3. Analyze the two triangles shown.



a. Determine the transformation used to create triangle ZQV.

Triangle  $TRG$  was reflected over the  $x$ -axis to create triangle  $ZQV$ .

b. Does the transformation preserve the size and shape of the triangle in this problem situation? Why or why not?

Yes. Reflections preserve both the size and shape of figures.

c. Write a triangle congruence statement for the triangles shown.

$\triangle TRG \cong \triangle ZQV$



d. Identify the congruent angles.

$\angle T \cong \angle Z$

$\angle R \cong \angle Q$

$\angle G \cong \angle V$

e. Identify the congruent sides.

$\overline{TR} \cong \overline{ZQ}$

$\overline{RG} \cong \overline{QV}$

$\overline{TG} \cong \overline{ZV}$

## Talk the Talk

Students will describe the characteristics of congruent triangles.

## Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

### Talk the Talk



1. Given any triangle on a coordinate plane, how can you create a different triangle that you know will be congruent to the original triangle?

I can translate, rotate, or reflect any triangle to create a congruent triangle.

2. Describe the properties of congruent triangles.

Congruent triangles have corresponding angles and corresponding side lengths that have equal measures.



Be prepared to share your solutions and methods.

## Check for Students' Understanding

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1. Given:

$$\angle B \cong \angle K$$

$$\angle W \cong \angle M$$

$$\angle P \cong \angle C$$

Write a triangle congruency statement based on the congruent corresponding angles given.

$$\triangle BWP \cong \triangle KMC$$

2. Given:

$$\overline{OV} \cong \overline{SR}$$

$$\overline{VT} \cong \overline{RX}$$

$$\overline{OT} \cong \overline{SX}$$

Write a triangle congruency statement based on the congruent corresponding sides given.

$$\triangle OVT \cong \triangle SRX$$

3. Given:

$$\angle H \cong \angle Z$$

$$\overline{HM} \cong \overline{ZG}$$

$$\angle Y \cong \angle D$$

Write a triangle congruency statement based on the congruent corresponding angles and congruent corresponding sides given.

$$\triangle YHM \cong \triangle DZG$$

# It's All About the Sides

## Side-Side-Side Congruence Theorem

### LEARNING GOALS

In this lesson, you will:

- Explore the Side-Side-Side Congruence Theorem through constructions.
- Explore the Side-Side-Side Congruence Theorem on the coordinate plane.
- Prove the Side-Side-Side Congruence Theorem.

### ESSENTIAL IDEAS

- The Side-Side-Side Congruence Theorem states “If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.”
- Translations preserve the size and shape of geometric figures.
- Reflections preserve the size and shape of geometric figures.

### COMMON CORE STATE STANDARDS FOR MATHEMATICS

#### G-CO Congruence

##### Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

### KEY TERM

- Side-Side-Side Congruence Theorem

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

#### Prove geometric theorems

10. Prove theorems about triangles.

#### Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

## Overview

The Side-Side-Side Congruence Theorem is stated and students use construction tools to informally prove the SSS Congruence Theorem. Rigid motion is used to explore the SSS Congruence Theorem in this lesson. When three sides of a triangle are translated and later reflected over the  $x$ -axis, students will compare the lengths of the corresponding sides and the measures of the corresponding angles of the image and pre-image. They conclude that the image and the pre-image always result in congruent triangles. Because the triangles are situated on a coordinate plane, the Distance Formula can be used to show the pairs of corresponding sides of the triangles are congruent. A protractor is used to verify the pairs of corresponding angles are congruent. The lesson ends with a formal two-column proof of the SSS Congruence Theorem.



## Warm Up

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1. Given:  $\triangle BHX \cong \triangle KRC$

a. Rewrite the triangle congruency statement five different ways.

$$\triangle XHB \cong \triangle CRK$$

$$\triangle BXH \cong \triangle KCR$$

$$\triangle XBH \cong \triangle CKR$$

$$\triangle HBX \cong \triangle RKC$$

$$\triangle HXB \cong \triangle RCK$$

b. List the pairs of congruent corresponding angles.

$$\angle B \cong \angle K$$

$$\angle H \cong \angle R$$

$$\angle X \cong \angle C$$

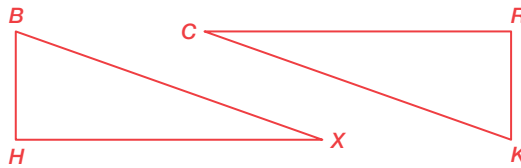
c. List the pairs of congruent corresponding sides.

$$\overline{BH} \cong \overline{KR}$$

$$\overline{HX} \cong \overline{RC}$$

$$\overline{BX} \cong \overline{KC}$$

d. Draw a diagram of the two congruent triangles.





# It's All About the Sides

## Side-Side-Side Congruence Theorem

### LEARNING GOALS

In this lesson, you will:

- Explore the Side-Side-Side Congruence Theorem through constructions.
- Explore the Side-Side-Side Congruence Theorem on the coordinate plane.
- Prove the Side-Side-Side Congruence Theorem.

### KEY TERM

- Side-Side-Side Congruence Theorem

**H**ave you ever tried to construct something from scratch—a model car or a bird house, for example? If you have, you have probably discovered that it is a lot more difficult than it looks. To build something accurately, you must have a plan in place. You must think about materials you will need, measurements you will make, and the amount of time it will take to complete the project. You may need to make a model or blueprint of what you are building. Then, when the actual building begins, you must be very precise in all your measurements and cuts. The difference of half an inch may not seem like much, but it could mean the wall of your bird house is too small and now you may have to start again!

You will be constructing triangles throughout the next four lessons. While you won't be cutting or building anything, it is still important to measure accurately and be precise. Otherwise, you may think your triangles are accurate even though they're not!

## Problem 1

The Side-Side-Side Congruence Theorem is stated. Students are given three line segments and using a compass and straightedge, they construct a triangle and compare their triangle to their classmates' triangles. They conclude that only one unique triangle can be constructed from the three given sides and the corresponding angles are also congruent. Therefore the triangles are congruent thus showing SSS is a valid method for proving two triangles congruent.

## Grouping

- Ask students to read introduction. Discuss as a class.
- Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 3

- What is the first step in your construction?
  - Which point did you use first?
  - Which line segment did you duplicate on the starter line?
  - Did your classmates duplicate the same line segment on their starter line?
  - Which line segment did you duplicate next?
  - Did you have to duplicate the third line segment to complete the triangle? Why or why not?
- How does the orientation of your triangle compare to the orientation of your classmates' triangles?
  - How many different triangles could result from this construction? Explain.
  - If the triangles are oriented differently, are they still congruent?
  - If the triangles are oriented differently, describe a rigid motion that could map one triangle onto the other triangle?

### PROBLEM 1 Can You Build It?

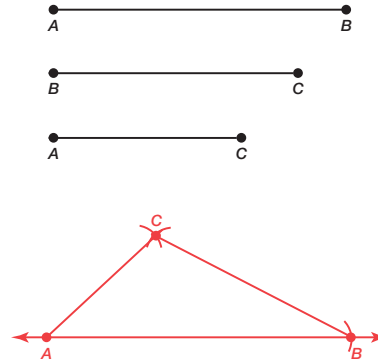


While you can assume that all duplicated or transformed triangles are congruent, mathematically, you need to use a theorem to prove it.

The *Side-Side-Side Congruence Theorem* is one theorem that can be used to prove triangle congruence. The **Side-Side-Side Congruence Theorem** states: "If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent."



1. Use the given line segments to construct triangle  $ABC$ . Then, write the steps you performed to construct the triangle.



Construct a starter line.

Locate point  $A$  on the starter line.

Duplicate  $\overline{AB}$  on the starter line to locate point  $B$ .

Using point  $A$  as the endpoint, cut an arc duplicating  $\overline{AC}$ .

Using point  $B$  as the endpoint, cut an arc duplicating  $\overline{BC}$ .

Label the point at which the two arcs intersect point  $C$ .

Connect points  $A$ ,  $B$ , and  $C$  to form  $\triangle ABC$ .

- What type of rigid motion is needed to map your triangle onto a classmate's triangle that is oriented differently?
- Is Side-Side-Side a valid shortcut for proving two triangles congruent? Why?

2. Analyze the triangle you created.

a. Classify  $\triangle ABC$ . Explain your reasoning.

Triangle  $ABC$  is an obtuse scalene triangle. I know this because all three side lengths are different and  $\angle C$  is obtuse.

b. Compare your triangle to your classmates' triangles. Are the triangles congruent? Why or why not?

Yes. All the triangles are congruent. I know this is true because everyone used sides that are the same length. Also, the measures of the corresponding angles formed by the sides are the same for everyone's triangle.

c. How many different triangles can be formed given the lengths of three distinct sides?

Given the lengths of three sides of a triangle, only one unique triangle can be formed.

3. Rico compares his triangle with his classmate Annette's. Rico gets out his ruler and protractor to verify that the triangles are congruent. Annette states he does not need to do that. Who is correct? Explain your reasoning.

Annette is correct. Because construction was used to duplicate the line segments, the line segments must be accurate. Also, because all three side lengths were given, there is only one way they can come together to form the triangle. So, the angles must also be congruent.



## Problem 2

Students graph three given coordinates and connect the points to form a triangle. Next, they translate the triangle on the coordinate plane and use the distance formula to calculate the lengths of the sides of the image and pre-image. They conclude the image and pre-image are congruent triangles using the SSS Congruence Theorem.

## Grouping

Have students complete Questions 1 through 7 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 7

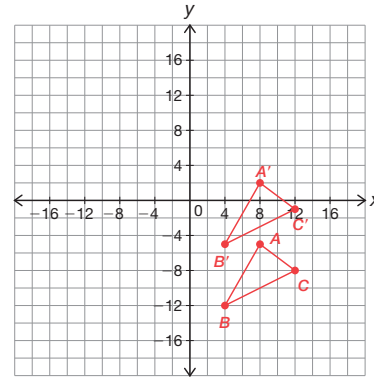
- How would you classify triangle  $ABC$ ?
- In which quadrant is triangle  $ABC$  located?
- In which quadrant is triangle  $A'B'C'$  located?
- How did you determine the length of sides  $AB$  and  $A'B'$ ?
- How did you determine the length of side  $BC$  and  $B'C'$ ?
- How did you determine the length of side  $AC$  and  $A'C'$ ?
- Does knowing the lengths of three pairs of corresponding sides are equal enough information to determine two triangles are congruent?
- Is the length written in radical form an exact measurement?

## PROBLEM 2 Get Back on the Plane

In the previous problem, you proved that two triangles are congruent if three sides of one triangle are congruent to the corresponding sides of another triangle. When dealing with triangles on the coordinate plane, measurement must be used to prove congruence.



1. Graph triangle  $ABC$  by plotting the points  $A(8, -5)$ ,  $B(4, -12)$ , and  $C(12, -8)$ .



2. How can you determine the length of each side of this triangle?

I can determine the length of each side by using the Distance Formula.

3. Calculate the length of each side of triangle  $ABC$ . Record the measurements in the table.

Use exact measurements when determining the lengths.



Side of Triangle $ABC$	Length of Side
$\overline{AB}$	$\sqrt{65}$
$\overline{BC}$	$\sqrt{80}$
$\overline{AC}$	5

$$\begin{aligned} AB &= \sqrt{(4 - 8)^2 + [-12 - (-5)]^2} \\ &= \sqrt{(-4)^2 + 7^2} \\ &= \sqrt{16 + 49} \\ &= \sqrt{65} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(12 - 4)^2 + [-8 - (-12)]^2} \\ &= \sqrt{8^2 + 4^2} \\ &= \sqrt{64 + 16} \\ &= \sqrt{80} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(12 - 8)^2 + [-8 - (-5)]^2} \\ &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

- Is the length written in decimal form an exact measurement?
- Do translations preserve size and shape?
- How can the SSS Congruence Theorem be applied to this situation?
- If the triangle was horizontally translated, do you think the image and the pre-image would be that of congruent triangles?
- In what situation would it be appropriate to use the SSS Congruence Theorem?
- In what situation would it not be appropriate to use the SSS Congruence Theorem?

4. Translate line segments  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  up 7 units to form triangle  $A'B'C'$ . Graph the image.  
See coordinate plane.
5. Calculate the length of each side of triangle  $A'B'C'$ . Record the measurements in the table.

Side of Triangle $A'B'C'$	Length of Side
$\overline{A'B'}$	$\sqrt{65}$
$\overline{B'C'}$	$\sqrt{80}$
$\overline{A'C'}$	5

$$\begin{aligned} A'B' &= \sqrt{(4-8)^2 + (-5-2)^2} \\ &= \sqrt{(-4)^2 + (-7)^2} \\ &= \sqrt{16 + 49} \\ &= \sqrt{65} \end{aligned}$$

$$\begin{aligned} B'C' &= \sqrt{(12-4)^2 + [-1-(-5)]^2} \\ &= \sqrt{8^2 + 4^2} \\ &= \sqrt{64 + 16} \\ &= \sqrt{80} \end{aligned}$$

$$\begin{aligned} A'C' &= \sqrt{(12-8)^2 + (-1-2)^2} \\ &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

6. Are the corresponding sides of the pre-image and image congruent? Explain your reasoning.  
Yes. The lengths of the corresponding sides of the pre-image and image are equal, so the corresponding sides of the pre-image and image are congruent.



7. Do you need to determine the measures of the angles to verify that the triangles are congruent? Explain why or why not.  
No. According to the SSS Theorem, if three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.

### Problem 3

Students graph the three given coordinates from Problem 1 and reflect the triangle over the  $x$ -axis. Using the distance formula to calculate the lengths of the sides of the image and pre-image, they conclude the image and pre-image are congruent triangles using the SSS Congruence Theorem.

### Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

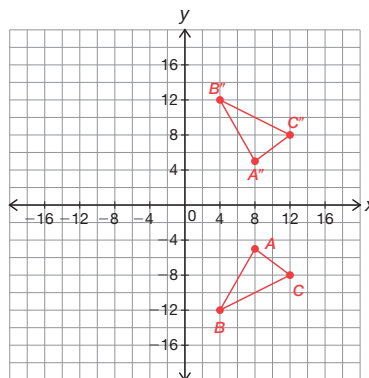
### Guiding Questions for Share Phase, Questions 1 through 4

- In which quadrant is triangle  $A'B'C'$  located?
- In which direction was each coordinate moved to perform the reflection?
- How does the reflection affect the coordinates of the vertices?
- Do reflections preserve size and shape?
- How can the SSS Congruence Theorem be applied to this situation?
- If the triangle was reflected over the  $y$ -axis, do you think the image and the pre-image would be congruent triangles? Why or why not?

### PROBLEM 3 Flipping for Congruence



1. Graph triangle  $ABC$  by plotting the points  $A(8, 25)$ ,  $B(4, 212)$ , and  $C(12, 28)$ .



2. Reflect line segments  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  over the  $x$ -axis to form triangle  $A'B'C'$ .  
See coordinate plane.
3. Calculate the length of each side of triangle  $A'B'C'$ . Record the measurements in the table.

