Polynomial Models

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Chapter 7 Overview

This chapter provides opportunities for students to solve polynomial inequalities algebraically and graphically. Lessons present various problem situations and ask students to use a graphing calculator to determine the polynomial regression function that best models the data. Students then use their regression functions to answer questions. Piecewise functions are introduced for situations where a single polynomial function is not the most appropriate model for a set of data. At the end of the chapter, the lesson provides opportunities for students to compare properties of two functions each represented in a different way. Questions present functions that are represented using a graph, table of values, equation, or description of its key characteristics.

Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
7.1	Solving Polynomial Inequalities	A.CED.1 A.CED.3	1	This lesson presents a real-world situation represented by a fourth degree function for students to explore intervals where the function is greater than, less than, or equal to zero. Questions ask students to analyze various methods to solve polynomial inequalities algebraically and graphically. Students will then write and solve polynomial inequalities for problem situations.	x	x	x		x
7.2	Modeling with Polynomials	A.CED.3 F.IF.4 F.IF.5 F.BF.1 S.ID.6.a	1	This lesson provides opportunities for students to determine the appropriate regression equation to model various problem situations. Questions ask students to use regression equations to make predictions and sketch functions that appropriately model a problem situation.	x				x

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Lesson		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
7.3	Piecewise Functions	A.CED.1 A.CED.3 F.IF.7.b S.ID.6.a	1	This lesson provides opportunities for students to understand that sometimes a single polynomial function is not the most appropriate model for a set of data. Questions ask students to explore intervals of a data set and consider different functions to model corresponding intervals of the domain of the function. Questions then ask students to use a graphing calculator to write piecewise functions to model real-world problems.	x	x		x	x
7.4	Modeling Polynomial Data	A.CED.2 A.CED.3 A.REI.11 F.LE.3 S.ID.6.a	1	This lesson provides opportunities for students to analyze data and use a graphing calculator to determine the regression function that best models the data. Students will use their regression functions to answer questions.	x		x		x
7.5	Comparing Polynomials in Different Representations	F.IF.9	1	This lesson provides opportunities for students to compare properties of two functions each represented in a different way. Questions present functions that are represented using a graph, table of values, equation, or description of its key characteristics.		x	x		

Skills Practice Correlation for Chapter 7

Lesson		Problem Set	Objectives
7.1	Solving Polynomial Inequalities	1 – 6	Analyze graphs and identify sets of *x*-values to represent polynomial inequalities
		7 – 12	Use a graphing calculator to solve polynomial inequalities
		13 – 18	Solve polynomial inequalities by factoring and sketching
7.2	Modeling with Polynomials		Vocabulary
		1 – 6	Create scatter plots and predict the type of polynomial that best fits the data
		7 – 12	Use a graphing calculator to determine regression equations and how well they model the data
		13 – 20	Use data and regression equations to make predictions for problem situations
7.3	Piecewise Functions		Vocabulary
		1 – 6	Sketch piecewise functions on the coordinate plane
		7 – 12	Write equations of piecewise functions given graphs
		13 – 18	Analyze scatter plots and determine regression equations for given intervals to write piecewise functions
7.4	Modeling Polynomial Data	1 – 6	Create scatter plots for data sets
		7 – 12	Analyze data sets and scatter plots to describe polynomial functions that model the data
		13 – 18	Use a graphing calculator to determine regression equations that model data
		19 – 24	Use regression equations to answer questions
7.5	Comparing Polynomials in Different Representations	1 – 12	Analyze pairs of representations and answer questions

7.1

Unequal Equals

Solving Polynomial Inequalities

LEARNING GOALS

In this lesson, you will:

- Determine all roots of polynomial equations.
- Determine solutions to polynomial inequalities algebraically and graphically.

ESSENTIAL IDEAS

- Solving polynomial inequalities is similar to solving linear inequalities.
- The solutions to a polynomial inequality are intervals of *x*-values that satisfy the inequality.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

A.CED Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Overview

Solving polynomial inequalities is very much like solving linear inequalities. Students will solve polynomial inequalities both graphically and algebraically. Problem situations include profit models, vertical motion, and glucose levels in the bloodstream. A graphing calculator is used in this lesson.

Warm Up

Given: $x^4 - 13x^2 + 36 = 0$

1. Factor the polynomial equation.

$x^4 - 13x^2 + 36 = 0$

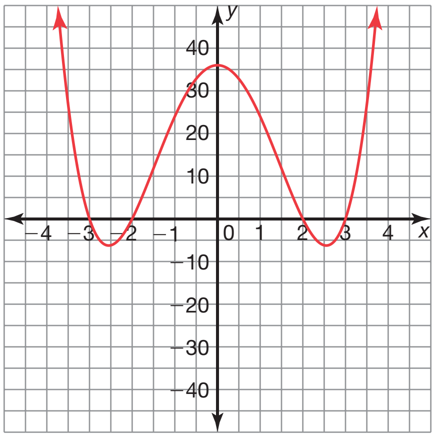
$(x^2 - 4)(x^2 - 9) = 0$

$(x - 2)(x + 2)(x - 3)(x + 3) = 0$

2. Write the roots of the polynomial equation.

$x = 2, -2, 3, -3$

3. Graph the polynomial equation.



7.1

Unequal Equals

Solving Polynomial Inequalities

LEARNING GOALS

In this lesson, you will:

- Determine all roots of polynomial equations.
- Determine solutions to polynomial inequalities algebraically and graphically.

Income Inequality is a term used to describe the gap or difference between the amount of money that wealthy people possess as compared to the amount people without wealth possess. From the 1950s through the 1970s, the trend in the United States was toward *more* income equality. In other words, non-wealthy people earned money at a faster rate than the wealthiest segment of the population, creating a smaller gap between these two social classes. Many economists attribute this trend towards equality to the industrial boom leading up to and following World War II. Millions of soldiers returning from active war duty after World War II received low interest loans for housing, and money for college and career-training. This helped non-wealthy people earn a greater share of the country's wealth. In the 1970s the wealthiest 1% of the population owned approximately 9% of America's total wealth.

Since the 1970s, the United States has become a nation with much more income inequality. Wages in the middle and lower classes have remained fairly stagnant while the wealth of the top 1% has increased from 9% in the 1970s to nearly 25% today.

Why do you think the income inequality changed after the 1970s? Do you think this trend will continue for the foreseeable future? What factors play a part in determining wealthy and non-wealthy classes?

Problem 1

Students are given profit model for a business which is represented by a fourth degree function. The graph of the function is also provided. They will analyze the graph of the function over a period of time by answering questions related to increasing and decreasing intervals.

Grouping

Ask a student to read the information and worked example. Discuss as a class.

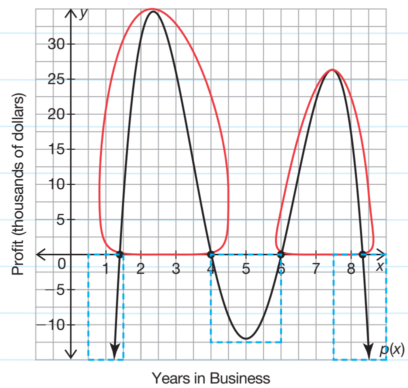
PROBLEM 1 Analyzing Profits

Lawn Enforcement is a small landscaping company. It has a profit model that can be represented by the function,

$$p(x) = -x^4 + 19.75x^3 - 133.25x^2 + 351.25x - 280.75$$

where profit, in thousands of dollars, is a function of time, in years, the company has been in business. Let's analyze $p(x)$ represented on a graph.

The graph shown represents the change in profit as a function of the number of years that Lawn Enforcement has been in business.



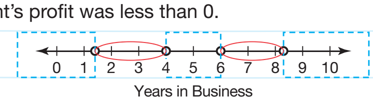
The points identified on the graph represent the zeros of the function where Lawn Enforcement's profit was 0.

Each point on the number line represents the years in business when Lawn Enforcement's profit was 0.

The function $p(x) = 0$ when $x = 1.4, 4, 6, 8.3$.

The regions enclosed in dashed boxes on the coordinate plane represent Lawn Enforcement's profit less than 0.

The regions on the number line enclosed in dashed boxes represent the years in business when Lawn Enforcement's profit was less than 0.



The function $p(x) < 0$ when $\left\{\begin{array}{r} x < 1.4 \\ 4 < x < 6 \\ x > 8.3 \end{array}\right\}$.

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Grouping

Have students complete Question 1 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- What do the x-intercepts on the graph of the function represent with respect to this problem situation?
- Is the profit greater than or less than zero at each x-intercept?
- What is the scale on the y-axis?
- Does the graph of the function reach a $35,000 profit? When?
- Does the graph of the function go over a $35,000 profit?

1. Analyze the worked example.

a. Why were the points changed to open circles on the number line to represent the years in business when $p(x) < 0$.

The points were changed to open circles on the number line because the set represents all the time in years when the profit was less than 0, not less than or equal to 0.

b. Circle the parts of the graph on the coordinate plane that represent where $p(x) > 0$. Then circle the intervals on the number line that represent the years in business where $p(x) > 0$. Finally identify the set of x-values to complete the sentence and explain your answer in terms of this problem situation.

See worked example.

The circled regions on the number line represent the years in business when Lawn Enforcement's profit was greater than 0.

The function $p(x) > 0$ when $\left\{\begin{matrix} 1.4 < x < 4 \\ 6 < x < 8.3 \end{matrix}\right\}$.

c. Draw a solid box around the segment(s) where $p(x) > 35{,}000$. Then identify the set of x-values to complete the sentence. Finally, explain your answer in terms of this problem situation.

The function $p(x)$ is never greater than 35,000.

Lawn Enforcement never reached a profit of $35,000.

The function $p(x) > 35{,}000$ when there is no solution.

Problem 2

Solving polynomial inequalities is similar to solving linear inequalities. Students will analyze three different methods of solving the same polynomial inequality. Using the method of their choice, they then solve a polynomial inequality.

Grouping

- Ask a student to read the introduction. Discuss as a class.
- Have students complete Questions 1 and 2 with a partner. Then have students share their responses as a class.

PROBLEM 2 Analyzing Methods for Solving Polynomial Inequalities

In this lesson, you will solve polynomial inequalities, which are very similar to solving linear inequalities. Recall from your experience of solving linear inequalities graphically, that $<$ or $>$ is represented with a dotted line, and $\leq$ or $\geq$ is represented with a solid line. Also remember that when you are determining which region(s) to shade, look at y-values above or below the boundary line depending on the inequality sign. It is always a good idea to check your work by selecting test points as well.

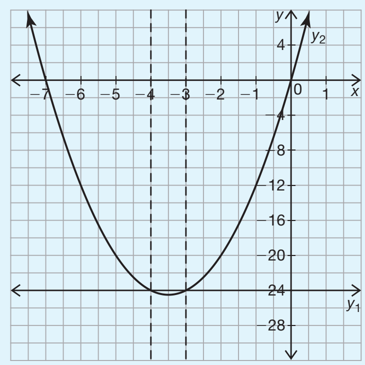
1. Samson, Kaley, Paco, and Sal each solved the quadratic inequality $-24 > 2x^2 + 14x$.

Samson

I graphed both sides of the inequality.

$y_1 = -24$

$y_2 = 2x^2 + 14x$



I drew vertical dashed lines at the two points where the graphs intersect.

I can then determine from the graph that the x-values of $2x^2 + 14x$ that generate values less than -24 are between -4 and -3.

Therefore the solution to the inequality is $-4 < x < -3$.

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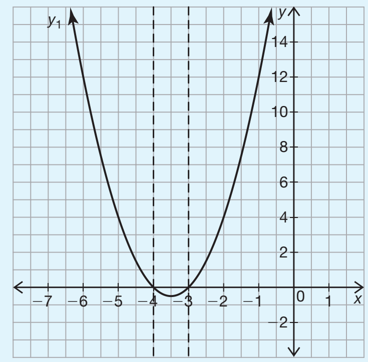
Paco

I added 24 to both sides of the inequality because I wanted one side to be equal to 0. Then, I graphed that inequality.

$y_1 = 2x^2 + 14x + 24$



I drew vertical dashed lines where the graph crosses the x-axis.

I can then determine from the graph that the x-values of $2x^2 + 14x$ that generate values less than 0 are between −4 and −3.

Therefore the solution to the inequality is $-4 < x < -3$.

a. Explain why the graphs of Samson and Paco are different, yet generate the same answers.

Samson graphed both sides of his equation as it was given in the problem. Then he analyzed the intersection points of the two graphs to correctly determine the solution set of the original inequality.

Paco rewrote the inequality to equal zero and graphed the equation. Then he analyzed the zeros of the graph to correctly determine the solution set of the original inequality.

Guiding Questions for Share Phase, Question 1, part (a)

- Who solved the problem by graphing both sides of the polynomial inequality?
- Who solved the problem by first moving all of the terms to one side of the polynomial inequality?
- Did both Samson and Paco graph both sides of the polynomial inequality?
- Did both Samson and Paco graph equivalent forms of the polynomial inequality?

Guiding Questions for Share Phase, Question 1, part (b)

- Who solved the problem by factoring?
- What is the Zero Product Property?
- How did Sal use the Zero Product Property incorrectly?

b. Explain the error in Sal's work.

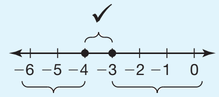
Kaley

I remember from solving linear inequalities that I can first treat the inequality as an equation and solve:

$0 = 2x^2 + 14x + 24$
$0 = 2(x^2 + 7x + 12)$
$0 = 2(x + 3)(x + 4)$
$x = -4, -3$

This means that the x-intercepts are -4 and -3. Breaking up the number line into 3 parts and testing each section in the original inequality $-24 > 2x^2 + 14x$, I can determine the solution:

Test $x = -3.5$
$-24 > 2(-3.5)^2 + 14(-3.5)$
$-24 > -24.5$
✓



Test $x = -5$
$-24 > 2(-5)^2 + 14(-5)$
$-24 > -20$
✗

Test $x = 0$
$-24 > 2(0)^2 + 14(0)$
$-24 > 0$
✗

The only section that satisfies the original inequality is when x is between -4 and -3 so the solution to the inequality is $-4 < x < -3$.

Sal

I remember from solving linear inequalities that I can treat the inequality as an equation and solve:

$2x^2 + 14x = -24$
$2x(x + 7) = -24$
$2x = -24 \quad (x + 7) = -24$
$x = -12 \quad x = -31$

This means that the x-intercepts are -12 and -31, so the solution to the inequality is $-31 < x < -12$.

Sal used the Zero Product Property incorrectly; if $ab = 0$, then $a = 0$ or $b = 0$. He would first need to add 24 to both sides before factoring like Kaley.

Guiding Questions for Share Phase, Question 1, part (c)

- Whose method requires the use of a graphing calculator?
- Whose method requires testing points?

Guiding Questions for Share Phase, Question 2

- Which method did you use to solve the polynomial inequality?
- Did it require the use of a graphing calculator?
- How many different methods could be used to solve this polynomial inequality?
- Which method is the most efficient? Why?

c. Compare Samson's method to Kaley's method. List advantages and disadvantages of each method.

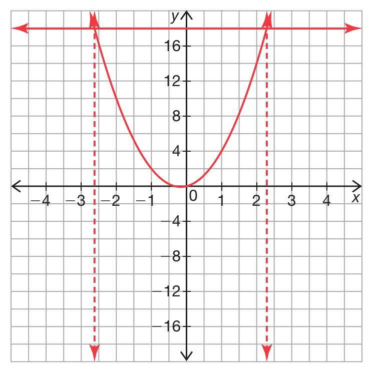
Answers will vary.

Student responses could include:

- Samson's method seems quicker, but requires a graphing calculator.
- Kaley's method seems like it would take longer because it requires testing points especially if the function is a higher degree or not easily factorable, but it does not require a calculator.

2. Solve $18 \le 3x^2 + x$ using any method. Explain why you chose the method.

Answers will vary.



I determined the intersection points of the two graphs and drew vertical dashed lines.

The solution to the inequality is $x \le -2.62$ or $x \ge 2.29$ because that is where $3x^2 + x$ is greater than or equal to 18.

Problem 3

Polynomial inequalities are used to model everyday situations. Students will write and solve each real-world inequality. The vertical motion function is provided and used in the first situation. The second situation uses a profit inequality and in the third situation, a polynomial function that represents the glucose levels of one individual over the span of 72 hours is given. Students will use a graphing a calculator to solve two inequalities. The last set of problems involves solving inequalities by factoring and sketching.

Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1 through Question 2, part (b)

- Using the formula for initial velocity, what value is V_0 with respect to this problem situation?
- Using the formula for initial velocity, what value is h_0 with respect to this problem situation?
- Using the formula for initial velocity, what value is $h(t)$ with respect to this problem situation?

PROBLEM 3 Real Life Kids

Polynomial inequalities can be used to represent everyday situations. Write and solve each real-world inequality.

1. Get Your Kicks is an indoor soccer complex. The roof's height at the facility is 80 feet. If a soccer ball is kicked and touches the ceiling during a game, the team that kicked the ball must have a player sit out for two minutes. Michael kicks a ball straight up in the air with an initial velocity of 73 feet per second.

   a. Write an inequality to represent this problem situation.