Side of Triangle $A'B'C'$	Length of Side
$\overline{A'B'}$	$\sqrt{65}$
$\overline{B'C'}$	$\sqrt{80}$
$\overline{A'C'}$	5

$$\begin{aligned} A'B' &= \sqrt{(4-8)^2 + (212-25)^2} \\ &= \sqrt{(-4)^2 + 7^2} \\ &= \sqrt{16+49} \\ &= \sqrt{65} \end{aligned}$$

$$\begin{aligned} B'C' &= \sqrt{(12-4)^2 + (28-212)^2} \\ &= \sqrt{8^2 + (-4)^2} \\ &= \sqrt{64+16} \\ &= \sqrt{80} \end{aligned}$$

$$\begin{aligned} A'C' &= \sqrt{(12-8)^2 + (28-25)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$





4. Are the triangles congruent? Explain your reasoning.

**Yes. The triangles are congruent because the lengths of the corresponding sides are congruent.**

## Grouping

Have students complete Problem 4 with a partner. Then have students share their responses as a class.

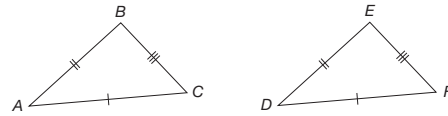
## Guiding Questions for Share Phase

- How can you determine the given information?
- What does  $\overline{AB}$  represent?
- What does  $AB$  represent?
- What does  $\sim$  represent?
- What does  $\cong$  represent?

### PROBLEM 4 And Finally the Proof . . .



Complete the proof of the Side-Side-Side Congruence Theorem.



Given:  $\overline{AB} \cong \overline{DE}$ ,  $\overline{BC} \cong \overline{EF}$ ,  $\overline{AC} \cong \overline{DF}$

Prove:  $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$ , $\overline{BC} \cong \overline{EF}$ , $\overline{AC} \cong \overline{DF}$	1. Given
2. $AB = DE$ , $BC = EF$ , $AC = DF$	2. Definition of congruence
3. $\frac{AB}{DE} = 1$ , $\frac{BC}{EF} = 1$ , $\frac{AC}{DF} = 1$	3. Division property of equality
4. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	4. Transitive property of equality
5. $\triangle ABC \sim \triangle DEF$	5. Side-Side-Side Similarity Theorem
6. $\angle A \cong \angle D$ , $\angle B \cong \angle E$ , $\angle C \cong \angle F$	6. Definition of similar triangles
7. $\triangle ABC \cong \triangle DEF$	7. Definition of congruent triangles



Be prepared to share your solutions and methods.

## Check for Students' Understanding

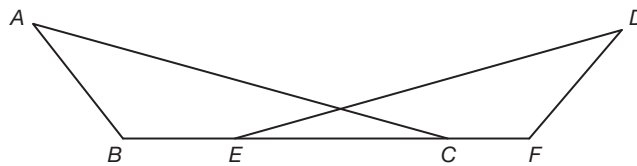
1. Suppose  $\overline{AB} \cong \overline{BC}$  and  $\overline{BD} \cong \overline{CD}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $ABD$  is congruent to triangle  $ACD$  by SSS Congruence Theorem. Side  $AD$  is congruent to itself (reflexive property), and that is the third pair of corresponding sides in the triangles.**

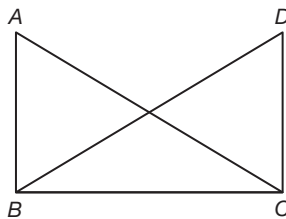
2. Suppose  $\overline{AB} \cong \overline{DF}$ ,  $\overline{AC} \cong \overline{DE}$  and  $\overline{BE} \cong \overline{FC}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $ABC$  is congruent to triangle  $DFE$  by SSS Congruence Theorem. Side  $BC$  ( $BE + EC$  by segment addition) is congruent to side  $FE$  ( $FC + EC$  by segment addition) because adding the same segment length ( $EC$ ) to two segments of equal length ( $BE$  and  $FC$ ) result in two new segments of equal length, and that is the third pair of corresponding sides in the triangles.**

3. Suppose  $\overline{AB} \cong \overline{DC}$  and  $\overline{AC} \cong \overline{DB}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $ABC$  is congruent to triangle  $DCB$  by SSS Congruence Theorem. Side  $BC$  is congruent to itself (reflexive property), and that is the third pair of corresponding sides in the triangles.**

# Make Sure the Angle Is Included

## Side-Angle-Side Congruence Theorem

### LEARNING GOALS

In this lesson, you will:

- Explore the Side-Angle-Side Congruence Theorem using constructions.
- Explore the Side-Angle-Side Congruence Theorem on the coordinate plane.
- Prove the Side-Angle-Side Congruence Theorem.

### ESSENTIAL IDEAS

- The Side-Angle-Side Congruence Theorem states “If two sides and the included angle of one triangle are congruent to the corresponding sides and the included angle of another triangle, then the triangles are congruent.”
- Rotations preserve the size and shape of geometric figures.

### COMMON CORE STATE STANDARDS FOR MATHEMATICS

#### G-CO Congruence

##### Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

### KEY TERM

- Side-Angle-Side Congruence Theorem

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

#### Prove geometric theorems

10. Prove theorems about triangles.

#### Make geometric constructions

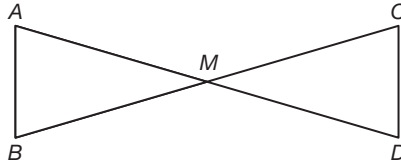
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

## Overview

The Side-Angle-Side Congruence Theorem is stated and students use construction tools to informally prove the SAS Congruence Theorem. Rigid motion is used to explore the SAS Congruence Theorem in this lesson. They rotate a triangle on the coordinate plane and algebraically prove the image and pre-image are congruent. Then, a formal two-column proof of the SAS Congruence Theorem is presented. Lastly, students determine if enough information is given to conclude two triangles are congruent using either the SSS or the SAS Congruence Theorem.

## Warm Up

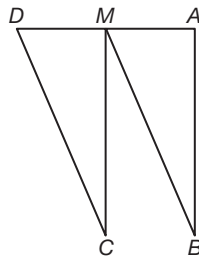
1. Suppose  $\overline{AB} \cong \overline{CD}$  and  $M$  is the midpoint of  $\overline{AD}$  and  $\overline{BC}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $ABM$  is congruent to triangle  $CDM$  by SSS Congruence Theorem. Since  $M$  is the midpoint of line segments  $AD$  and  $BC$ , then line segment  $AM$  is congruent to line segment  $DM$  and line segment  $BM$  is congruent to line segment  $CM$ .**

2. Suppose  $\overline{AB} \cong \overline{MC}$ , and  $\overline{BM} \cong \overline{CD}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**No. There is not enough information to determine if triangle  $ABM$  is congruent to triangle  $MCD$ . We do not know if  $M$  is the midpoint of line segment  $AD$ .**



# Make Sure the Angle Is Included

## Side-Angle-Side Congruence Theorem

### LEARNING GOALS

In this lesson, you will:

- Explore the Side-Angle-Side Congruence Theorem using constructions.
- Explore the Side-Angle-Side Congruence Theorem on the coordinate plane.
- Prove the Side-Angle-Side Congruence Theorem.

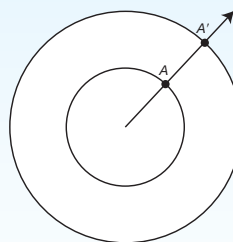
### KEY TERM

- Side-Angle-Side Congruence Theorem

The smaller circle you see here has an infinite number of points. The larger circle also has an infinite number of points. But since the larger circle is, well, larger, shouldn't it have more points than the smaller circle?

Mathematicians use one-to-one correspondence to determine if two sets are equal. If you can show that each object in a set corresponds to one and only one object in another set, then the two sets are equal.

Look at the circles. Any ray drawn from the center will touch only two points—one on the smaller circle and one on the larger circle. This means that both circles contain the same number of points! Can you see how correspondence was used to come up with this answer?



## Problem 1

The Side-Angle-Side Congruence Theorem is stated. Students are given two sides of a triangle and the included angle and using a construction tools, they construct a triangle and compare their triangle to their classmates'. They conclude that all corresponding sides and angles are also congruent, therefore the triangles are congruent thus showing SAS is a valid method for proving two triangles congruent.

## Grouping

- Ask students to read introduction. Discuss as a class.
- Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 6

- What is the first step in your construction?
- Did you begin the construction by duplicating a line segment or an angle first? Why?
- Which point did you use first?
- Which line segment did you duplicate on the starter line?
- Did your classmates duplicate the same line segment on their starter line?
- Which line segment did you duplicate next?

## PROBLEM 1 Using Constructions to Support Side-Angle-Side



So far in this chapter, you have determined the congruence of two triangles by proving that if the sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.

There is another way to determine if two triangles are congruent that does not involve knowledge of three sides. You will prove the *Side-Angle-Side Congruence Theorem*.

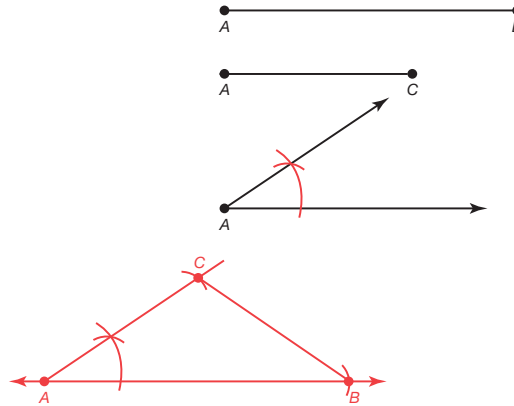
The **Side-Angle-Side Congruence Theorem** states: "If two sides and the included angle of one triangle are congruent to the corresponding sides and the included angle of the second triangle, then the triangles are congruent."

First, let's prove this theorem through construction.



1. Construct  $\triangle ABC$  using the two line segments and included angle shown. Then, write the steps you performed to construct the triangle.

Recall that an included angle is the angle formed by two sides of a triangle.



Construct a starter line.

Plot point  $A$  on the starter line.

Duplicate  $\overline{AB}$  on the starter line to locate point  $B$ .

Duplicate  $\angle A$  using point  $A$  on the starter line.

Duplicate  $\overline{AC}$  on the left ray of  $\angle A$  to locate point  $C$ .

Connect points  $A$ ,  $B$ , and  $C$  to form  $\triangle ABC$ .

- Did you have to duplicate the third line segment to complete the triangle? Why or why not?
- How does the orientation of your triangle compare to the orientation of your classmates' triangles?
- How many different triangles could result from this construction? Explain.
- If the triangles are oriented differently, are they still congruent?
- If the triangles are oriented differently, could rigid motion map one triangle onto the other triangle?



- What type of rigid motion is needed to map your triangle onto your classmates' triangle that is oriented differently?
- Is Side-Angle-Side a valid shortcut for proving two triangles congruent? Explain.

2. How does the length of side  $\overline{BC}$  compare to the length of your classmates' side  $\overline{BC}$ ?  
**Side  $BC$  is the same length in everyone's triangle.**

3. Use a protractor to measure angle  $B$  and angle  $C$  in triangle  $ABC$ .  
 $m\angle B = 34^\circ$   
 $m\angle C = 111.5^\circ$

4. How do the measures of your corresponding angles compare to the measures of your classmates' corresponding angles?  
**We all have congruent corresponding angles.**

5. Is your triangle congruent to your classmates' triangles? Why or why not?  
**Yes. All of the triangles are congruent because the corresponding sides and the corresponding angles are congruent.**



6. If you were given one of the non-included angles,  $\angle C$  or  $\angle B$ , instead of  $\angle A$ , do you think everyone in your class would have constructed an identical triangle? Explain your reasoning.  
**No. Each student would probably construct different triangles because the measures of  $\angle C$  and  $\angle B$  do not determine the length of side  $CB$ .**

## Problem 2

Students graph three given coordinates and connect the points to form a triangle. Next, they rotate two sides and the included angle of the triangle on the coordinate plane  $270^\circ$  counterclockwise. Points  $A'$  and  $C'$  are connected to form a second triangle, and the distance formula is used to calculate the lengths of the sides of the image and pre-image. Students conclude the image and pre-image are congruent triangles using the SSS Congruence Theorem thus validating SAS Congruence.

## Grouping

Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 6

- How would you classify triangle  $ABC$ ?
- In which quadrant is triangle  $ABC$  located?
- Did you rotate the side lengths and the angle about the origin?
- In which quadrant is triangle  $A'B'C'$  located?
- How can the SSS Congruence Theorem be applied to this situation?

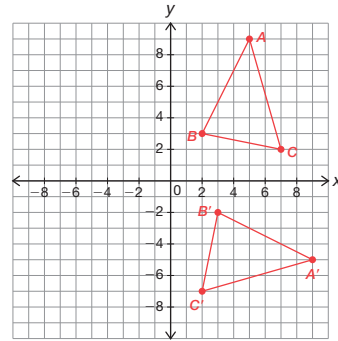
## PROBLEM 2 Using Rotation to Support Side-Angle-Side

Through your construction, you and your classmates constructed congruent triangles using two given sides and the included angle of a triangle.

Let's now try to prove the Side-Angle-Side Theorem on the coordinate plane using algebra.



1. Graph triangle  $ABC$  by plotting the points  $A(5, 9)$ ,  $B(2, 3)$ , and  $C(7, 2)$ .



2. Calculate the length of each side of triangle  $ABC$  and record the measurements in the table. Record exact measurements.

Side of Triangle $ABC$	Length of Side
$\overline{AB}$	$\sqrt{45}$
$\overline{BC}$	$\sqrt{26}$
$\overline{AC}$	$\sqrt{53}$

$$\begin{aligned} AB &= \sqrt{(5-2)^2 + (9-3)^2} \\ &= \sqrt{3^2 + 6^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(7-2)^2 + (2-3)^2} \\ &= \sqrt{5^2 + (-1)^2} \\ &= \sqrt{25 + 1} \\ &= \sqrt{26} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(5-7)^2 + (9-2)^2} \\ &= \sqrt{(-2)^2 + 7^2} \\ &= \sqrt{4 + 49} \\ &= \sqrt{53} \end{aligned}$$

Do you remember the difference between exact and approximate solutions?



- If the triangle was rotated clockwise, do you think the image and the pre-image would be that of congruent triangles? Why or why not?
- In what situations would it be appropriate to use the SAS Congruence Theorem?
- In what situations would it not be appropriate to use the SAS Congruence Theorem?

3. Rotate side  $\overline{AB}$ , side  $\overline{BC}$ , and included angle  $B$ , in triangle  $ABC$   $270^\circ$  counterclockwise about the origin. Then, connect points  $A'$  and  $C'$  to form triangle  $A'B'C'$ . Use the table to record the image coordinates.

Coordinates of Triangle $ABC$	Coordinates of Triangle $A'B'C'$
$A (5, 9)$	$A' (9, -5)$
$B (2, 3)$	$B' (3, -2)$
$C (14, 3)$	$C' (2, -7)$

Before you graph, think! In which quadrant will the rotated image appear?



4. Calculate the length of each side of triangle  $A'B'C'$  and record the measurements in the table. Record exact measurements.

Side of Triangle $A'B'C'$	Length of Side
$\overline{A'B'}$	$\sqrt{45}$
$\overline{B'C'}$	$\sqrt{26}$
$\overline{A'C'}$	$\sqrt{53}$

$$\begin{aligned} A'B' &= \sqrt{(9-3)^2 + [-5-(-2)]^2} \\ &= \sqrt{6^2 + (-3)^2} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

$$\begin{aligned} B'C' &= \sqrt{(3-2)^2 + [-2-(-7)]^2} \\ &= \sqrt{1^2 + 5^2} \\ &= \sqrt{1+25} \\ &= \sqrt{26} \end{aligned}$$

$$\begin{aligned} A'C' &= \sqrt{(9-2)^2 + [-5-(-7)]^2} \\ &= \sqrt{7^2 + 2^2} \\ &= \sqrt{49+4} \\ &= \sqrt{53} \end{aligned}$$

5. What do you notice about the corresponding side lengths of the pre-image and the image?

The side lengths of triangle  $ABC$  are the same length as the corresponding side lengths of triangle  $A'B'C'$ .

6. Use a protractor to measure angle  $B$  of triangle  $ABC$  and angle  $B'$  of triangle  $A'B'C'$ .

a. What are the measures of each angle?

The angle measures of both angle  $B$  and angle  $B'$  are  $76^\circ$ .



b. What does this information tell you about the corresponding angles of the two triangles?

This tells me that the angles are congruent.



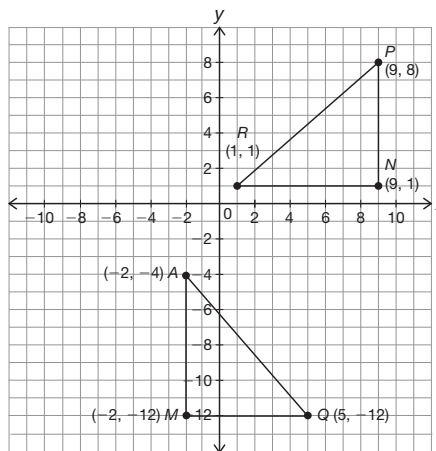
You have shown that the corresponding sides of the image and pre-image are congruent. Therefore, the triangles are congruent by the SSS Congruence Theorem.

You have also used a protractor to verify that the corresponding included angles of each triangle are congruent.

In conclusion, when two side lengths of one triangle and the measure of the included angle are equal to the two corresponding side lengths and the measure of the included angle of another triangle, the two triangles are congruent by the SAS Congruence Theorem.



7. Use the SAS Congruence Theorem and a protractor to determine if the two triangles drawn on the coordinate plane shown are congruent. Use a protractor to determine the measures of the included angles.



The lengths of sides  $AM$  and  $RN$  are equal.

The lengths of sides  $MQ$  and  $NP$  are equal.

The measures of  $\angle M$  and  $\angle N$  are equal.

Triangle  $AMQ$  and triangle  $RNP$  are congruent by the SAS Congruence Theorem.



## Grouping

- Ask students to read the information. Discuss as a class.
- Have students complete Question 7 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Question 7

- How did you determine the length of side  $PN$  and the length of side  $AM$ ?
- How did you determine the length of side  $RN$  and the length of side  $QM$ ?
- How did you determine the length of side  $PR$  and the length of side  $AQ$ ?
- How does the length of side  $AM$  compare to the length of side  $RN$ ?
- How does the length of side  $MQ$  compare to the length of side  $NP$ ?
- How does the measure of angle  $M$  compare to the measure of angle  $N$ ?

- Does knowing the lengths of two pairs of corresponding sides and the measures of the included angles are equal enough information to determine the two triangles are congruent? Why or why not?
- Identify the corresponding sides of the two triangles.
- Write at least 2 valid triangle congruence statements.

### Problem 3

Students graph the three given coordinates from Problem 1 and reflect the triangle over the  $x$ -axis. Using the distance formula to calculate the lengths of the sides of the image and pre-image, they conclude the image and pre-image are congruent triangles using the SSS Congruence Theorem.

### Grouping

Have students complete Problem 3 with a partner. Then have students share their responses as a class.

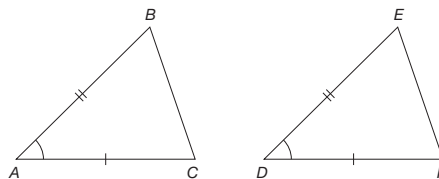
### Guiding Questions for Share Phase

- How can you conclude triangle  $BAC$  to be congruent to triangle  $DAC$  if you only know the length of two pair of corresponding sides?
- Do the two triangles share a common side?
- Do the two triangles share a common angle?
- Is angle  $A$  shared by triangle  $ABC$  and triangle  $ADC$ ? Why not?
- Is  $AC$  perpendicular to  $BD$ ? How do you know?
- Is the line segment  $FC$  shared by side  $AC$  and side  $DF$ ?
- Is line segment  $AC$  the same length as line segment  $DF$ ? How do you know?
- Are angle  $A$  and angle  $D$  considered included angles in triangle  $ABC$  and triangle  $DCB$ ?

### PROBLEM 3 And Finally the Proof . . .



Prove the Side-Angle-Side Congruence Theorem.



Given:  $\overline{AB} \cong \overline{DE}$ ,  $\overline{AC} \cong \overline{DF}$ , and  $\angle A \cong \angle D$

Prove:  $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$ , $\overline{AC} \cong \overline{DF}$ , and $\angle A \cong \angle D$	1. Given
2. $AB = DE$ , $AC = DF$	2. Definition of congruence
3. $\frac{AB}{DE} = 1$ , $\frac{AC}{DF} = 1$	3. Division Property of Equality
4. $\frac{AB}{DE} = \frac{AC}{DF}$	4. Transitive Property of Equality
5. $\triangle ABC \sim \triangle DEF$	5. SAS Similarity Postulate
6. $\angle B \cong \angle E$ and $\angle C \cong \angle F$	6. Definition of similar triangles
7. $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$	7. Definition of similar triangles
8. $\frac{BC}{EF} = 1$	8. Substitution Property of Equality
9. $BC = EF$	9. Multiplicative Property of Equality
10. $\overline{BC} \cong \overline{EF}$	10. Definition of congruence
11. $\triangle ABC \cong \triangle DEF$	11. Definition of congruent triangles



## Grouping

Ask students to read worked example and complete Question 1. Discuss as a class.

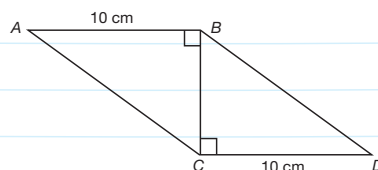
### PROBLEM 4 Writing Congruence Statements



You can analyze diagrams and use SAS and SSS to determine if triangles are congruent.

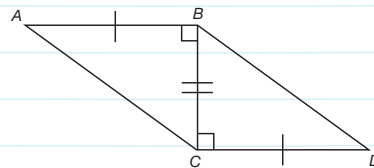


Analyze the figure shown to determine if  $\triangle ABC$  is congruent to  $\triangle DCB$ .



Notice,  $m\overline{AB} = 10$  cm and  $m\overline{DC} = 10$  cm, and the two line segments are corresponding sides of the two triangles. Also, notice that  $\angle ABC$  and  $\angle DCB$  are right angles, and they are corresponding angles of the two triangles.

In order to prove that the two triangles are congruent using SAS, you need to show that another side of triangle  $ABC$  is congruent to another side of triangle  $DCB$ . Notice that the two triangles share a side. Because  $\overline{BC}$  is the same as  $\overline{CB}$ , you know that these two line segments are congruent.



So,  $\triangle ABC \cong \triangle DCB$  by the SAS Congruence Theorem.

1. Write the three congruence statements that show  $\triangle ABC \cong \triangle DCB$  by the SAS Congruence Theorem.

$$\overline{AB} \cong \overline{DC}$$

$$\overline{BC} \cong \overline{CB}$$

$$\angle ABC \cong \angle DCB$$

## Grouping

Have students complete Questions 2 and 3 with a partner. Then have students share their responses as a class.

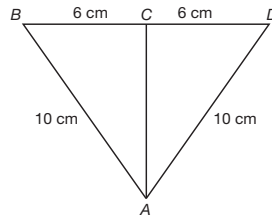
## Guiding Questions for Share Phase, Questions 2 and 3

- Explain how the reflexive property helps prove that the triangles in Question 2 part (a) are congruent.
- Explain how segment addition helps prove that the triangles in Question 2 part (b) are congruent.
- How do you know whether or not an angle is an included angle?



2. Determine if there is enough information to prove that the two triangles are congruent by SSS or SAS. Write the congruence statements to justify your reasoning.

a.  $\triangle ABC \stackrel{?}{\cong} \triangle ADC$



The triangles are congruent by SSS.

$$\overline{BC} \cong \overline{DC}$$

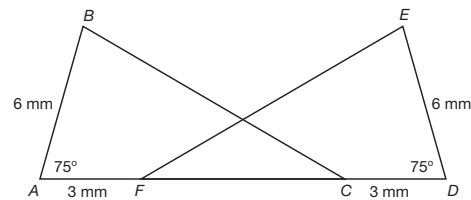
$$\overline{AB} \cong \overline{AD}$$

$$\overline{AC} \cong \overline{AC}$$

Use markers to identify all congruent line segments and angles.



b.  $\triangle ABC \stackrel{?}{\cong} \triangle DEF$



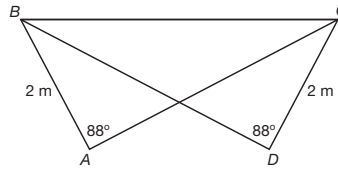
The triangles are congruent by SAS.

$$\overline{AB} \cong \overline{DE}$$

$$\angle BAC \cong \angle EDF$$

$$\overline{AC} \cong \overline{DF}$$

3. Simone says that since triangle  $ABC$  and triangle  $DCB$  have two pairs of congruent corresponding sides and congruent corresponding angles, then the triangles are congruent by SAS. Is Simone correct? Explain your reasoning.



Simone is not correct. The congruent angles are not formed by two pairs of congruent sides, so they are not the included angles. There is not enough information to determine if the triangles are congruent by SAS or SSS.



Be prepared to share your solutions and methods.



## Check for Students' Understanding

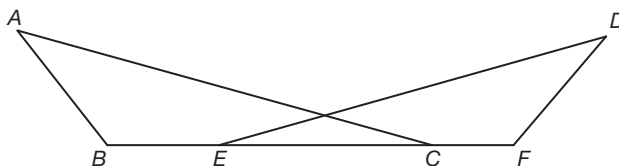
1. Suppose  $\overline{AD} \perp \overline{BC}$  and  $\overline{BD} \cong \overline{CD}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes.** Triangle  $ABD$  is congruent to triangle  $ACD$  by SAS Congruence Theorem. Side  $AD$  is congruent to itself (reflexive property). Side  $AD$  is perpendicular to side  $BC$  such that  $\angle ADB$  and  $\angle ADC$  are right angles and all right angles are congruent.

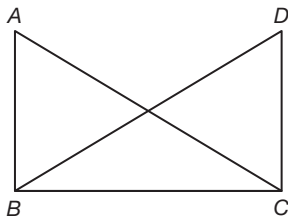
2. Suppose  $\overline{AB} \cong \overline{DF}$ ,  $\angle A \cong \angle D$  and  $\overline{BE} \cong \overline{FC}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**No.** There is not enough information to determine that triangle  $ABC$  congruent to triangle  $DFE$  because  $\angle A$  and  $\angle D$  are not the included angles.

3. Suppose  $\overline{AB} \cong \overline{DC}$ ,  $\overline{AB} \perp \overline{BC}$  and  $\overline{DC} \perp \overline{CB}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes.** Triangle  $ABC$  is congruent to triangle  $DCB$  by SAS Congruence Theorem. Side  $BC$  is congruent to itself (reflexive property). Side  $AB$  is perpendicular to side  $BC$  and side  $DC$  is perpendicular to side  $CB$  such that  $\angle ABC$  and  $\angle DCB$  are right angles and all right angles are congruent.



# Angle to the Left of Me, Angle to the Right of Me

## Angle-Side-Angle Congruence Theorem

### LEARNING GOALS

In this lesson, you will:

- Explore the Angle-Side-Angle Congruence Theorem using constructions.
- Explore the Angle-Side-Angle Congruence Theorem on the coordinate plane.
- Prove the Angle-Side-Angle Congruence Theorem.

### ESSENTIAL IDEAS

- The Angle-Side-Angle Congruence Theorem states “If two angles and the included side of one triangle are congruent to the corresponding angles and the included side of another triangle, then the triangles are congruent.”

### COMMON CORE STATE STANDARDS FOR MATHEMATICS

#### G-CO Congruence

##### Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

### KEY TERM

- Angle-Side-Angle Congruence Theorem

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

#### Prove geometric theorems

10. Prove theorems about triangles.

#### Make geometric constructions

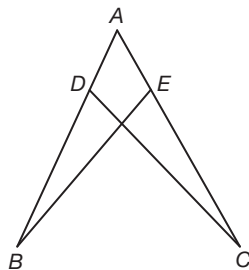
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

## Overview

The Angle-Side-Angle Congruence Theorem is stated and students use construction tools to informally prove the ASA Congruence Theorem. Rigid motion is used to explore the ASA Congruence Theorem in this lesson. Three triangles are drawn on a coordinate plane and students consider two triangles at a time, describing the possible transformations necessary to map one triangle onto the other, then use the distance formula and a protractor to measure the lengths of the sides and the measures of the angles to conclude the triangles are congruent in the first situation but not in the second situation. The lesson ends with a formal two-column proof of the ASA Congruence Theorem.

## Warm Up

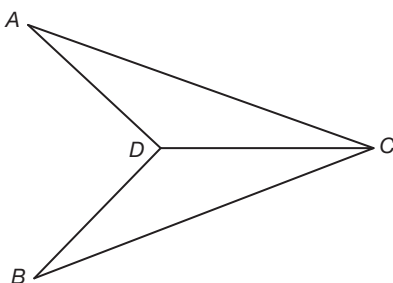
1. Suppose  $\overline{AB} \cong \overline{AC}$  and  $\overline{AD} \cong \overline{AE}$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $ABE$  is congruent to triangle  $ACD$  by SAS Congruence Theorem. Angle  $A$  is shared by both triangles and is congruent to itself (reflexive property).**

2. Suppose  $\overline{AC} \cong \overline{BC}$ , and  $\overline{DC}$  bisects  $\angle C$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $CDA$  is congruent to triangle  $CDB$  by SAS Congruence Theorem. Side  $CD$  is shared by both triangles and is congruent to itself (reflexive property), and  $\angle ACD$  and  $\angle BCD$  are congruent by definition of angle bisector.**



# Angle to the Left of Me, Angle to the Right of Me

## Angle-Side-Angle Congruence Theorem

### LEARNING GOALS

In this lesson, you will:

- Explore the Angle-Side-Angle Congruence Theorem using constructions.
- Explore the Angle-Side-Angle Congruence Theorem on the coordinate plane.
- Prove the Angle-Side-Angle Congruence Theorem.