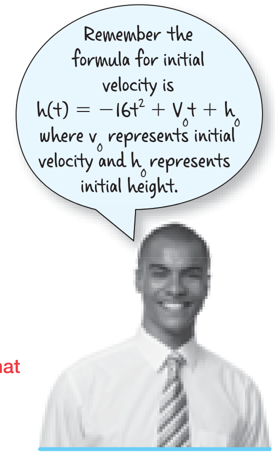
   Let t represent the time in seconds.

   $-16t^2 + 73t < 80$

   b. Use your inequality to determine whether Michael's team will be penalized for hitting the ceiling. Explain your reasoning.

   The ball Michael kicked hit the ceiling after 1.83 seconds, so the team would be penalized.

   I graphed $y_1 = -16t^2 + 73t$ and $y_2 = 80$. Using the intersection function of the calculator I determined that $-16t^2 + 73t < 80$ when $x < 1.83$ and $x \geq 2.73$. This means that the ball did hit the roof at 1.83 seconds.



2. Glen High School's student council is hosting a dance to raise money for panda bears. The dance will cost \$2250. At the current ticket price of \$10, the council knows that they will have 185 people attend the dance. This is not enough people to cover the cost of the dance, so they estimate that for every \$0.25 decrease in ticket price, 15 more people will attend the dance.

   a. Write an equation that will represent the profit that the dance will make.

   Let x = the change in ticket price per person

   $P(x)$ = (number of people)(cost of ticket) − (cost of dance)

   $P(x) = (185 + 15x)(10 - 0.25x) - 2250$

   or

   $P(x) = -3.75x^2 + 103.75x - 400$

   b. Write an inequality to represent the dance making a profit.

   $(185 + 15x)(10 - 0.25x) > 2250$

- Is $-16t^2 + 73t < 80$, or is $-16t^2 + 73t > 80$? How do you know?
- When did Michael's ball hit the ceiling? How do you know?
- Does the variable in your inequality represent the change in ticket price per person?
- How did you set up your profit equation for this situation?
- Will the (number of people) times the (cost of the ticket) minus the (cost of the dance) equal the profit?
- How did you set up your profit inequality for this situation?

Guiding Questions for Share Phase, Question 2, parts (c) and (d)

- What is the difference between the profit equation and the profit inequality?
- To determine the maximum price the council can charge for tickets and still make a profit, did you use the profit equation or the profit inequality?
- What does the ticket price have to be to break even?
- Why does the highest cost of the ticket have to be less than $8.84?
- How did you use the graphing calculator to determine the price of the ticket that will maximize profit? What is the value of *x*?

Guiding Questions for Share Phase, Questions 3 and 4

- Did you use the same method to solve the inequalities in part (a) and part (b)? Did you graph both sides of the inequality?
- How did you determine the hours the glucose levels were greater than 120 mb/dL?
- How did you determine the hours the glucose levels were less than 120 mb/dL?

c. Determine the maximum price the council can charge for tickets and still make a profit.

$(185 + 15x)(10 - 0.25x) - 2250 > 0$

$-3.75x^2 + 103.75x - 400 > 0$

$-3.75x^2 + 103.75x - 400 = 0$

$$x = \frac{-103.75 \pm \sqrt{103.75^2 - 4(-3.75)(-400)}}{2(-3.75)}$$

$$\approx \frac{-103.75 \pm \sqrt{4764.06}}{-7.5} \approx \frac{-103.75 \pm 69}{-7.5}$$

$x \approx 4.63, 23.04$

Therefore, the price of the ticket will be [10 − 0.25(4.63)] = $8.84 to break even. So to make a profit the cost of a ticket should not exceed $8.84.

d. Determine the price of the ticket that will maximize profit. What is the maximum profit?

Using a graphing calculator, $317.60 is the maximum profit if $x \approx 13.83$. Therefore, the cost of the ticket to maximize profit would be [10 − 0.25(13.83)] ≈ $6.54.

3. Use a graphing calculator to solve each inequality.

a. $-5 \geq x^3 - 9x$

I graphed $y_1 = x^3 - 9x$ and $y_2 = -5$.

Using the intersection function of the calculator, I determined that $x^3 - 9x \leq -5$ when $x \leq -3.25$ or $0.58 \leq x \leq 2.67$.

b. $0 < 2x^3 - 3x^2 - 3x + 2$

I graphed $y_1 = 2x^3 - 3x^2 - 3x + 2$.

Using the zero function of the calculator, I determined that $0 < 2x^3 - 3x^2 - 3x + 2$ when $-1 < x < \frac{1}{2}$ or $x > 2$.

4. The average blood sugar (also known as glucose) levels in a person's blood should be between 70 and 100 mg/dL (milligrams per deciliter) one hour after eating. A person with Type 2 diabetes strives to keep glucose levels under 120 mg/dL with diet and exercise in order to avoid insulin injections. Glucose levels of one individual over the span of 72 hours can be represented with the polynomial function,

$$b(t) = 0.000139x^4 - 0.0188x^3 + 0.8379x^2 - 13.55x + 176.51$$

where glucose levels is a function of the number of hours.

a. For what hours were the glucose levels greater than 120 mg/dL?

$23.9 < t < 45.2$

$t > 59.9$

$t < 6.3$

b. For what hours were the glucose levels less than 120 mg/dL?

$6.3 < t < 23.9$

$45.2 < t < 59.9$

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Grouping

Have students complete Question 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 5

- What degree is the polynomial inequality?
- The Fundamental Theorem of Algebra tells you this polynomial has how many zeros?
- Can the terms be grouped together before factoring them?
- Do all of the terms have something in common?
- Can the quadratic formula be used in this situation?
- What is the end behavior of the function?

5. Solve each inequality by factoring and sketching. Use the coordinate plane to sketch the general graph of the polynomial in order to determine which values satisfy the inequality.

a. $2x^3 - 8x^2 - 8x + 32 > 0$

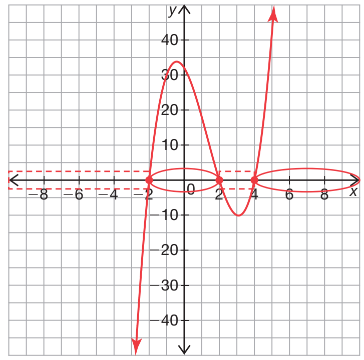
$2x^2(x - 4) - 8(x - 4) = 0$

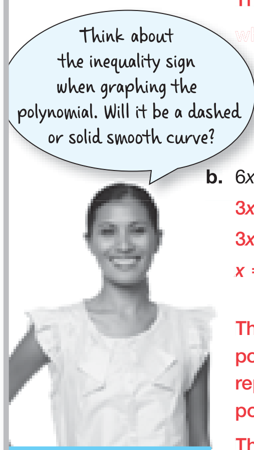
$(2x^2 - 8)(x - 4) = 0$

$x = \pm 2, 4$

The boxes represent the x-values when the polynomial is less than zero. The ovals represent the x-values when the polynomial is greater than zero.

The function $2x^3 - 8x^2 - 8x + 32 > 0$ when $\left\{ \begin{array}{c} -2 < x < 2 \\ x > 4 \end{array} \right\}$.





b. $6x^3 - 21x^2 - 12x > 0$

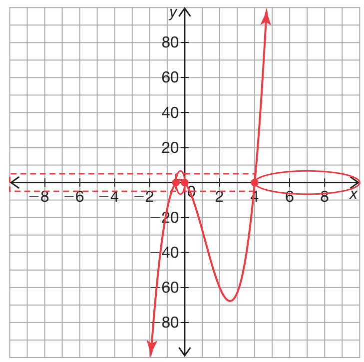
$3x(2x^2 - 7x - 4) = 0$

$3x(2x + 1)(x - 4) = 0$

$x = 0, -\frac{1}{2}, 4$

The boxes represent the x-values when the polynomial is less than zero. The ovals represent the x-values when the polynomial is greater than zero.

The function $6x^3 - 21x^2 - 12x > 0$ when $\left\{ \begin{array}{c} -\frac{1}{2} < x < 0 \\ x > 4 \end{array} \right\}$.



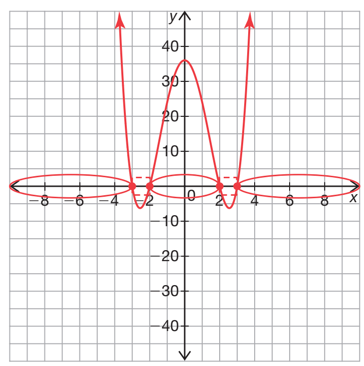
c. $x^4 - 13x^2 + 36 \leq 0$

$(x^2 - 4)(x^2 - 9) = 0$

$x = \pm 2, \pm 3$

The boxes represent the x-values when the polynomial is less than zero. The ovals represent the x-values when the polynomial is greater than zero.

The function $x^4 - 13x^2 + 36 \leq 0$ when $\left\{ \begin{array}{c} -3 \leq x \leq -2 \\ 2 \leq x \leq 3 \end{array} \right\}$.



Be prepared to share your solutions and methods.