### KEY TERM

- Angle-Side-Angle Congruence Theorem

“Don’t judge a book by its cover.” What does this saying mean to you? Usually it is said to remind someone not to make assumptions. Just because something (or someone!) looks a certain way on the outside, until you really get into it, you don’t know the whole story. Often in geometry, it is easy to make assumptions. You assume that two figures are congruent because they look congruent. You assume two lines are perpendicular because they look perpendicular. Unfortunately, mathematics and assumptions do not go well together. Just as you should not judge a book by its cover, you should not assume anything about a measurement just because it looks a certain way.

Have you made any geometric assumptions so far in this chapter? Was your assumption correct or incorrect? Hopefully, it will only take you one incorrect assumption to learn not to assume!

## Problem 1

Using construction tools, students will construct a triangle using two given angles and the included side. They then compare their triangle to their classmates' triangles and conclude that all constructed triangles are congruent to each other supporting the ASA Congruence Theorem. The Angle-Side-Angle Congruence Theorem is stated.

## Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 3

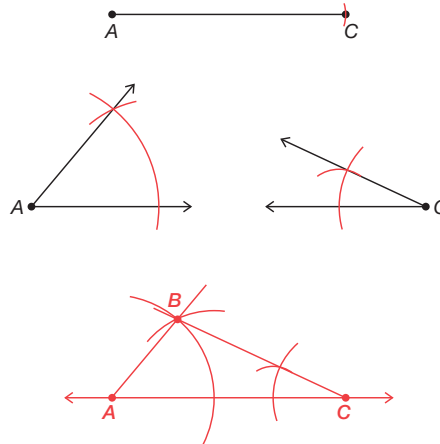
- What is the first step in your construction?
- Which point did you locate first?
- Did you duplicate the line segment or an angle on the starter line first?
- Which angle did you duplicate first?
- How did you determine the location of the third interior angle?
- How did you determine the length of the other two sides?
- How does the orientation of your triangle compare to the orientation of your classmates' triangles?

### PROBLEM 1 Putting the Pieces Together

So far you have looked at the Side-Side-Side Congruence Theorem and the Side-Angle-Side Congruence Theorem. But are there other theorems that prove triangle congruence as well?



1. Use the given two angles and included line segment to construct triangle  $ABC$ . Then, write the steps you performed to construct the triangle.



Construct a starter line.

Duplicate the line segment on the line.

Duplicate each angle using the two endpoints of the line segment as the vertices of the angles.

Extend the sides of the angles to form a triangle.

- How many different triangles could result from this construction? Explain.
- If the triangles are oriented differently, are they still congruent?
- If the triangles are oriented differently, could rigid motion map one triangle onto the other triangle?
- What type of rigid motion is needed to map your triangle onto your classmates' triangle that is oriented differently?
- Is Angle-Side-Angle a valid shortcut for proving two triangles congruent? Explain.



2. Compare your triangle to your classmates' triangles. Are the triangles congruent? Why or why not?

Yes. All the triangles are congruent. I know this is true because everyone used the same line segment and two angles to form the triangle. Because there was only one way to put these pieces together, all the triangles must be congruent.

3. Wendy says that if the line segment and angles in the construction had not been labeled, then all the triangles would not have been congruent. Ian disagrees, and says that there is only one way to put two angles and a side together to form a triangle, whether they are labeled or not. Who is correct? Explain your reasoning.

Wendy is correct. Had the line segment and angles not been labeled, there would be three different ways these pieces could have gone together. Because they were labeled, there is only one way.

Recall that an included side is the side between two angles of a triangle.



You just used construction to prove the *Angle-Side-Angle Congruence Theorem*. The **Angle-Side-Angle Congruence Theorem** states: "If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the triangles are congruent."



## Problem 2

Three triangles are graphed on the coordinate plane. Students focus on two triangles at one time and describe the possible transformations needed to map one triangle onto another. The distance formula is used to calculate the lengths of the sides of the image and pre-image and a protractor is used to determine the measure of the angles. Students conclude the image and pre-image are congruent triangles using the ASA Congruence Theorem for the first pair of triangles, but the second pair of triangles are not congruent.

## Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

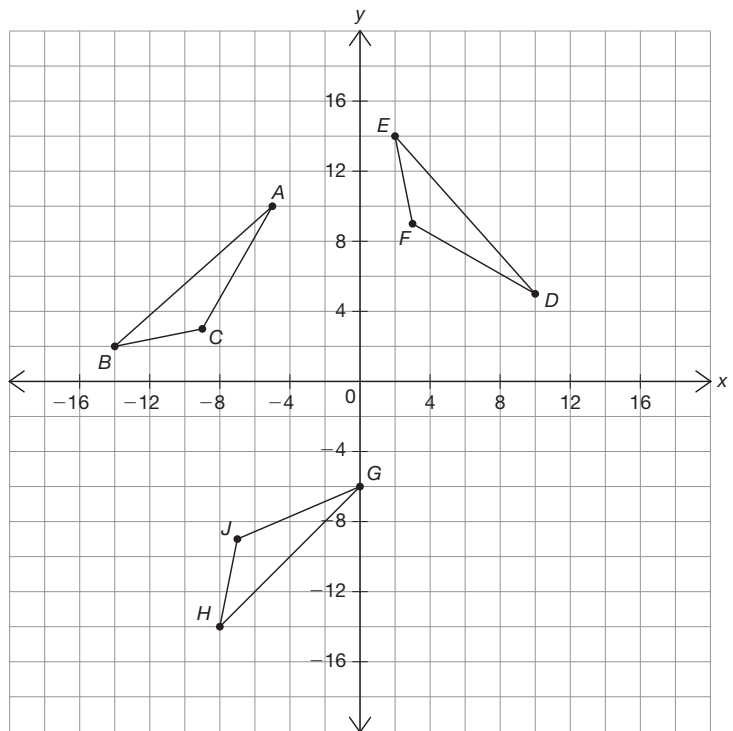
## Guiding Questions for Share Phase, Questions 1 through 3

- In what situations would it be appropriate to use the ASA Congruence Theorem?
- In what situations would it not be appropriate to use the ASA Congruence Theorem?
- How did you determine the type of transformation(s) used to create each image?
- Could a different transformation(s) be used to create the same image?
- Is triangle  $ABC$  congruent to triangle  $DEF$ ? Explain.

### PROBLEM 2 How Did You Get There?



1. Analyze triangles  $ABC$  and  $DEF$ .



- a. Measure the angles and calculate the side lengths of each triangle.

$\angle A = 20^\circ$	$\angle D = 20^\circ$
$\angle B = 30^\circ$	$\angle E = 30^\circ$
$\angle C = 130^\circ$	$\angle F = 130^\circ$
$m\overline{AB} = \sqrt{145}$	$m\overline{DE} = \sqrt{145}$
$m\overline{BC} = \sqrt{26}$	$m\overline{EF} = \sqrt{26}$
$m\overline{AC} = \sqrt{65}$	$m\overline{DF} = \sqrt{65}$

- b. Describe the possible transformation(s) that could have occurred to transform pre-image  $ABC$  into image  $DEF$ .

Answers will vary.

The pre-image looks like it was rotated  $90^\circ$  clockwise around the origin to create the image.

- If a counterclockwise rotation were performed, how many degree rotation about the origin would be needed to map triangle  $ABC$  onto triangle  $DEF$ ?
- How does the length of side  $AB$  compare to the length of side  $DE$ ?
- How does the measure of angle  $A$  compare to the measure of angle  $D$ ?
- How does the measure of angle  $B$  compare to the measure of angle  $E$ ?
- To map triangle  $DEF$  onto triangle  $GHJ$ , was it reflected over the  $x$ -axis or was it reflected over the  $y$ -axis?

- To map triangle  $DEF$  onto triangle  $GHJ$ , was it translated to the left or translated to the right?
- How does the length of side  $DE$  compare to the length of side  $GJ$ ?
- How does the measure of angle  $D$  compare to the measure of angle  $G$ ?
- How does the measure of angle  $E$  compare to the measure of angle  $J$ ?
- What property did you use to answer Question 3?

- c. Identify two pairs of corresponding angles and a pair of corresponding included sides that could be used to determine congruence through the ASA Congruence Theorem.

Answers will vary.

Angle  $A$ , angle  $B$ , and side  $AB$  correspond with angle  $D$ , angle  $E$ , and side  $DE$ .

- d. Use the ASA Congruence Theorem to determine if the two triangles are congruent.

Answers will vary.

The lengths of sides  $AB$  and  $DE$  are congruent.

The measure of  $\angle A$  and  $\angle D$  are congruent.

The measure of  $\angle B$  and  $\angle E$  are congruent.

Triangle  $ABC$  and triangle  $DEF$  are congruent by the ASA Congruence Theorem.

2. Analyze triangles  $DEF$  and  $GHJ$ .

- a. Measure the angles and calculate the side lengths of triangle  $GHJ$ .

$$\angle G = 22^\circ$$

$$\angle H = 33^\circ$$

$$\angle J = 125^\circ$$

$$m\overline{GH} = \sqrt{128}$$

$$m\overline{HJ} = \sqrt{26}$$

$$m\overline{GJ} = \sqrt{58}$$

- b. Describe the possible transformation(s) that could have occurred to transform pre-image  $DEF$  to image  $GHJ$ .

Answers will vary.

The pre-image looks like it was reflected over the  $x$ -axis, then translated 10 units to the left.

- c. Identify two pairs of corresponding angles and a pair of corresponding included sides that could be used to determine congruence through the ASA Congruence Theorem.

Answers will vary.

Angle  $D$ , angle  $F$ , and side  $DF$  correspond with angle  $G$ , angle  $J$ , and side  $GJ$ .

- d. Use the ASA Congruence Theorem to determine if the two triangles are congruent.

Answers will vary.

The lengths of sides  $DE$  and  $GJ$  are not congruent.

The measure of  $\angle D$  and  $\angle G$  are not congruent.

The measure of  $\angle E$  and  $\angle J$  are not congruent.

Triangle  $DEF$  and triangle  $GHJ$  are not congruent by the ASA Congruence Theorem.



3. What can you conclude about the relationship between triangle  $ABC$  and triangle  $GHJ$ ? Explain your reasoning.

I know that triangle  $ABC$  and triangle  $GHJ$  are not congruent.

I know that triangle  $ABC$  is congruent to triangle  $DEF$ , but triangle  $DEF$  is not congruent to triangle  $GHJ$ . Therefore, triangle  $ABC$  is not congruent to triangle  $GHJ$ .

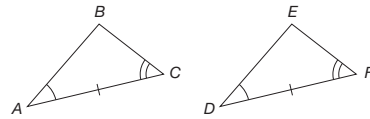
### PROBLEM 3 And Finally the Proof . . .



Prove the Angle-Side-Angle Congruence Theorem.

Given:  $\angle A \cong \angle D, \overline{AC} \cong \overline{DF}, \angle C \cong \angle F$

Prove:  $\triangle ABC \cong \triangle DEF$



Statements

Reasons

1. $\angle A \cong \angle D, \overline{AC} \cong \overline{DF}, \angle C \cong \angle F$	1. Given
2. $\triangle ABC \sim \triangle DEF$	2. AA Similarity Postulate
3. $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$	3. Definition of Similar Triangles
4. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	4. Definition of Similar Triangles
5. $AC = DF$	5. Definition of Congruence
6. $\frac{AC}{DF} = 1$	6. Division Property of Equality
7. $\frac{AB}{DE} = \frac{BC}{EF} = 1$	7. Substitution
8. $AB = DE, BC = EF$	8. Multiplication Property of Equality
9. $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$	9. Definition of Congruence
10. $\triangle ABC \cong \triangle DEF$	10. Definition of Congruent Triangles



Be prepared to share your solutions and methods.

## Grouping

Have students complete Problem 3 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase

- How can you determine the first statement of the proof?
- How can you determine the last statement of the proof?
- What is the difference between similar triangles and congruent triangles?
- What do the proofs of the SSS, SAS, and ASA Triangle Congruence Theorems have in common?

## Check for Students' Understanding

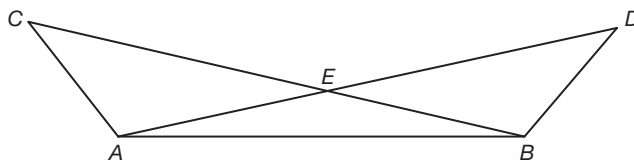
1. Suppose  $\overline{AD} \perp \overline{BC}$ , and  $\overline{AD}$  bisects  $\angle A$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $ABD$  is congruent to triangle  $ACD$  by ASA Congruence Theorem. Side  $AD$  is congruent to itself (reflexive property). Side  $AD$  is perpendicular to side  $BC$  such that  $\angle ADB$  and  $\angle ADC$  are right angles and all right angles are congruent. Angle  $BAD$  is congruent to angle  $CAD$  because segment  $AD$  bisects angle  $A$ .**

2. Use the diagram to answer the questions.



- a. What additional information is needed to conclude  $\triangle CAB \cong \triangle DBA$  using the ASA Congruence Theorem?

**Answers will vary.**

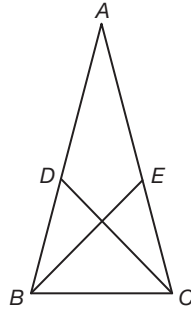
**I know segment  $AB$  is congruent to itself because of the reflexive property. In addition, I would need to know that the following angle pairs are congruent:  $\angle CAD$  and  $\angle DBA$ ,  $\angle CDA$  and  $\angle DAB$ .**

- b. What additional information is needed to conclude  $\triangle CAE \cong \triangle DBE$  using the ASA Congruence Theorem?

**Answers will vary.**

**I know angles  $CEA$  and  $DEB$  are congruent because they are vertical angles. In addition, I would need to know that segments  $CE$  and  $DE$  are congruent, and that angles  $C$  and  $D$  are congruent.**

3. Suppose  $\angle DBC \cong \angle ECB$ , and  $\angle DCB \cong \angle ECB$  in the diagram shown.



Are there congruent triangles in this diagram? Explain your reasoning?