Check for Students' Understanding

Solve: $x^5 - 4x^3 + x^2 - 4 \leq 0$.

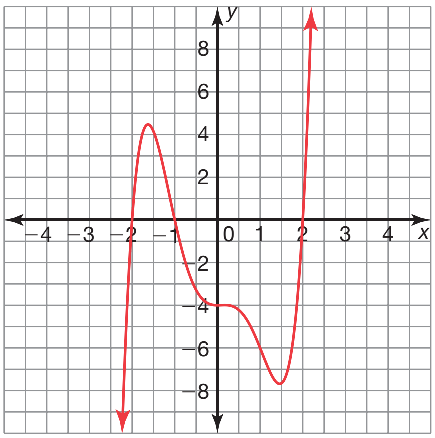
$x^5 - 4x^3 + x^2 - 4 = 0$

$x^3(x^2 - 4) + 1(x^2 - 4) = 0$

$x = -1, -2, 2$

The function $x^5 - 4x^3 + x^2 - 4 \leq 0$ when

$x \leq -2$ or $-1 \leq x \leq 2$.



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7.2

America's Next Top Polynomial Model

Modeling with Polynomials

LEARNING GOALS

In this lesson, you will:

- Determine the appropriate regression equation to model a problem situation.
- Predict outcomes using a regression equation.
- Sketch polynomial functions that appropriately model a problem situation.

KEY TERMS

- regression equation
- coefficient of determination

ESSENTIAL IDEAS

- A regression equation is a function that models the relationship between two variables in a scatter plot.
- The coefficient of determination or R^2, measures the strength of the relationship between the original data and its regression equation. The value ranges from 0 to 1 with a value of 1 indicating a perfect fit between the regression equation and the original data.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

A-CED Creating Equations

Create equations that describe numbers or relationships

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

F-IF Interpreting Functions

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-BF Building Functions

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

S-ID Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

Overview

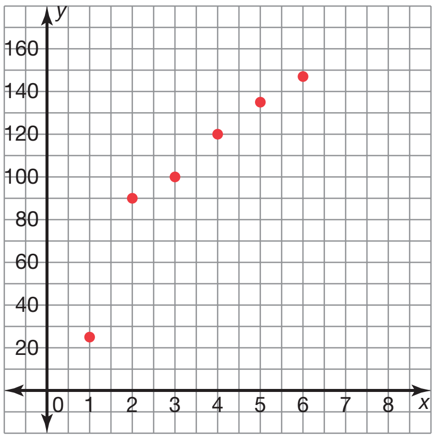
Traffic patterns in a downtown area and the federal minimum wage are contexts for modeling polynomial functions. Data is organized in a table of values for each situation and students will use a graphing calculator to create a scatter plot and determine regression equations. The coefficient of determination, R^2, is used to determine which regression equation best describes the data. The regression equations are used to make predictions and students then construct graphs over different periods of time. The behaviors of the functions are not always the best model of the problem situation.

Warm Up

The lacrosse players sold raffle tickets for their annual fundraiser for six weeks. The table of values is a record of how many tickets were sold each week.

x	y
1	25
2	90
3	100
4	120
5	135
6	147

1. Create a scatter plot.



2. Perform a linear regression. What is the linear regression equation?

   $y = 21.857x + 26.333$

   $R^2 \approx 0.938$

3. Perform a quadratic regression. What is the quadratic regression equation?

   $y = -4.375x^2 + 52.482x - 14.5$

   $R^2 \approx 0.954$

4. Perform a cubic regression. What is the cubic regression equation?

   $y = 1.991x^3 - 25.278x^2 + 115.589x - 64.667$

   $R^2 \approx 0.981$

5. Which regression equation appears to be a better fit?

   The cubic regression equation appears to be the better fit. The coefficient of determination is closer to 1 in the cubic regression.

America's Next Top Polynomial Model

Modeling with Polynomials

7.2

LEARNING GOALS

In this lesson, you will:

- Determine the appropriate regression equation to model a problem situation.
- Predict outcomes using a regression equation.
- Sketch polynomial functions that appropriately model a problem situation.

KEY TERMS

- regression equation
- coefficient of determination

Transportation plans are an essential part of any large urban development project. Whether designing residential blocks, shopping districts, or stadiums, part of the planning process is determining how to move large groups of people in and out of an area quickly. Building new highways, bus stations, bike lanes, or railways may be necessary for some large-scale developments.

Part of urban development projects is monitoring existing conditions in a specific area. Planners must determine how well the current traffic infrastructure meets the community's needs before modeling and predicting what transportation processes may work best for a future project.

What things do you consider when planning projects? What type of predictions or considerations do you make when planning projects?

Problem 1- Feeling a Little Congested

A table of values describes the average number of vehicles on a typical weekday entering and exiting a downtown area at various hours of the day. Students will describe patterns in the data and predict the type of polynomial that best fits the data. They then use the data and a graphing calculator to create a scatter plot and determine a regression equation. They determine a cubic regression equation is a better fit than a quadratic regression equation because the coefficient of determination is closer to 1. The regression equation is used to make several predictions. Students will identify the intervals when the model is appropriate for the problem situation and the intervals when it is not. Finally, students sketch a curve that predicts the number of vehicles on the road over a longer period of time and answer related questions.

Grouping

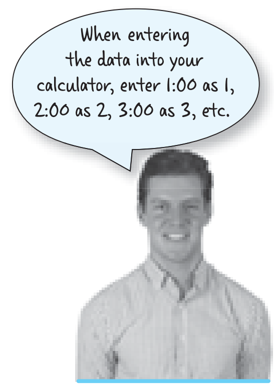
- Ask a student to read the information. Discuss as a class.
- Have students complete Questions 1 through 6 with a partner. Then have students share their responses as a class.

PROBLEM 1 Feeling a Little Congested

City planners consider building a new stadium on several acres of land close to the downtown of a large city. They monitored the number of cars entering and exiting downtown from a major highway between 1:00 PM and 7:00 PM to determine current traffic conditions.

1. Analyze the table of values that represent the average number of cars entering and exiting downtown during the given hours of a typical weekday. The value for time represents the start-time for the full hour over which the vehicles were monitored.

Time (PM)	Average Number of Vehicles on a Typical Weekday (thousands)
1:00	7.0
2:00	10.8
3:00	14.5
4:00	21.1
5:00	23.9
6:00	19.0
7:00	10.0



a. Describe any patterns you notice. Explain the patterns in the context of this problem situation.

The number of cars increase, reach a maximum at 5:00 PM, and then decrease again. This pattern makes sense in the context of this problem because rush hour occurs around 5:00 PM.

b. Predict the type of polynomial that best fits the data. Explain your reasoning.

Answers will vary.

The data increases and then decreases. The curve appears to be quadratic.

Guiding Questions for Share Phase, Question 1

- Between which hours do the number of cars increase?
- Between which hours do the number of cars decrease?
- When is rush hour?
- Which polynomial functions have behaviors that increase and then decrease?

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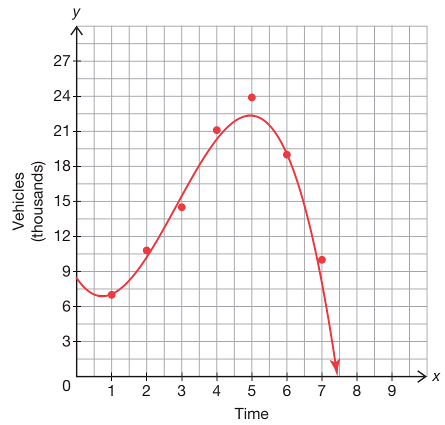
Guiding Questions for Share Phase, Questions 2 and 3

- Are all of the data points on your scatter plot?
- How did you decide which regression equation was the better fit?
- Does the regression equation of best fit always go through the most data points? Why not?
- Why is the cubic regression equation the best fit?
- What are the coefficients of determination for the cubic and quadratic regression equations?

2. Create a scatter plot of the data.



Recall that a **regression equation** is a function that models the relationship between two variables in a scatter plot. The regression equation can be used to make predictions about future events. Any degree polynomial can model a scatter plot, but data generally has one curve that best fits the data. You may also recall that the **coefficient of determination** (R^2) measures the "strength" of the relationship between the original data and its regression equation. The value ranges from 0 to 1 with a value of 1 indicating a perfect fit between the regression equation and the original data.

3. Use a graphing calculator to determine the regression equation for the average number of cars entering and exiting downtown on a typical weekday. Sketch the regression equation on the coordinate plane in Question 2. How well does the regression equation model the data? Was your prediction about the type of polynomial that best fits the data correct? Explain your reasoning.

See graph.

The quadratic regression equation is approximately $y = -1.36x^2 + 12.58x - 6.16$ with a coefficient of determination of 0.83.