**Yes. Triangle  $DBC$  is congruent to triangle  $ECB$  by ASA Congruence Theorem. Side  $BC$  is congruent to itself (reflexive property).**

# Sides Not Included

## Angle-Angle-Side Congruence Theorem

### LEARNING GOALS

In this lesson, you will:

- Explore the Angle-Angle-Side Congruence Theorem using constructions.
- Explore the Angle-Angle-Side Congruence Theorem on the coordinate plane.
- Prove the Angle-Angle-Side Congruence Theorem.

### ESSENTIAL IDEAS

- The Angle-Angle-Side Congruence Theorem states “If two angles and the non-included side of one triangle are congruent to the corresponding angles and the non-included side of another triangle, then the triangles are congruent.”

### COMMON CORE STATE STANDARDS FOR MATHEMATICS

#### G-CO Congruence

##### Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

### KEY TERM

- Angle-Angle-Side Congruence Theorem

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

#### Prove geometric theorems

10. Prove theorems about triangles.

#### Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

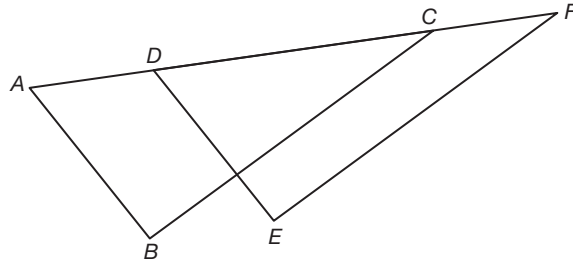
## Overview

Rigid motion is used to explore the AAS Congruence Theorem in this lesson. When two angles and the non-included side of a triangle are translated and later reflected, students will compare the lengths of the corresponding sides and the measures of the corresponding angles of the image and pre-image. They then conclude that the image and the pre-image always result in congruent triangles. Because the triangles are situated on a coordinate plane, the Distance Formula in combination a protractor can be used to show the triangles are congruent. A formal two-column proof of the ASA Congruence Theorem is presented. The lesson ends with questions asking students to determine whether triangles are congruent using ASA or AAS.



## Warm Up

1. Suppose  $\overline{AB} \parallel \overline{DE}$  in the diagram shown.

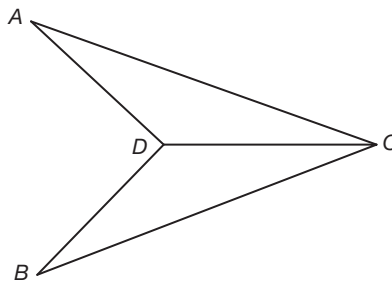


What additional information is needed to conclude  $\triangle ABC \cong \triangle DEF$  using the ASA Congruence Theorem?

Answers will vary.

I would need to know that  $\angle ACB \cong \angle F$  and  $\overline{AD} \cong \overline{CF}$  to conclude  $\triangle ABC \cong \triangle DEF$  using the ASA Congruence Theorem.

2. Suppose  $\overline{DC}$  bisects  $\angle C$  in the diagram shown.



What additional information is needed to conclude  $\triangle ACD \cong \triangle BCD$  using the ASA Congruence Theorem?

Answers will vary.

I would need to know that  $\angle ADC \cong \angle BDC$  to conclude  $\triangle ACD \cong \triangle BCD$  using the ASA Congruence Theorem.



## Sides Not Included

### Angle-Angle-Side Congruence Theorem

#### LEARNING GOALS

In this lesson, you will:

- Explore the Angle-Angle-Side Congruence Theorem using constructions.
- Explore the Angle-Angle-Side Congruence Theorem on the coordinate plane.
- Prove the Angle-Angle-Side Congruence Theorem.

#### KEY TERM

- Angle-Angle-Side Congruence Theorem

Sometimes, good things must come to an end, and that can be said for determining if triangles are congruent, given certain information.

You have used many different theorems to prove that two triangles are congruent based on different criteria. Specifically,

- Side-Side-Side Congruence Theorem
- Side-Angle-Side Congruence Theorem
- and Angle-Side-Angle Congruence Theorem.

So, do you think there are any other theorems that can be used to prove that two triangles are congruent? Here's a hint: we have another lesson—so there must be at least one more congruence theorem!

## Problem 1

Using construction tools, students will construct a triangle using two given angles and the non-included side. They compare their triangle to their classmates' triangles and conclude that all constructed triangles are congruent to each other supporting the AAS Congruence Theorem.

### Grouping

Ask students to read introduction. Discuss as a class.

### PROBLEM 1 Using Constructions to Support AAS



There is another way to determine if two triangles are congruent that is different from the congruence theorems you have already proven. You will prove the *Angle-Angle-Side Congruence Theorem*.

The **Angle-Angle-Side Congruence Theorem** states: "If two angles and a non-included side of one triangle are congruent to the corresponding angles and the corresponding non-included side of a second triangle, then the triangles are congruent."

First, you will prove this theorem through construction.

Recall that a non-included side is a side that is not located between the two angles.



## Grouping

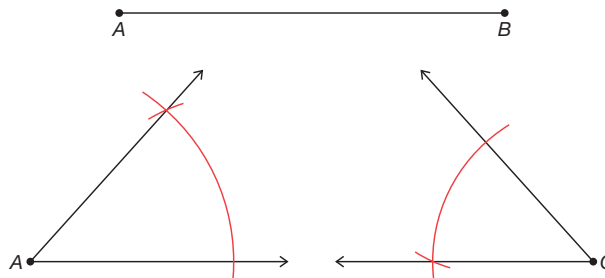
Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 5

- What is the first step in your construction?
- Which point did you locate first?
- Did you duplicate the line segment first or an angle first?
- Which angle did you duplicate first?
- How did you determine the location of the third interior angle?
- How did you determine the length of the other two sides?
- How does the orientation of your triangle compare to the orientation of your classmates' triangles?
- How many different triangles could result from this construction? Explain.
- If the triangles are oriented differently, are they still congruent?
- If the triangles are oriented differently, which rigid motion could map one triangle onto the other triangle?
- Describe the rigid motion that will map your triangle onto a classmate's triangle that is oriented differently.



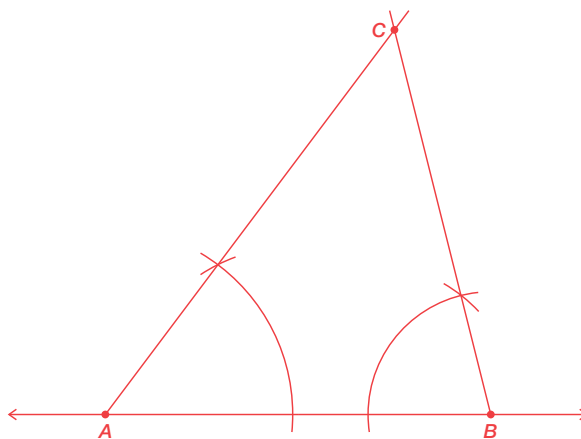
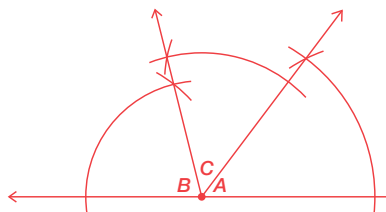
1. Construct triangle  $ABC$  given  $\overline{AB}$  and angles  $A$  and  $C$ . Then, write the steps you performed to construct the triangle.



It might be helpful to construct angle  $B$  first and use that to create your triangle.



Construct a starter line. Duplicate angle  $A$  and angle  $C$  on this starter line adjacent to one another. The remaining angle must be  $\angle B$ .



Construct a second starter line. Duplicate line segment  $AB$  on this line. Duplicate  $\angle A$  and  $\angle B$  on this line using the endpoints. Extend the sides of each angle until they intersect. Label the intersection  $C$ . This angle is congruent to the given  $\angle C$ .

- Is Angle-Angle-Side a valid shortcut for proving two triangles congruent? Explain.
- How are the SSS, SAS, ASA, and AAS constructions similar?
- How are the SSS, SAS, ASA, and AAS constructions different?

2. How does the length of side  $\overline{AB}$  compare to the length of your classmates' side  $\overline{AB}$ ?  
Side  $\overline{AB}$  is the same length in everyone's triangle.

3. Use a protractor to measure angle  $A$  and angle  $C$  in triangle  $ABC$ . What do you notice about your angle measures and your classmates' angle measures?  
The angle measures of angle  $A$  and angle  $C$  are the same as my classmates.

4. Thomas claims that his constructed triangle is not congruent because he drew a vertical starter line that created a triangle that has side  $AB$  being vertical rather than horizontal. Denise claims that all the constructed triangles are congruent even though Thomas's triangle looks different. Who's correct? Why is this student correct?  
Denise is correct because she recognized the given angles and the given line segment are duplicated through construction. Therefore, if all the students duplicated the same angles and side, the triangles must be congruent no matter what the triangle's orientation is.



5. Is your triangle congruent to your classmates' triangles? Why or why not?  
All of the triangles are congruent because the sets of corresponding sides and the sets of the pairs of corresponding angles are congruent.

## Problem 2

Students will reflect two sides and the non-included angle of a triangle over the  $x$ -axis on the coordinate plane. They then use the Distance Formula to determine the lengths of the sides of the image and the pre-image and use a protractor to verify the corresponding angles of the pre-image and image congruent. It can be concluded that since all of the corresponding angles and all of the corresponding sides are congruent, the triangles must be congruent.

## Grouping

Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 6

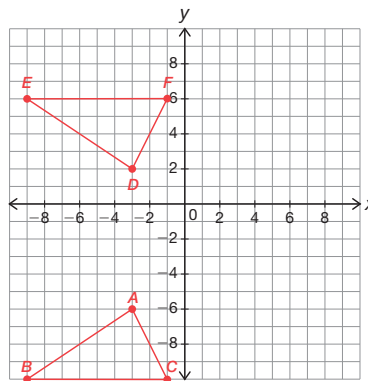
- If the triangle was reflected over the  $y$ -axis, do you think the image and the pre-image would be that of congruent triangles? Why or why not?
- Do reflections preserve size and shape of geometric figures?
- What is the difference between the AAS Congruence Theorem and the ASA Congruence Theorem?
- How is the AAS Congruence Theorem similar to the ASA Congruence Theorem?

## PROBLEM 2 Using Reflection to Support AAS



If two angles and the non-included side of a triangle are reflected, is the image of the triangle congruent to the pre-image of the triangle?

1. Graph triangle  $ABC$  by plotting the points  $A(-3, -6)$ ,  $B(-9, -10)$ , and  $C(-1, -10)$ .



2. Calculate the length of each side of triangle  $ABC$ . Record the exact measurements in the table.

Side of Triangle $ABC$	Length of Side
$\overline{AB}$	$\sqrt{52}$
$\overline{BC}$	8
$\overline{AC}$	$\sqrt{20}$

$$\begin{aligned} AB &= \sqrt{[-3 - (-9)]^2 + [-6 - (-10)]^2} \\ &= \sqrt{6^2 + 4^2} \\ &= \sqrt{36 + 16} \\ &= \sqrt{52} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{[-3 - (-1)]^2 + [-6 - (-10)]^2} \\ &= \sqrt{(-2)^2 + 4^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{[-9 - (-1)]^2 + [-10 - (-10)]^2} \\ &= \sqrt{(-8)^2 + 0^2} \\ &= \sqrt{64} \\ &= 8 \end{aligned}$$

3. Reflect angle  $A$ , angle  $B$ , and side  $\overline{BC}$  over the line of reflection  $y = -2$  to form angle  $D$ , angle  $E$ , and side  $\overline{EF}$ . Then, connect points  $D$  and  $E$  to form triangle  $DEF$ . Record the image coordinates in the table.

Coordinates of Triangle $ABC$	Coordinates of Triangle $DEF$
$A (-3, -6)$	$D (-3, 2)$
$B (-9, -10)$	$E (-9, 6)$
$C (-1, -10)$	$F (-1, 6)$

4. Calculate the length of each side of triangle  $DEF$ . Record the exact measurements in the table.

Side of Triangle $DEF$	Length of Side
$\overline{DE}$	$\sqrt{52}$
$\overline{EF}$	8
$\overline{DF}$	$\sqrt{20}$

$$\begin{aligned}
 DE &= \sqrt{[-3 - (-9)]^2 + (2 - 6)^2} \\
 &= \sqrt{6^2 + (-4)^2} \\
 &= \sqrt{36 + 16} \\
 &= \sqrt{52}
 \end{aligned}$$

$$\begin{aligned}
 EF &= \sqrt{[-9 - (-1)]^2 + (6 - 6)^2} \\
 &= \sqrt{(-8)^2 + 0^2} \\
 &= \sqrt{64} \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 DF &= \sqrt{[-3 - (-1)]^2 + (2 - 6)^2} \\
 &= \sqrt{(-2)^2 + (-4)^2} \\
 &= \sqrt{4 + 16} \\
 &= \sqrt{20}
 \end{aligned}$$



5. Compare the corresponding side lengths of the pre-image and image. What do you notice?

The side lengths of triangle  $ABC$  are the same as the corresponding side lengths of  $DEF$ .

You have shown that the corresponding sides of the image and pre-image are congruent. Therefore, the triangles are congruent by the SSS Congruence Theorem. However, you are proving the Angle-Angle-Side Congruence Theorem. Therefore, you need to verify that angle  $A$  and angle  $C$  are congruent to the corresponding angles in triangle  $DEF$ .

6. Use a protractor to determine the angle measures of each triangle.
- a. What is the measure of angle  $A$  and angle  $C$ ?

The measure of angle  $A$  is  $83^\circ$ , and the measure of angle  $C$  is  $63^\circ$ .

- b. Which angles in triangle  $DEF$  correspond to angle  $A$  and angle  $C$ ?

Angle  $D$  corresponds to angle  $A$ , and angle  $F$  corresponds to angle  $C$ .



- c. What do you notice about the measures of the corresponding angles in the triangles? What can you conclude from this information?

The measure of angle  $D$  is equal to the measure of angle  $A$ .

The measure of angle  $F$  is equal to the measure of angle  $C$ .

I can conclude that angle  $A$  is congruent to angle  $D$ , and angle  $C$  is congruent to angle  $F$ .



You have used a protractor to verify that the corresponding angles of the two triangles are congruent.

In conclusion, when the measure of two angles and the length of the non-included side of one triangle are equal to the measure of the two corresponding angles and the length of the non-included side of another triangle, the two triangles are congruent by the AAS Congruence Theorem.

## Guiding Questions for Discuss Phase

- What is the difference between an included angle and a non-included angle?
- What is the difference between the AAS and ASA Triangle Congruence Theorems?

## Grouping

Have students complete Problem 3 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase

- What information is given?
- What are you trying to prove?
- Describe the overarching flow of the proof.
- What do the proofs for the SSS, SAS, ASA, and AAS congruence theorems have in common?

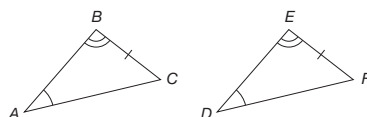
### PROBLEM 3 And Finally the Proof . . .