The cubic regression equation is approximately $y = -0.41x^3 + 3.50x^2 - 4.47x + 8.44$ with a coefficient of determination of 0.98.

The cubic regression is the better fit.

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Guiding Questions for Share Phase, Question 4

- What horizontal equation can be used to determine when downtown will be most congested?
- How can the x-values be converted to minutes?
- At which times is the graph above 20,000?
- At which times is the output less than 10,000?
- What x-value represents noon?
- How many cars does the table of values for the cubic regression predict are on the highway at noon?
- Do a negative number of cars make sense in this problem situation?

4. Use the regression equation that best models the data to make predictions.

a. Downtown is congested when more than 20,000 cars are on the streets and highway. Predict when the downtown will be congested. Explain your reasoning.

Downtown will be congested between 3:55 PM and 5:51 PM.

I graphed the function $f(x) = 20$ and determined the points of intersection $x = 3.92$ and $x = 5.85$. I converted these points to time by multiplying 0.92 and 0.85 by 60 to calculate the minutes. The graph is above 20,000 for the time interval 3:55 PM $< x <$ 5:51 PM.

b. Predict the hours when the number of cars that enter and exit downtown is less than 10,000. Explain your reasoning.

Fewer than 10,000 cars are on the highway before 2:00 PM or after 7:00 PM. I used the graph to estimate the times when the output was less than 10,000.

c. Predict the number of vehicles that enter or exit downtown during the hour starting at noon.

An x-value of 0 represents noon. The table of values for the cubic regression predicts that on average approximately 8440 cars were on the highway at noon. I can also read the graph to predict the value.

d. Predict the number of vehicles that enter or exit downtown during the hour starting at 9 PM.

The cubic regression equation predicts that on average approximately −47,250 cars were on the highway at 9 PM. This doesn't make sense in terms of this problem situation. The end behavior of the cubic function approaches negative infinity as the x-values increase. I would have to change to a higher degree polynomial to predict beyond the data set given.

e. Predict the number of cars that enter or exit downtown during the hour starting at midnight the previous evening.

The value $x = -12$ represents midnight the previous evening. The number of cars on the highway last midnight was approximately 1,267,000. This does not make sense in terms of this problem situation. The cubic regression equation is the best fit for the data set provided in the table, and is not the best fit going beyond this data set.

Guiding Questions for Share Phase, Questions 5 and 6

- As the *x*-values increase, what happens to the *y*-values?
- Why isn't the cubic regression equation a good fit for predictions?
- Does your graph have repeated intervals of increase and decrease over the two day period?
- Does your graph show rush hour?
- How would you describe the shape of your graph?
- What is a cyclical graph?
- Is the curve representing this situation continuous and smooth?

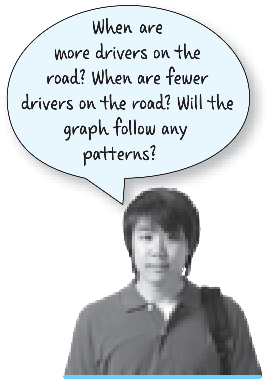
5. Consider the data and your regression equation.

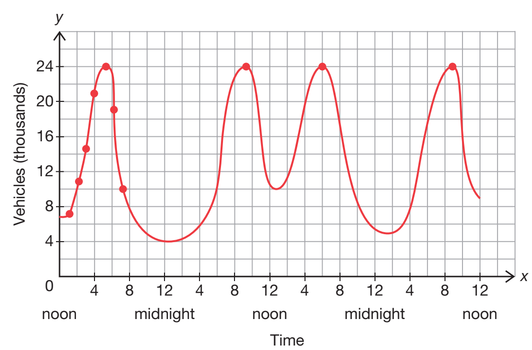
a. For what intervals is the model appropriate for this problem situation? For what intervals is the model inappropriate? Explain your reasoning.

Answers will vary depending on how students interpret or predict traffic patterns.

The model predicts $f(5) = 22{,}300$. This is the maximum value, so the number of vehicles for $x \leq -2$ wouldn't make sense. The output values $f(x) < 0$ are not in the range for the problem situation. The interval $(-1, 7)$ is appropriate for this problem situation.

b. Sketch a curve that you believe accurately predicts the number of vehicles on the road over a 2-day period. Explain your reasoning.





Answers will vary. Graphs should show fluctuations with peaks at rush hour (8 AM–9 AM and 4 PM–5 PM) and valleys during non-peak times such as midnight).
The output values are higher during rush hour and lower during odd times of the day and night. The graph is cyclical with the same approximate values repeating.

6. Do you think a polynomial function could accurately model this problem situation over the next 2 months before the next planning phase? Explain your reasoning.

Yes. The curve representing this situation is a smooth and continuous function, so a polynomial can model this situation.

Problem 2

A table of values describes the minimum wage and the years since 1950 they were enacted by Congress. Students will describe patterns in the data and predict the type of polynomial that best fits the data. They use the data and a graphing calculator to create a scatter plot and determine a regression equation. Students determine a cubic regression equation is the best fit with the coefficient of determination close to 1. The regression equation is used to make several predictions. They conclude that the end behavior of the cubic model is inappropriate because it approaches either positive or negative infinity. Students will sketch a curve that accurately models the minimum wage for the time interval 1900 through 1955 and conclude that no polynomial function can accurately model the situation because the graph is not continuous.

Grouping

Have students complete Question 1 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

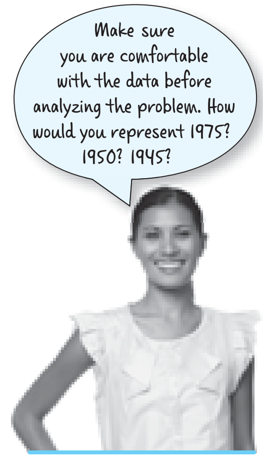
- Do the wages increase over time?
- Is the rate of change constant?

PROBLEM 2 Keep It to a Minimum

Although the minimum wage may vary from state to state, the U.S. federal government sets an absolute minimum wage for the nation every few years.

1. Analyze the table of values that shows the absolute minimum wage, and the years they were enacted by Congress.

Time Since 1950 (years)	Absolute Minimum Wage (dollars)
5	0.75
6	1.00
11	1.15
13	1.25
17	1.40
18	1.60
24	2.00
25	2.10
28	2.65
29	2.90
30	3.10
31	3.35
40	3.80
41	4.25
46	4.75
47	5.15
57	5.85
58	6.55
59	7.25



a. Describe any patterns you notice.

Answers will vary.

The wages are increasing over time.

The rate of change is different over different intervals.

b. Predict the type of polynomial that best fits this data. Explain your reasoning.

Answers will vary.

The data increases as x approaches infinity and decreases as x approaches negative infinity, so I know the polynomial that represents this data has an odd degree.

- As the x-values approach infinity, what happens to the y-values?
- As the x-values approach negative infinity, what happens to the y-values?
- Is the polynomial representing the data an even degree or an odd degree polynomial?

Grouping

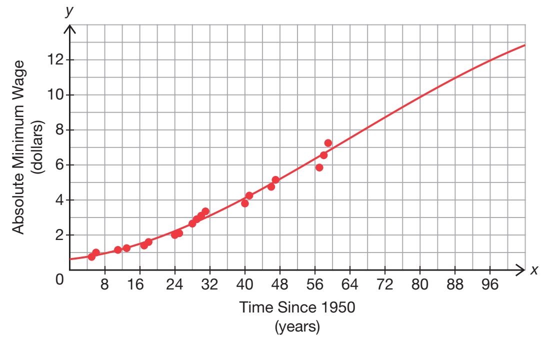
Have students complete Questions 2 through 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 2

- Which regression equation best fits this data?
- What is the coefficient of determination?

2. Analyze the data graphically.

a. Use a graphing calculator to determine the best regression function $f(x)$ to model the changes in the minimum wage over the years since 1950. Sketch the regression equation on the coordinate plane.



The regression equation is approximately
$f(x) = -0.0000099x^3 + 0.0019x^2 + 0.027x + 0.620$.

b. How well does the regression function model this data? Explain your reasoning.

The R^2 value is 0.98 so I know it fits the data well.

3. Use the regression equation that best models the data to make predictions.

a. Predict the absolute minimum wage in 2020. Explain your reasoning.

I predict the minimum wage in 2020 will be slightly more than $8.00.
I estimated the wage using the graph. I also calculated $f(70) = 8.42$.

Guiding Questions for Share Phase, Questions 3 and 4

- Why is $f(70)$ an indicator of the minimum wage in the year 2020?
- Why is $f(-5)$ an indicator of the minimum wage in the year 1945?
- $f(102)$ represents what year?
- $f(161)$ represents what year?
- What value is used to determine the minimum wage in the year 1865?
- $f(-85)$ represents what year?
- Does the model reach a local maximum? When?
- What happens on the graph in the year 2083?
- Why is a cubic model inappropriate for this situation?

b. Predict the minimum wage in 1945. Explain your reasoning.