Prove the Angle-Angle-Side Congruence Theorem.

Given:  $\angle A \cong \angle D$ ,  $\angle B \cong \angle E$ ,  $\overline{BC} \cong \overline{EF}$

Prove:  $\triangle ABC \cong \triangle DEF$



Statements	Reasons
1. $\angle A \cong \angle D$ , $\angle B \cong \angle E$ , $\overline{BC} \cong \overline{EF}$	1. Given
2. $\triangle ABC \sim \triangle DEF$	2. AA Similarity Postulate
3. $\angle A \cong \angle D$ , $\angle B \cong \angle E$ , $\angle C \cong \angle F$	3. Definition of similar triangles
4. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	4. Definition of similar triangles
5. $BC = EF$	5. Definition of congruence
6. $\frac{BC}{EF} = 1$	6. Division Property of Equality
7. $\frac{AB}{DE} = \frac{AC}{DF} = 1$	7. Substitution
8. $AB = DE$ , $AC = DF$	8. Multiplication Property of Equality
9. $\overline{AB} \cong \overline{DE}$ , $\overline{AC} \cong \overline{DF}$	9. Definition of congruence
10. $\triangle ABC \cong \triangle DEF$	10. Definition of congruent triangles



## Problem 4

Students will determine if there is enough information to show two triangles are congruent by ASA or AAS. They will write congruence statements to justify their reasoning.

## Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 4

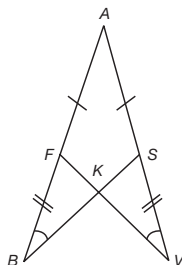
- In Question 1, is side  $AB$  congruent to side  $AV$ ?
- In Question 2, how can ASA or AAS be used if there is only one pair of congruent angle markers in the diagram?
- In Question 2, how can ASA or AAS be used if there are no congruent line segment markers in the diagram?
- In Question 3, is there enough information to determine triangle  $EQD$  is congruent to triangle  $DWE$ ? Why or why not?
- In Question 4, are line segments  $AR$  and  $PC$  sides of triangle  $BAC$  and  $QPR$ ?
- In Question 4, why is side  $AC$  congruent to side  $PR$ ?

### PROBLEM 4 ASA Congruence or AAS Congruence

Determine if there is enough information to prove that the two triangles are congruent by ASA or AAS. Write the congruence statements to justify your reasoning.



1.  $\triangle ABS \stackrel{?}{\cong} \triangle AVF$



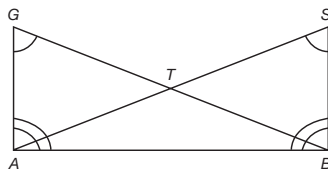
$\triangle ABS \cong \triangle AVF$  by ASA

$\angle B \cong \angle V$

$\angle A \cong \angle A$

$\overline{AB} \cong \overline{AV}$

2.  $\triangle GAB \stackrel{?}{\cong} \triangle SBA$



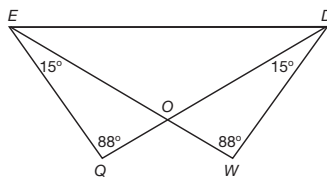
$\triangle GAB \cong \triangle SBA$  by AAS

$\angle G \cong \angle S$

$\angle GAB \cong \angle SBA$

$\overline{AB} \cong \overline{BA}$

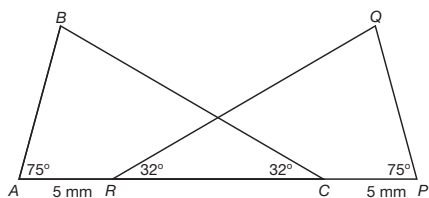
3.  $\triangle EQD \stackrel{?}{\cong} \triangle DWE$



There is not enough information to prove that the triangles are congruent by ASA or AAS.



4.  $\triangle ABC \stackrel{?}{\cong} \triangle PQR$



$$\angle A \cong \angle P$$

$$\angle BCA \cong \angle QRP$$

$$\overline{AC} \cong \overline{PR}$$

The triangles are congruent by ASA.

## Talk the Talk

This chapter focused on four theorems that can be used to prove that two triangles are congruent. Students will complete a graphic organizer in which they illustrate each theorem.

## Grouping

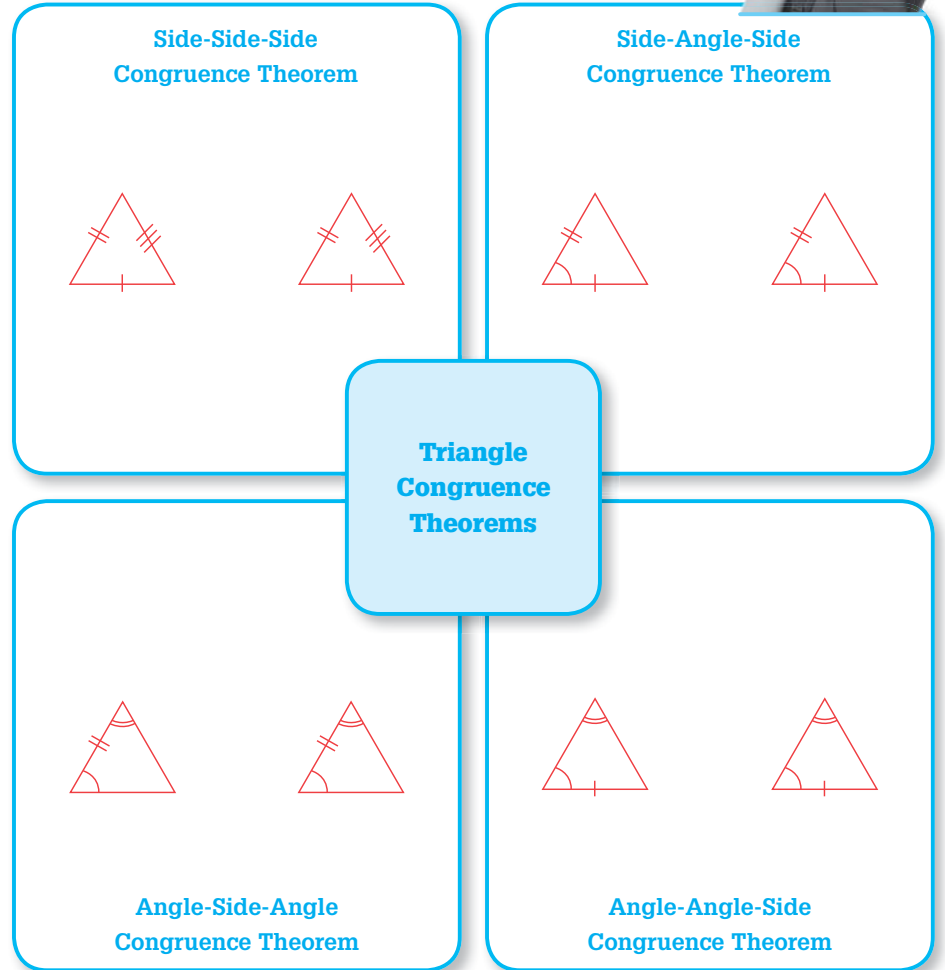
Ask students to read instructions and complete the graphic organizer. Discuss as a class.

### Talk the Talk



This chapter focused on four methods that you can use to prove that two triangles are congruent. Complete the graphic organizer by providing an illustration of each theorem.

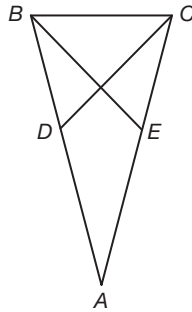
Use markers to show congruent sides and congruent angles.



Be prepared to share your solutions and methods.

## Check for Students' Understanding

1. Suppose  $\overline{AB} \cong \overline{AC}$  in the diagram shown.



- a. What additional information is needed to conclude  $\triangle ABE \cong \triangle ACD$  using the ASA Congruence Theorem?

Answers will vary.

I would need to know that  $\angle ABE \cong \angle ACD$  to conclude  $\triangle ABE \cong \triangle ACD$  using the ASA Congruence Theorem.

- b. What additional information is needed to conclude  $\triangle ABE \cong \triangle ACD$  using the AAS Congruence Theorem?

Answers will vary.

I would need to know that  $\angle AEB \cong \angle ADC$  to conclude  $\triangle ABE \cong \triangle ACD$  using the AAS Congruence Theorem.

- c. What additional information is needed to conclude  $\triangle ABE \cong \triangle ACD$  using the SAS Congruence Theorem?

Answers will vary.

I would need to know that  $\overline{AE} \cong \overline{AD}$  to conclude  $\triangle ABE \cong \triangle ACD$  using the SAS Congruence Theorem.

- d. What additional information is needed to conclude  $\triangle ABE \cong \triangle ACD$  using the SSS Congruence Theorem?

Answers will vary.

I would need to know that  $\overline{AE} \cong \overline{AD}$  and  $\overline{BE} \cong \overline{CD}$  to conclude  $\triangle ABE \cong \triangle ACD$  using the SSS Congruence Theorem.

# Any Other Theorems You Forgot to Mention?

## Using Congruent Triangles

### LEARNING GOALS

In this lesson, you will:

- Prove that the points on a perpendicular bisector of a line segment are equidistant to the endpoints of the line segment.
- Show that AAA for congruent triangles does not work.
- Show that SSA for congruent triangles does not work.
- Use the congruence theorems to determine triangle congruency.

### ESSENTIAL IDEAS

- The points on the perpendicular bisector of a line segment are equidistant to the endpoints of the line segment.
- AAA and SSA are not triangle congruence theorems.
- Triangle congruency can be determined by using the SSS, SAS, ASA, or AAS theorem.
- In order to determine triangle congruence using SSS, SAS, ASA, or AAS, the specific criteria of the selected congruence theorem must be satisfied.

of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

### COMMON CORE STATE STANDARDS FOR MATHEMATICS

#### G-CO Congruence

##### Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition

##### Prove geometric theorems

9. Prove theorems about lines and angles.

##### Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

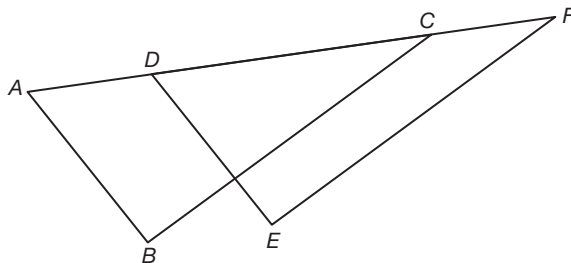
## Overview

Students use triangle congruence theorems to prove points on a perpendicular bisector of a line segment are equidistant to the endpoints of the segment. Students explore why AAA and ASS are not valid reasons for proving triangles congruent. Next, students determine which sets of given information results in the congruency of two triangles and state the appropriate congruency theorem. In the last activity, students analyze a diagram containing multiple sets of triangles writing appropriate triangle congruency statements, and identifying characteristics such as common sides, common angles, and vertical angles.



## Warm Up

1. Suppose  $\overline{BC} \cong \overline{EF}$  and  $\overline{BC} \parallel \overline{EF}$  in the diagram shown.



- a. What other information is needed to conclude  $\triangle ABC \cong \triangle DEF$  using the SAS Congruence Theorem?

I would need to know that  $\overline{AC} \cong \overline{DF}$  to conclude  $\triangle ABC \cong \triangle DEF$  using the SAS Congruence Theorem.

- b. What other information is needed to conclude  $\triangle ABC \cong \triangle DEF$  using the ASA Congruence Theorem?

I would need to know that  $\angle B \cong \angle E$  to conclude  $\triangle ABC \cong \triangle DEF$  using the ASA Congruence Theorem.

- c. What other information is needed to conclude  $\triangle ABC \cong \triangle DEF$  using the AAS Congruence Theorem?

I would need to know that  $\angle A \cong \angle FDE$  to conclude  $\triangle ABC \cong \triangle DEF$  using the AAS Congruence Theorem.

- d. What other information is needed to conclude  $\triangle ABC \cong \triangle DEF$  using the SSS Congruence Theorem?

I would need to know that  $\overline{AC} \cong \overline{DF}$  and  $\overline{AB} \cong \overline{DE}$  to conclude  $\triangle ABC \cong \triangle DEF$  using the SSS Congruence Theorem.



# Any Other Theorems You Forgot to Mention? Using Congruent Triangles

## LEARNING GOALS

In this lesson, you will:

- Prove that the points on a perpendicular bisector of a line segment are equidistant to the endpoints of the line segment.
- Show that AAA for congruent triangles does not work.
- Show that SSA for congruent triangles does not work.
- Use the congruence theorems to determine triangle congruency.

**N***ame That Tune* was a popular game show that aired from 1974 to 1981. Contestants played against each other based on their knowledge of popular songs. One of the rounds was named Bid-a-Note in which contestants took turns stating, “I can name that tune in X notes,” each time lowering the number of notes. Eventually one of the contestants would declare, “Name that tune!”

The goal was to name the tune in the fewest number of notes. You have been exploring congruent triangles and determining the fewest measurements that are needed to name that triangle. Although, *Name That Triangle* probably wouldn't be as popular a game show as *Name That Tune*!

## Problem 1

Using construction tools, students will construct a perpendicular bisector to a given line segment. Locating points above and below the given line segment on the bisector, students form triangles and prove those triangles congruent, thus concluding points located on the perpendicular bisector of a segment are equidistant from the endpoints of the segment. They also explain how rigid motion could be used to prove this theorem as well.

## Grouping

Have students complete Questions 1 through 10 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 through 10

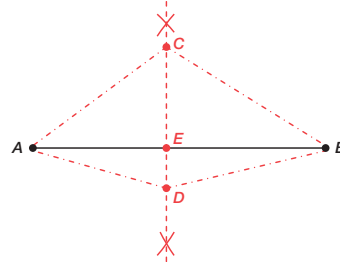
- If segment  $CD$  is the perpendicular bisector of segment  $AB$ , what two relationships exist between the two segments?
- When proving this theorem, what information is considered given?
- When proving this theorem, what information is to be proven?
- When proving  $C$  is equidistant to point  $A$  and  $B$ , what two triangles are used?
- What is the definition of perpendicular lines?

### PROBLEM 1 It's All Equal

Theorem: Points on a perpendicular bisector of a line segment are equidistant to the endpoints of the segment.



1. Construct the perpendicular bisector of line segment  $AB$ .



2. Locate point  $C$  on the perpendicular bisector above line segment  $AB$ .
3. Locate point  $D$  on the perpendicular bisector below line segment  $AB$ .
4. Locate point  $E$  on the perpendicular bisector where it intersects line segment  $AB$ .
5. Connect point  $A$  and point  $C$  to form triangle  $AEC$ .
6. Connect point  $B$  and point  $C$  to form triangle  $BEC$ .
7. Use a triangle congruence theorem to prove point  $C$  is equidistant to points  $A$  and  $B$ . Write each statement and reason.

Statements	Reasons
1. $\overline{AB} \perp \overline{CD}$ , $\overline{CD}$ bisects $\overline{AB}$	1. Definition of perpendicular bisector
2. $\angle AEC$ and $\angle BEC$ are right angles	2. Definition of perpendicular lines
3. $\angle AEC \cong \angle BEC$	3. All right angles are congruent
4. $\overline{AE} \cong \overline{BE}$	4. Definition of bisect
5. $\overline{CE} \cong \overline{CE}$	5. Identity
6. $\triangle AEC \cong \triangle BEC$	6. SAS Congruence Theorem
7. $\overline{AC} \cong \overline{BC}$	7. Corresponding sides of congruent triangles are congruent
8. Point $C$ is equidistant to points $A$ and $B$	8. Definition of equidistant

- What is the definition of segment bisector?
- Are all right angles congruent? Explain.
- What side is shared by both triangles?
- What triangle congruence theorem is helpful when proving the theorem?

8. Use a triangle congruence theorem to prove point  $D$  is equidistant to points  $A$  and  $B$ . Write each statement and reason.

Statements	Reasons
1. $\overline{AB} \perp \overline{CD}$ , $\overline{CD}$ bisects $\overline{AB}$	1. Definition of perpendicular bisector
2. $\angle AED$ and $\angle BED$ are right angles	2. Definition of perpendicular lines
3. $\angle AED \cong \angle BED$	3. All right angles are congruent
4. $\overline{AE} \cong \overline{BE}$	4. Definition of bisect
5. $\overline{DE} \cong \overline{DE}$	5. Identity
6. $\triangle AED \cong \triangle BED$	6. SAS Congruence Theorem
7. $\overline{AD} \cong \overline{BD}$	7. Corresponding sides of congruent triangles are congruent
8. Point $D$ is equidistant to points $A$ and $B$	8. Definition of equidistant

9. How many other points on the perpendicular bisector can be proven equidistant from the endpoints of line segment  $AB$  using the same strategy?

There are an infinite number of points located on the perpendicular bisector and they can all be proven equidistant from the endpoints of the line segment using the same strategy.



10. Explain how rigid motion could be used to prove points on the perpendicular bisector are equidistant from the endpoints of line segment  $AB$ .

A right triangle could be constructed with the right angle at the origin. Then, the triangle could be reflected over the  $x$ -axis or  $y$ -axis to create an image of the triangle that is congruent to the original pre-image.

## Problem 2

Students provide a counterexample to show why Angle-Angle-Angle is not considered a valid congruence theorem.

## Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

## Guiding Questions for Share Phase, Questions 1 and 2

- What is the measure of the interior angles of an equilateral triangle?
- Is it possible to draw one large equilateral triangle and one small equilateral triangle? Are the two triangles always congruent? Explain.
- What is the measure of the interior angles of an equiangular triangle?
- Is it possible to draw one large equiangular triangle and one small equiangular triangle? Are the two triangles always congruent? Explain.
- Is it possible to draw a larger version of any triangle? If so, are the triangles congruent? Are the triangles similar?

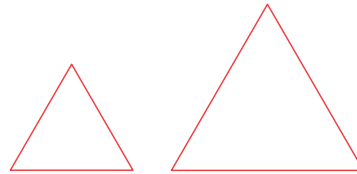
### PROBLEM 2 Help Juno Understand...

Thus far, you have explored and proven each of the triangle congruence theorems:

- Side-Side-Side (SSS)
- Side-Angle-Side (SAS)
- Angle-Side-Angle (ASA)
- Angle-Angle-Side (AAS)



1. Juno wondered why AAA isn't on the list of congruence theorems. Provide a counterexample to show Juno why Angle-Angle-Angle (AAA) **is not** considered a congruence theorem.



The measure of each interior angle of any equilateral triangle is equal to  $60^\circ$ .

Shown are two different equilateral triangles.

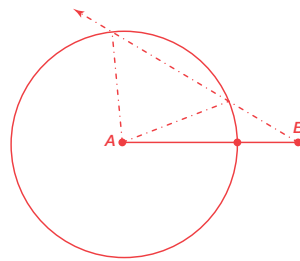
All six angles are equal in measure but the triangles are not congruent because the pairs of corresponding sides are not congruent.

The pairs of corresponding sides are in fact proportional, but not congruent.

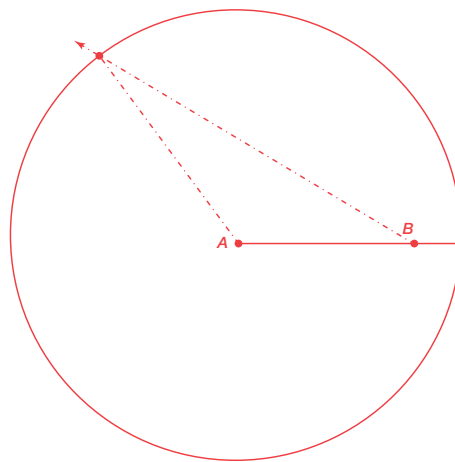
2. Juno also wondered why SSA isn't on the list of congruence theorems.

Provide a counterexample to show Juno why Side-Side-Angle (SSA) *is not* considered a triangle congruence theorem.

CASE 1:



CASE 2:



Consider line segment  $AB$  as the second 'S' in SSA. Point  $A$  is used as the center of a circle. The length of the circle radius is the first 'S' in SSA.

The length of the radius of circle  $A$  is either longer (Case 2) or shorter (Case 1) than line segment  $AB$ . Both cases are drawn.

In Case 2, if a ray is drawn at point  $B$  forming the given angle or the 'A' in SSA, only one triangle is possible when the third vertex of the triangle is located on the circle.

In Case 1, if a ray is drawn at point  $B$  forming the given angle or the 'A' in SSA, there are two possible locations for the third vertex of the triangle, both on circle  $A$ . Both triangles have SSA, but they are not congruent.



We can conclude that if the middle 'S' is equal to or longer than the first 'S', and the middle 'S' is the shorter side, then SSA does not result in two congruent triangles.

### Problem 3

Students determine which sets of given information results in the congruency of two triangles. They are also asked to state the appropriate congruence theorem if the triangles can be proven congruent, or state that there is not enough information if additional givens are needed to determine congruent triangles.

### Grouping

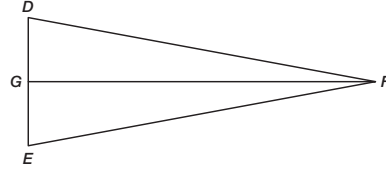
Have students complete the Questions 1 through 6 with a partner. Then have students share their responses as a class.

### Guiding Questions for Share Phase, Questions 1 through 6

- Do the two triangles share a common angle?
- Do the two triangles share a common side?
- Can we assume segment  $FG$  is perpendicular to segment  $DE$ ? Why or why not?
- Can we assume any information?
- What is the definition of midpoint?
- What is the definition of perpendicular segments?
- What is the definition of angle bisector?
- What is an isosceles triangle?
- What is the definition of a right angle?

### PROBLEM 3 Congruent or Not Congruent, That Is the Question!

Determine which given information results in  $\triangle DFG \cong \triangle EFG$ . State the appropriate congruence theorem if the triangles can be proven congruent, or state that there is not enough information if additional givens are needed to determine congruent triangles.



1. Given:  $G$  is the midpoint of  $\overline{DE}$   
**There is not enough information to determine  $\triangle DFG \cong \triangle EFG$ .**

2. Given:  $\overline{DE} \perp \overline{FG}$   
**There is not enough information to determine  $\triangle DFG \cong \triangle EFG$ .**

3. Given:  $\overline{FG}$  bisects  $\angle DFE$ ,  $\overline{FD} \cong \overline{FE}$   
 **$\triangle DFG \cong \triangle EFG$  using the SAS Triangle Congruence Theorem.**

4. Given: Triangle  $DEF$  is isosceles with  $\overline{FD} \cong \overline{FE}$   
**There is not enough information to determine  $\triangle DFG \cong \triangle EFG$ .**

5. Given:  $\overline{FG}$  bisects  $\angle DFE$ ,  $\angle DGF$  is a right angle  
 **$\triangle DFG \cong \triangle EFG$  using the ASA Triangle Congruence Theorem.**



6. Given:  $\angle D \cong \angle E$ ,  $\overline{FG}$  bisects  $\angle DFE$   
 **$\triangle DFG \cong \triangle EFG$  using the AAS Triangle Congruence Theorem.**



## Problem 4

Students determine what additional information is needed to prove that the triangles are congruent, using the specified triangle congruence theorem.

## Grouping

Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

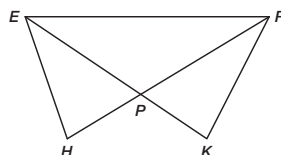
## Guiding Questions for Share Phase, Questions 1 and 2

- Do the triangles have anything in common?
- Do the triangles share a common angle?
- Do the triangles share a common side?
- Is any pair of corresponding angles considered vertical angles?
- Which pair of corresponding sides is considered the included side in each triangle?
- Which pair of corresponding angles is considered the included angle in each triangle?
- What is the difference between the ASA and AAS congruence theorems? Explain.
- Is it possible to prove two triangles congruent using more than one triangle congruence theorem?

## PROBLEM 4 Under What Circumstances?



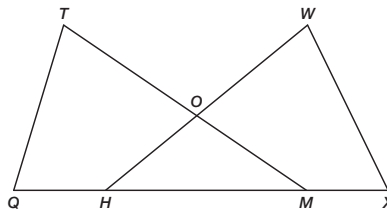
1. Determine what additional information is needed to prove the specified triangles congruent.



- a. Given:  $\angle H \cong \angle K$   
Prove:  $\triangle EPH \cong \triangle RPK$  by the ASA Congruence Triangle Theorem  
 $\overline{HP} \cong \overline{KP}$  is needed to determine triangle  $EPH \cong$  triangle  $RPK$  by the ASA Congruence Triangle Theorem.
- b. Given:  $\overline{HE} \cong \overline{KR}$   
Prove:  $\triangle EHR \cong \triangle RKE$  by the SSS Congruence Triangle Theorem  
 $\overline{HR} \cong \overline{KE}$  is needed to determine triangle  $EPH \cong$  triangle  $RPK$  by the SSS Congruence Triangle Theorem.
- c. Given: Triangle  $EPR$  is isosceles with  $\overline{EP} \cong \overline{RP}$   
Prove:  $\triangle EPH \cong \triangle RPK$  by the SAS Congruence Triangle Theorem  
 $\overline{HP} \cong \overline{KP}$  is needed to determine triangle  $EPH \cong$  triangle  $RPK$  by the SAS Congruence Triangle Theorem.

- Is there a benefit to proving two triangles congruent using more than one triangle congruence theorem?
- What do the four triangle congruence theorems (SSS, SAS, ASA, AAS) have in common?

2. Determine what additional information is needed to prove the specified triangles congruent.



- a. Given:  $\overline{QH} \cong \overline{XM}$ ,  $\overline{TQ} \cong \overline{WX}$

Prove:  $\triangle TQM \cong \triangle WXH$  by the SAS Congruence Triangle Theorem

$\angle Q \cong \angle X$  is needed to determine triangle  $TQM \cong$  triangle  $WXH$  by the SAS Congruence Triangle Theorem.



- b. Given:  $\angle Q \cong \angle X$ ,  $\angle T \cong \angle W$

Prove:  $\triangle TQM \cong \triangle WXH$  by the AAS Congruence Triangle Theorem

$\overline{TM} \cong \overline{WH}$  or  $\overline{QM} \cong \overline{XH}$  is needed to determine triangle  $TQM \cong$  triangle  $WXH$  by the AAS Congruence Triangle Theorem.

## Problem 5

A diagram containing three pairs of congruent triangles is provided. Students write triangle congruency statements, identify the common sides, common angles, vertical angles, and answer questions related to each pair of congruent triangles.

### Grouping

Have students complete Question 1 with a partner. Then have students share their responses as a class.

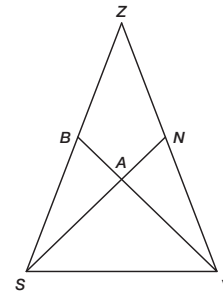
### Guiding Questions for Share Phase, Question 1

- How many different triangles are in this diagram?
- Do the triangles have anything in common?
- Do the triangles share a common angle?
- Do the triangles share a common side?
- Is any pair of corresponding angles considered vertical angles? Explain.
- Which pair of corresponding sides is considered the included side in each triangle?
- Which pair of corresponding angles is considered the included angle in each triangle?
- Which pairs of triangles overlap each other?

## PROBLEM 5 Too Many Triangles



1. Use this diagram to answer each question.



a. Name three possible sets of congruent triangles by writing triangle congruency statements.

$$\triangle BSY \cong \triangle NYS$$

$$\triangle BAS \cong \triangle NAY$$

$$\triangle ZSN \cong \triangle ZYB$$

b. Which set of triangles share a common side?

Triangle *BSY* and triangle *NYS*

c. Which set of triangles share a common angle?

Triangle *ZSN* and triangle *ZYB*

d. Which set of triangles contain a pair of vertical angles?

Triangle *BAS* and triangle *NAY*

- e. Knowing triangle  $ZSN$  is isosceles with  $\overline{ZS} \cong \overline{ZY}$  would be helpful in determining the congruence of which set of triangles?

It would be helpful in determining  $\triangle ZSN \cong \triangle ZYB$

- f. Knowing triangle  $SAY$  is equilateral would be helpful in determining the congruence of which set of triangles?

It would be helpful in determining  $\triangle BAS \cong \triangle NAY$

- g. Knowing  $\angle Z \cong \angle Z$  would be helpful in determining the congruence of which set of triangles?

It would be helpful in determining  $\triangle ZSN \cong \triangle ZYB$

- h. Knowing  $\overline{SY} \cong \overline{SY}$  would be helpful in determining the congruence of which set of triangles?

It would be helpful in determining  $\triangle BSY \cong \triangle NYS$

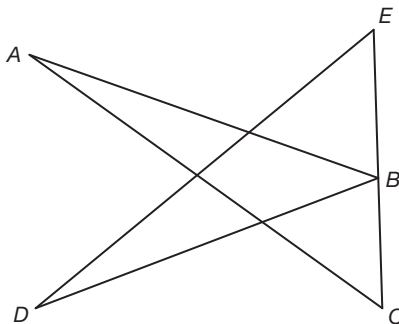


Be prepared to share your solutions and methods.

## Check for Students' Understanding

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Suppose  $\angle ABE \cong \angle DBC$  in the diagram shown.



Is  $\overline{AB} \cong \overline{DB}$  and  $\overline{BE} \cong \overline{BC}$  enough information to conclude  $\triangle ABC \cong \triangle DBE$ ? Explain your reasoning.