I predict that the minimum wage in 1945 was $0.53.

I calculated the output value $f(-5) = \$0.53$.

c. Predict when the minimum wage is greater than $12.50. Explain your reasoning.

The minimum wage function $f(x) > \$12.50$ for the interval $102 < x < 161$.

Therefore the minimum wage is greater than $12.50 between the years 2052 and 2111.

4. Use the regression function to make predictions about events in the distant past and distant future.

a. According to the regression equation, what was the minimum wage when the Civil War ended in 1865? Explain your reasoning.

I used $f(-85) = \$17.96$ to determine the minimum wage in 1865. This does not make sense in terms of this context.

b. Predict the years when the minimum wage will be greater than $15.00. Explain your reasoning.

According to the regression function the minimum wage never reaches $15.00 in the future. The model reaches a local maximum at $14.26 in the year 2083 and then decreases toward negative infinity as x approaches infinity.

c. Do you think that a cubic model is appropriate to predict minimum wages in the distant past and future? Explain your reasoning.

A cubic model is inappropriate because of its end behavior. The output values of a cubic function approach either positive or negative infinity.

Grouping

- Ask a student to read the information. Discuss as a class.
- Have students complete Question 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 5

- What year does this problem situation begin?
- At what *x*-value does the graph of the function begin?
- Does your graph increase over time?
- Is your graph considered a continuous graph? Why not?
- Are all polynomial functions continuous?

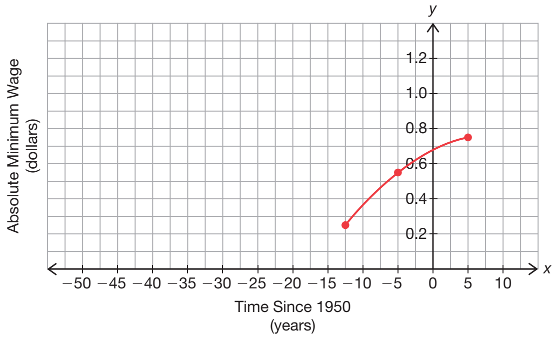
Let's take a closer look at the minimum wage in the early part of the 20th Century. A minimum wage did not exist until 1938 under the Fair Labor Standards Act. Before this time, employers could pay employees any hourly wage that employees were willing to accept. The initial hourly minimum wage in 1938 was $0.25 per hour. The wage increased steadily before reaching $0.75 in 1955.

5. Consider the minimum wage from 1900 to 1955.

a. Sketch a graph that you believe accurately models the minimum wage for the time interval (1900, 1955). Explain your reasoning.

Answers will vary.

Correct graphs will show a function that begins at $y = 0.25$ and gradually increases over time.



b. Do you think a polynomial function can accurately model the changes in minimum wage in the 20th Century? Explain your reasoning.

No. A polynomial cannot accurately model this situation. A polynomial is continuous and this problem situation begins in 1938.

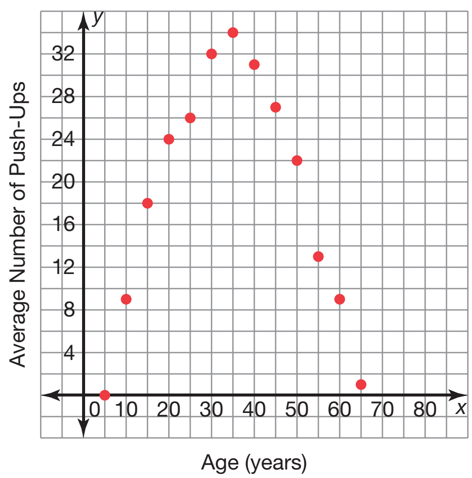
Be prepared to share your solutions and methods.

Check for Students' Understanding

The following data was gathered at a school during physical education awareness week. Both students and staff participated.

Age (years)	Average Number of Push-Ups
5	0
10	9
15	18
20	24
25	26
30	32
35	34
40	31
45	27
50	22
55	13
60	9
65	1

1. Create a scatter plot.



2. What type of regression equation appears to fit this data?

   A quadratic regression equation appears to fit this data.

3. Use the data and a graphing calculator to calculate a quadratic regression equation.

   $y = -0.035x^2 + 2.459x - 11.342$

   $R^2 \approx 0.978$

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4. Use the quadratic regression equation to answer each question.

a. What is the maximum value for y?

The maximum value for y is approximately equal to 31.

b. What is the value of x at the maximum value of y?

When y reaches its maximum value, the value of x is approximately equal to 35.1.

c. What do the values in parts (a) and (b) mean with respect to this problem situation?

The most push-ups any age group was capable of doing is 31 push-ups. That age group was the 35 year old age group.

d. How do the values in parts (a) and (b) compare to the values in the table?

The values resulting from the regression equation in parts (a) and (b) are slightly off when compared to the values in the table. According to the table of values, the maximum number of push-ups for age group was 34, not 31. The age group, however, is the same.

5. "As a person gets older, they are able to do more push-ups." Do you agree or disagree with this claim? Explain your reasoning.

I disagree with the statement. According to the table of values, after age 35, as a person gets older, they are able to do fewer push-ups.

7.3

Connecting Pieces

Piecewise Functions

LEARNING GOALS

In this lesson, you will:

- Write a piecewise function to model data.
- Graph a piecewise function.
- Determine intervals for a piecewise function to best model data.

KEY TERM

- piecewise function

ESSENTIAL IDEA

- A piecewise function is a function that is defined with different functional relationships between the independent and dependent variables over different domains.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

A-CED Creating Equations

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

F-IF Interpreting Functions

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

S-ID Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

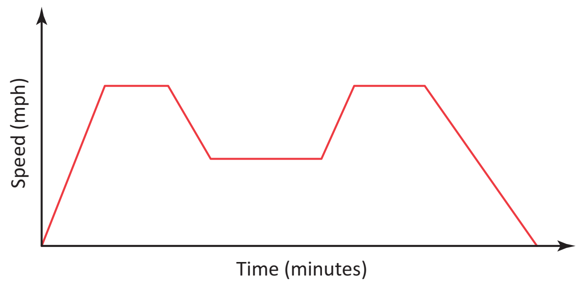
Overview

The federal minimum wage problem from the previous lesson is revisited. Data is organized in a table of values and a scatter plot using the data is shown. Since a single polynomial function is not the best model for this set of data, students divide the graph into five pieces, where each piece is modeled by a single polynomial function. Using a graphing calculator, students will determine regression equations for each of the five pieces of graph. They then create a piecewise function by writing each of the regression equations and their corresponding domains. Additional situations involve the salinity of an estuary in North Carolina, the average price of a gallon of regular unleaded gas from the years 1980 through 2008 and the average life expectancy of a person from the years 1910 to 1920. Students create scatter plots, divide the graphs into pieces and create a piecewise function to model each situation. The functions are used to make predictions.

Warm Up

A car is started and the driver accelerates to a speed of 25 mph. The driver slows down to 15 mph traveling through a school zone, then accelerates back up to 25 mph, and finally brakes to a complete stop.

1. Create a graph to model this situation.



2. If you were to divide this graph into pieces, how many pieces would you divide it into?

Answers will vary.

The graph appears to have seven pieces.

3. Where are the natural breaks in the data across the domain of this graph?

Answers will vary.

Each time the car changes speed is a natural break in the data across the domain of this graph.

4. Describe the type of function that best models this graph.

Answers will vary.

This graph cannot be modeled using only one function. The data needs to be separated into pieces and each piece will have a different linear model.

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7.3

Connecting Pieces

Piecewise Functions

LEARNING GOALS

In this lesson, you will:

- Write a piecewise function to model data.
- Graph a piecewise function.
- Determine intervals for a piecewise function to best model data.

KEY TERM

- piecewise function

Some of the most popular children's books from the 1980s and 1990s had an interesting format: the reader controlled the action of the story! At various key moments throughout the text, the reader was given an opportunity to make a decision about the main character's next move. Each choice led to a different outcome.

For example, in a dragon adventure, the reader may have to decide whether the knight should run and hide from a dragon, or grab a sword and try to slay the beast. One set of conditions led to one outcome, while another set of conditions led to a different outcome.

This idea is fairly common today, but at the time it was revolutionary for the same book to have multiple story lines and endings.

Have you ever read a book like this? If so, what did you like or dislike about it?

Problem 1

A table of values that describes the minimum wage and a scatter plot of the data are given. A single polynomial function is not the best model for a set of data. Students divide the data so that there are five pieces. They then determine the two cubic regression equations and three linear regression equations used to model the data in this situation. Next, students write the equation of the piecewise function $f(x)$, where $f(x)$ includes each regression equation and the domains used to model the absolute minimum wage data from the years 1955 through 2009. A piecewise function is defined and a worked example is given. Students are given piecewise functions and they will sketch them on the coordinate plane. Finally, they are given the graph of piecewise functions and they write the equations for each piece.