Yes. This would be enough information to conclude  $\triangle ABC \cong \triangle DBE$  using the SAS Congruence Theorem. From the given information, I know that two pairs of corresponding sides are congruent. Using the Angle Addition Postulate, I know that the corresponding included angles,  $\angle ABC$  and  $\angle DBE$ , are also congruent.



# Chapter 5 Summary

## POSTULATES AND THEOREMS

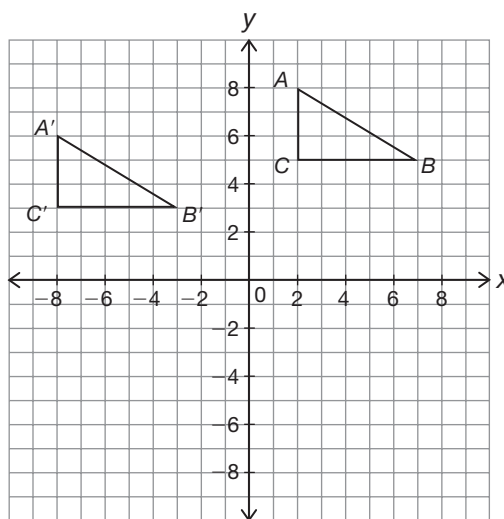
- Side-Side-Side Congruence Theorem (5.3)
- Side-Angle-Side Congruence Theorem (5.4)
- Angle-Side-Angle Congruence Theorem (5.5)
- Angle-Angle-Side Congruence Theorem (5.6)

### 5.1 Translating Triangles on the Coordinate Plane

A translation is a rigid motion that slides each point of a figure the same distance and direction.

#### Example

Triangle  $ABC$  has been translated 10 units to the left and 2 units down to create triangle  $A'B'C'$ .



The coordinates of triangle  $ABC$  are  $A(2, 8)$ ,  $B(7, 5)$ , and  $C(2, 5)$ .

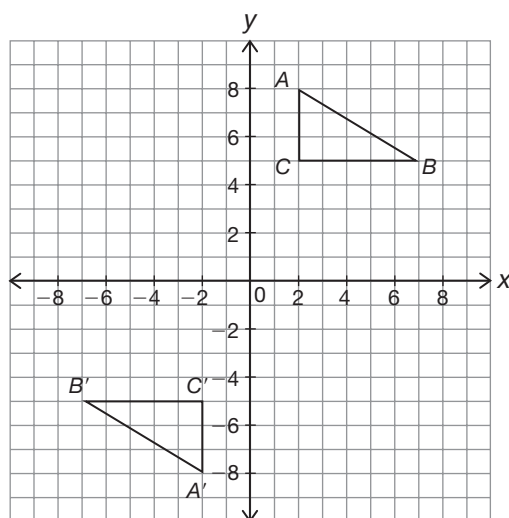
The coordinates of triangle  $A'B'C'$  are  $A'(-8, 6)$ ,  $B'(-3, 3)$ , and  $C'(-8, 3)$ .

## 5.1 Rotating Triangles in the Coordinate Plane

A rotation is a rigid motion that turns a figure about a fixed point, called the point of rotation. The figure is rotated in a given direction for a given angle, called the angle of rotation. The angle of rotation is the measure of the amount the figure is rotated about the point of rotation. The direction of a rotation can either be clockwise or counterclockwise. To determine the new coordinates of a point after a  $90^\circ$  counterclockwise rotation, change the sign of the  $y$ -coordinate of the original point and then switch the  $x$ -coordinate and the  $y$ -coordinate. To determine the new coordinates of a point after a  $180^\circ$  rotation, change the signs of the  $x$ -coordinate and the  $y$ -coordinate of the original point.

### Example

Triangle  $ABC$  has been rotated  $180^\circ$  counterclockwise about the origin to create triangle  $A'B'C'$ .



The coordinates of triangle  $ABC$  are  $A(2, 8)$ ,  $B(7, 5)$ , and  $C(2, 5)$ .

The coordinates of triangle  $A'B'C'$  are  $A'(-2, -8)$ ,  $B'(-7, -5)$ , and  $C'(-2, -5)$ .



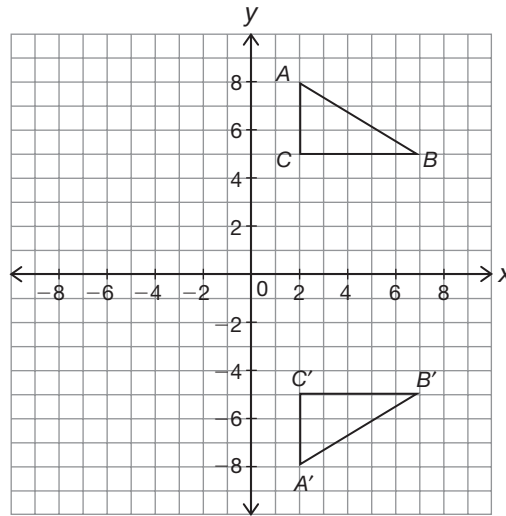
## 5.1

## Reflecting Triangles on a Coordinate Plane

A reflection is a rigid motion that reflects or “flips” a figure over a given line called a line of reflection. Each point in the new triangle will be the same distance from the line of reflection as the corresponding point in the original triangle. To determine the coordinates of a point after a reflection across the  $x$ -axis, change the sign of the  $y$ -coordinate of the original point. The  $x$ -coordinate remains the same. To determine the coordinates of a point after a reflection across the  $y$ -axis, change the sign of the  $x$ -coordinate of the original point. The  $y$ -coordinate remains the same.

**Example**

Triangle  $ABC$  has been reflected across the  $x$ -axis to create triangle  $A'B'C'$ .



The coordinates of triangle  $ABC$  are  $A(2, 8)$ ,  $B(7, 5)$ , and  $C(2, 5)$ .

The coordinates of triangle  $A'B'C'$  are  $A'(2, -8)$ ,  $B'(7, -5)$ , and  $C'(2, -5)$ .

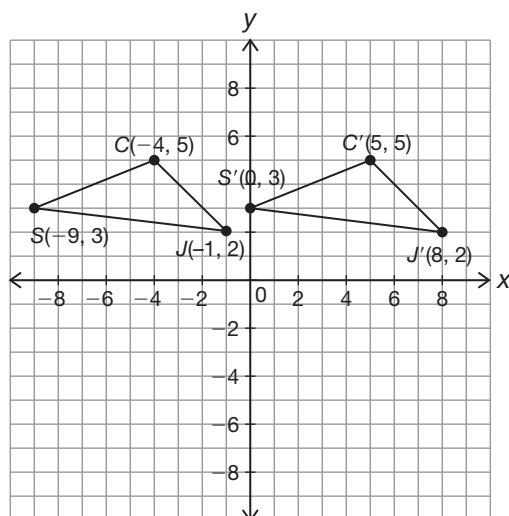
## 5.3

## Using the SSS Congruence Theorem to Identify Congruent Triangles

The Side-Side-Side (SSS) Congruence Theorem states that if three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.

**Example**

Use the SSS Congruence theorem to prove  $\triangle CJS$  is congruent to  $\triangle C'J'S'$ .



$$\begin{aligned} SC &= \sqrt{[-4 - (-9)]^2 + (5 - 3)^2} \\ &= \sqrt{5^2 + 2^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

$$\begin{aligned} S'C' &= \sqrt{(5 - 0)^2 + (5 - 3)^2} \\ &= \sqrt{5^2 + 2^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

$$\begin{aligned} CJ &= \sqrt{[-1 - (-4)]^2 + (2 - 5)^2} \\ &= \sqrt{3^2 + (-3)^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \end{aligned}$$

$$\begin{aligned} C'J' &= \sqrt{(8 - 5)^2 + (2 - 5)^2} \\ &= \sqrt{3^2 + (-3)^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \end{aligned}$$

$$\begin{aligned} SJ &= \sqrt{[-1 - (-9)]^2 + (2 - 3)^2} \\ &= \sqrt{8^2 + (-1)^2} \\ &= \sqrt{64 + 1} \\ &= \sqrt{65} \end{aligned}$$

$$\begin{aligned} S'J' &= \sqrt{(8 - 0)^2 + (2 - 3)^2} \\ &= \sqrt{8^2 + (-1)^2} \\ &= \sqrt{64 + 1} \\ &= \sqrt{65} \end{aligned}$$

The lengths of the corresponding sides of the pre-image and the image are equal, so the corresponding sides of the image and the pre-image are congruent. Therefore, the triangles are congruent by the SSS Congruence Theorem.

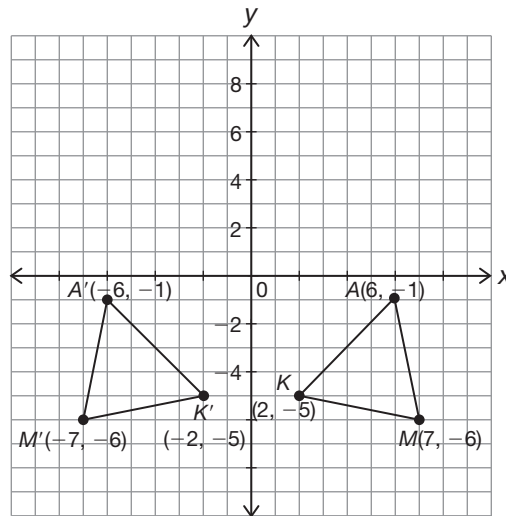
## 5.4

## Using the SAS Congruence Theorem to Identify Congruent Triangles

The Side-Angle-Side (SAS) Congruence Theorem states that if two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of a second triangle, then the triangles are congruent. An included angle is the angle formed by two sides of a triangle.

**Example**

Use the SAS Congruence Theorem to prove that  $\triangle AMK$  is congruent to  $\triangle A'M'K'$ .



$$\begin{aligned} KA &= \sqrt{(6-2)^2 + [-1-(-5)]^2} \\ &= \sqrt{4^2 + 4^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \end{aligned}$$

$$\begin{aligned} K'A' &= \sqrt{[-5-(-1)]^2 + [-2-(-6)]^2} \\ &= \sqrt{(-4)^2 + 4^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \end{aligned}$$

$$\begin{aligned} KM &= \sqrt{(7-2)^2 + [-6-(-5)]^2} \\ &= \sqrt{5^2 + (-1)^2} \\ &= \sqrt{25 + 1} \\ &= \sqrt{26} \end{aligned}$$

$$\begin{aligned} K'M' &= \sqrt{[-7-(-2)]^2 + [-6-(-5)]^2} \\ &= \sqrt{(-5)^2 + (-1)^2} \\ &= \sqrt{25 + 1} \\ &= \sqrt{26} \end{aligned}$$

$$m\angle K = 58^\circ$$

$$m\angle K' = 58^\circ$$

The lengths of the pairs of the corresponding sides and the measures of the pair of corresponding included angles are equal. Therefore, the triangles are congruent by the SAS Congruence Theorem.

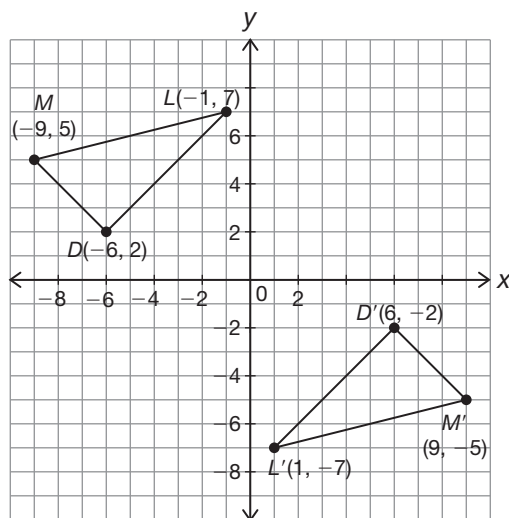
## 5.5

## Using the ASA Congruence Theorem to Identify Congruent Triangles

The Angle-Side-Angle (ASA) Congruence Theorem states that if two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the triangles are congruent. An included side is the line segment between two angles of a triangle.

**Example**

Use the ASA Congruence Theorem to prove that  $\triangle DLM$  is congruent to  $\triangle D'L'M'$ .



$$\begin{aligned} DM &= \sqrt{[-9 - (-6)]^2 + (5 - 2)^2} \\ &= \sqrt{(-3)^2 + 3^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \end{aligned}$$

$$m\angle D = 90^\circ$$

$$m\angle M = 60^\circ$$

$$\begin{aligned} D'M' &= \sqrt{(9 - 6)^2 + [-5 - (-2)]^2} \\ &= \sqrt{3^2 + (-3)^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \end{aligned}$$

$$m\angle D' = 90^\circ$$

$$m\angle M' = 60^\circ$$

The measures of the pairs of corresponding angles and the lengths of the corresponding included sides are equal. Therefore, the triangles are congruent by the ASA Congruence Theorem.

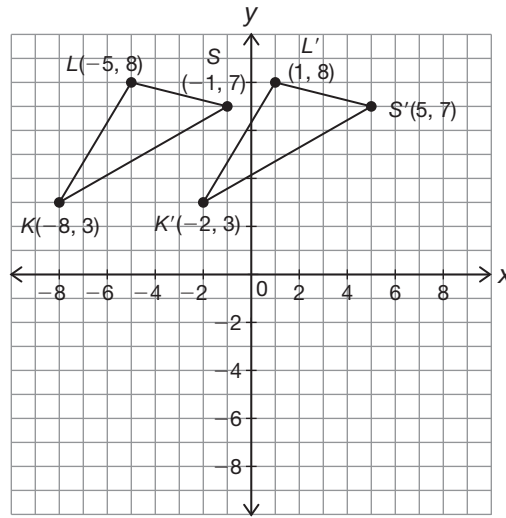
## 5.6

## Using the AAS Congruence Theorem to Identify Congruent Triangles

The Angle-Angle-Side (AAS) Congruence Theorem states that if two angles and a non-included side of one triangle are congruent to the corresponding two angles and the corresponding non-included side of a second triangle, then the triangles are congruent.

**Example**

Use the AAS Congruence Theorem to prove  $\triangle LSK$  is congruent to  $\triangle L'S'K'$ .



$$m\angle L = 108^\circ$$

$$m\angle L' = 108^\circ$$

$$m\angle K = 30^\circ$$

$$m\angle K' = 30^\circ$$

$$\begin{aligned} KS &= \sqrt{[-1 - (-8)]^2 + (7 - 3)^2} \\ &= \sqrt{7^2 + 4^2} \\ &= \sqrt{49 + 16} \\ &= \sqrt{65} \end{aligned}$$

$$\begin{aligned} K'S' &= \sqrt{[5 - (-2)]^2 + (7 - 3)^2} \\ &= \sqrt{7^2 + 4^2} \\ &= \sqrt{49 + 16} \\ &= \sqrt{65} \end{aligned}$$

The measures of the two pairs of corresponding angles and the lengths of the pair of corresponding non-included sides are equal. Therefore, the triangles are congruent by the AAS Congruence Theorem.