Grouping

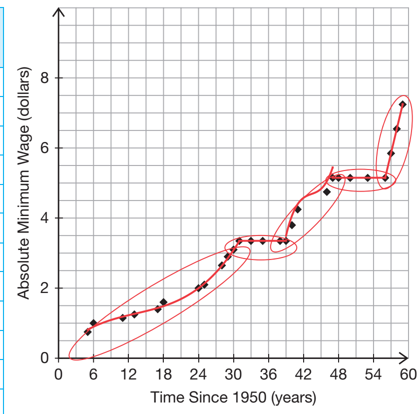
Ask a student to read the information and complete Question 1 as a class.

PROBLEM 1 Keep It to a Minimum, Revisited

Recall the minimum wage problem from the previous lesson.

The table shows the absolute minimum wage during various years. A scatter plot of this data is also shown.

Time Since 1950 (years)	Absolute Minimum Wage (dollars)
5	0.75
6	1.00
11	1.15
13	1.25
17	1.40
18	1.60
24	2.00
25	2.10
28	2.65
29	2.90
30	3.10
31	3.35
33	3.35
35	3.35
38	3.35
39	3.35
40	3.80
41	4.25
46	4.75
47	5.15
48	5.15
50	5.15
53	5.15
56	5.15
57	5.85
58	6.55
59	7.25



Sometimes, a single polynomial function is not the best model for a set of data. Analyze the graph of the minimum wage data. Instead of using a single polynomial function to model this data, consider separating the data into "pieces," where each piece is modeled by a single polynomial function.

To determine the pieces, look for breaks in the patterns that you see in the data. For example, the data in one part of the graph may appear to be linear, but then it may appear to be cubic in another part of the graph. Therefore, you can model these two parts of the graph with two different polynomial functions.

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Grouping

Have students complete Questions 2 through 8 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 2 and 3

- Does a linear function, quadratic function or a cubic function best model the data in the years 1955 through 1981? Why?
- Does the data increase at a constant rate for the years 1955 through 1981?
- Does the data have intervals of increase and decrease?
- Is the coefficient of determination for the cubic regression equation close to 1?
- How would you describe the graph of a constant linear function?
- Does a constant linear function best model the data between the years 1981 and 1989?
- What is the coefficient of determination for a constant linear function?

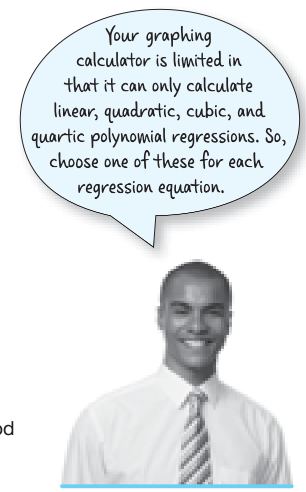
The number of pieces, or functions, that model the data can vary. For example, one person may look at the data and determine that it can be represented with two polynomial functions, while another person may see three, four, or even more functions.

Let's model the absolute minimum wage data by dividing it into five pieces, where each piece is modeled by a different polynomial function.

1. How do you think the data should be divided so that there are five pieces? Circle each piece on the given graph.

   Answers will vary.

   See graph for sample answer.



2. Consider the data from the years 1955 through 1981.

   a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.

   Answers will vary.

   A cubic function best models the data because the data increases rapidly, continues to increase less rapidly, and then increases rapidly again.

   b. Write the regression equation that best models the data over this interval.

   $y = 0.000248x^3 - 0.00988x^2 + 0.170x + 0.181$

   c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

   The coefficient of determination is 0.9962. The model is a very good fit to the data because the R^2-value is very close to 1.

3. Consider the data after the year 1981 and before the year 1989.

   a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.

   A constant linear function best models the data because the data appears to form a horizontal line.

   b. Determine a regression equation for this data over this interval.

   $y = 3.35$

   c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

   The coefficient of determination is 1. The model is a perfect fit.

Guiding Questions for Share Phase, Questions 4 through 6

- Does a linear function, quadratic function or a cubic function best model the data in the years 1989 through 1997? Why?
- Does the data increase at a constant rate for the years 1989 through 1997?
- Does the data have intervals of increase and decrease?
- Is the coefficient of determination for the cubic regression equation close to 1?
- Does a constant linear function best model the data between the years 1997 and 2006?
- What is the coefficient of determination for the constant linear function?
- Does a linear function, quadratic function or a cubic function best model the data in the years 2006 through 2009? Why?
- Does the data increase at a constant rate for the years 2006 through 2009?
- Does the data have intervals of increase and decrease?
- Is the coefficient of determination for the constant linear regression equation equal to 1?

4. Consider the data from the years 1989 through 1997.

a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.

Answers will vary.

Sample answer: A cubic function best models the data because the data increases rapidly, continues to increase less rapidly, and then increases rapidly again.

b. Determine a regression equation for this data over this interval.

$y = 0.01127x^3 - 1.466x^2 + 63.625x - 916.621$

c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

The coefficient of determination is 0.9955. The model is a very good fit because the R^2-value is very close to 1.

5. Consider the data after the year 1997 and before the year 2006.

a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.

A constant linear function best models the data because the data appears to form a horizontal line.

b. Determine a regression equation for this data over this interval.

$y = 5.15$

c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

The coefficient of determination is 1. The model is a perfect fit.

6. Consider the data from the years 2006 through 2009.

a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.

Answers will vary.

Sample answer: A linear function best models the data because the data appear to increase at a constant rate.

Guiding Questions for Share Phase, Questions 7 and 8

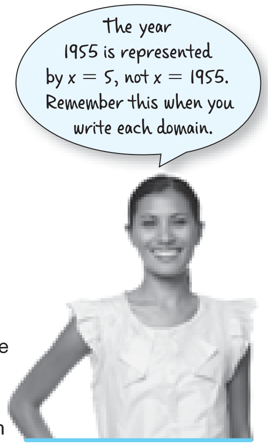
- How did you determine the domain for each of the pieces of $f(x)$?
- Overall, does the graph behave like a linear, quadratic, cubic, or quartic function?
- When looking at the separate pieces of the graph, does the graphic behavior appear to have cubic and linear patterns?

b. Determine a regression equation for this data over this interval.

$y = 0.7x - 34.05$

c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

The coefficient of determination is 1. The model is a perfect fit.



7. Write the equation of the function $f(x)$, where $f(x)$ includes each regression equation you used to model the absolute minimum wage data from the years 1955 through 2009. Write each equation on the line before the comma and write its corresponding domain on the line after the comma. Then, use a graphing calculator to sketch the graph of this function on the scatter plot at the beginning of the problem.

$$f(x) = \begin{cases} 0.000248x^3 - 0.00988x^2 + 0.170x + 0.181, & 5 \le x \le 31 \\ 3.35, & 31 < x < 39 \\ 0.01127x^3 - 1.466x^2 + 63.625x - 916.621, & 39 \le x \le 47 \\ 5.15, & 47 < x < 56 \\ 0.7x - 34.05, & 56 \le x \le 59 \end{cases}$$

See the graph at the beginning of this problem.

8. Explain why the function you wrote in Question 7 is a better fit for the data than a single linear, quadratic, cubic, or quartic function.

Answers will vary.

Sample answer: Overall, the graph does not behave like a linear, quadratic, cubic, or quartic function. But, when looking at certain chunks of the graph, I can see cubic and linear patterns.

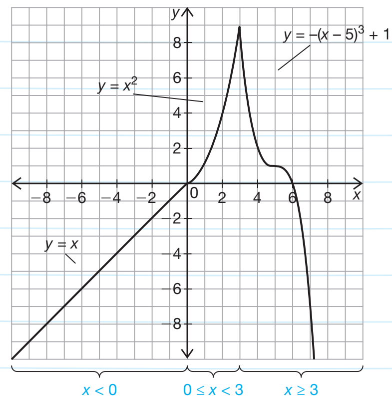
Grouping

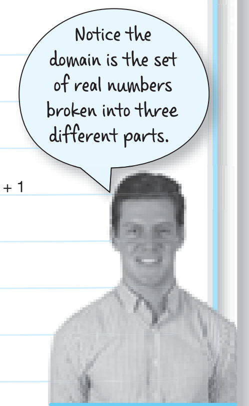
Ask a student to read the definition and the worked example. Discuss as a class.

You have just written the equation for a *piecewise function*. A **piecewise function** includes different functions that represent different parts of the domain.

A piecewise function and its graph are shown.

$$f(x) = \begin{cases} x, & x < 0 \\ x^2, & 0 \le x < 3 \\ -(x - 5)^3 + 1, & x \ge 3 \end{cases}$$





7

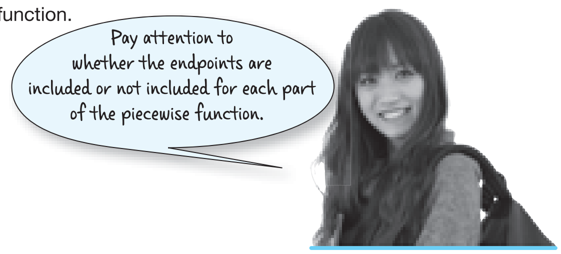
Grouping

Have students complete Questions 9 and 10 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 9

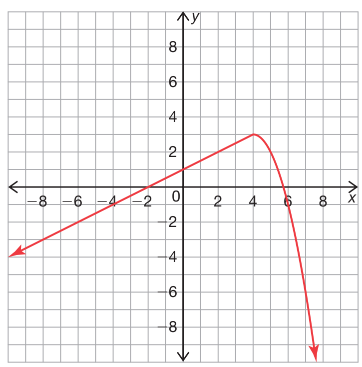
- Is the piece of the graph defined by the function $g(x) = \frac{1}{2}x + 1$ a linear, quadratic, or cubic function?
- What is the domain for the piece of the graph defined by the function $g(x) = \frac{1}{2}x + 1$?
- Is the piece of the graph defined by the function $g(x) = -x(x - 4)^2 + 3$ a linear, quadratic, or cubic function?
- What is the domain for the piece of the graph defined by the function $g(x) = -x(x - 4)^2 + 3$?
- Is the piece of the graph defined by the function $t(x) = x$ a linear, quadratic, or cubic function?
- What is the domain for the piece of the graph defined by the function $t(x) = x$?
- How can the function $t(x) = x^4 - 25x^2$ be factored?
- Is the piece of the graph defined by the function $t(x) = x^4 - 25x^2$ a linear, quadratic, or cubic function?
- What is the domain for the piece of the graph defined by the function $t(x) = x^4 - 25x^2$?

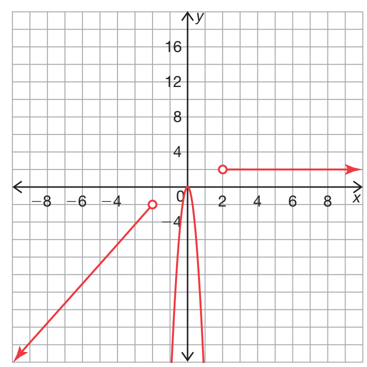
9. Sketch each piecewise function.



a. $g(x) = \begin{cases} \frac{1}{2}x + 1, & x < 4 \\ -(x - 4)^2 + 3, & x \geq 4 \end{cases}$

b. $t(x) = \begin{cases} x, & x < -2 \\ x^4 - 25x^2, & -2 \leq x \leq 2 \\ 2, & x > 2 \end{cases}$





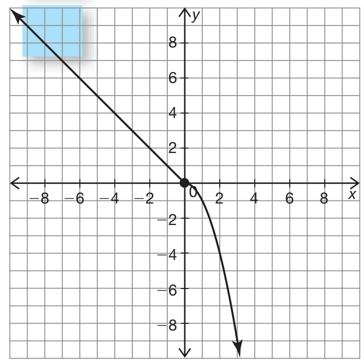
- Is the piece of the graph defined by the function $t(x) = 2$ a linear, quadratic, or cubic function?
- What is the domain for the piece of the graph defined by the function $t(x) = 2$?

7

Guiding Questions for Share Phase, Question 11

- What types of functions are used in the piecewise function $h(x)$?
- How many pieces are used in the piecewise function $h(x)$?
- What are the domains of each piece in the piecewise function $h(x)$? How do you know?
- What types of functions are used in the piecewise function $b(x)$?
- How many pieces are used in the piecewise function $b(x)$?
- What are the domains of each piece in the piecewise function $b(x)$? How do you know?

10. Billie, Kyle, and Avery were each asked to write the piecewise function to represent the graph shown.



Kyle

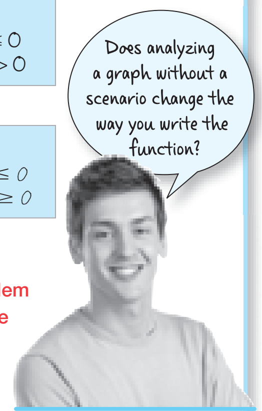
$f(x) = \begin{cases} -x, & x < 0 \\ -x^2, & x \geq 0 \end{cases}$

Avery

$f(x) = \begin{cases} -x, & x \leq 0 \\ -x^2, & x > 0 \end{cases}$

Billie

$f(x) = \begin{cases} -x, & x \leq 0 \\ -x^2, & x \geq 0 \end{cases}$



Who's correct? Explain your reasoning.

Both Kyle and Avery are correct. There is no specific problem situation to consider when writing this function so 0 can be included in either part of the domain.

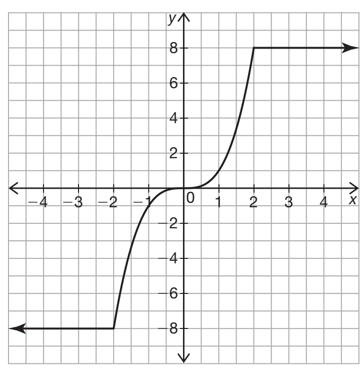
Billie is not correct because she included 0 in both parts of the domain.

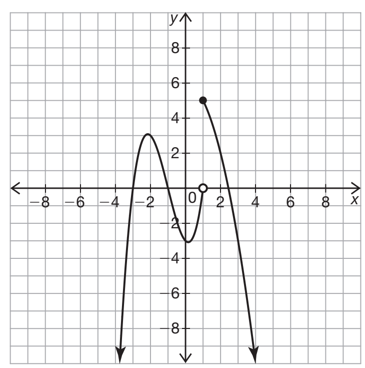
11. Write the equation for each piecewise function given its graph.

a. $h(x) = \begin{cases} -8, & x < -2 \\ x^3, & -2 \leq x \leq 2 \\ 8, & x > 2 \end{cases}$

b. $b(x) = \begin{cases} (x + 3)(x + 1)(x + 1), & x < 1 \\ -x^2 + 6, & x \geq 1 \end{cases}$

Note: Other domains are possible, such as $x \leq -2$, $-2 < x \leq 2$, and $x > 2$.





Problem 2

A table of values describes the salinity levels in an estuary in North Carolina over a period of 24 days. The data is displayed on a scatter plot. Students will use a graphing calculator to write regression equations representing pieces of the graph. Next, they write a piecewise function that models the salinity over the 24-day period and graph the function on the scatter plot. The regression equations are used to make predictions.

Grouping

- Ask a student to read the information. Discuss as a class.
- Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- Does a linear function, quadratic function, a cubic function, or a quartic function best model the data in the scatter plot?
- Does the data appear to increase or decrease at a constant rate?
- Does the data have intervals of increase and decrease?
- Which regression equation has the best fit with respect to the coefficient of determination?
- Is the coefficient of determination close to 1?
- Is a piecewise function a better fit?

PROBLEM 2 Salinity Now! Salinity Now!

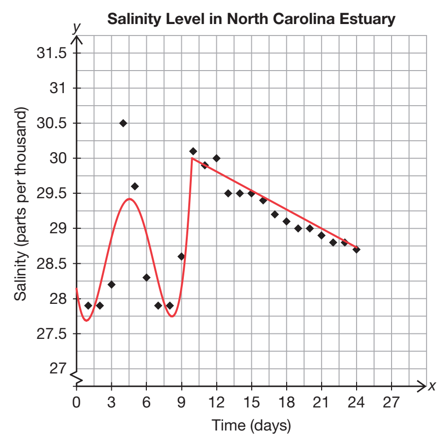
Salinity is the measure of saltiness, or dissolved salt content in water. Salinity in an estuary changes due to location, tidal functions, seasonal weather changes, and volume of freshwater runoff. Ecologists routinely measure salinity in estuaries because of its impact on plants, animals, and people. Too much salinity can reduce vegetation in surrounding areas.

The table shows the salinity levels in an estuary in North Carolina over a period of 24 days. A scatter plot of this data is also shown.

Time (days)	1	2	3	4	5	6	7	8
Salinity (parts per thousand)	27.9	27.9	28.2	30.5	29.6	28.3	27.9	27.9

Time (days)	9	10	11	12	13	14	15	16
Salinity (parts per thousand)	28.6	30.1	29.9	30	29.5	29.5	29.5	29.4

Time (days)	17	18	19	20	21	22	23	24
Salinity (parts per thousand)	29.2	29.1	29.0	29.0	28.9	28.8	28.8	28.7



1. Consider the data for the first ten days. Determine the regression equation that is the best fit for the data over this interval. Explain your reasoning.

   Answers will vary.

   A quartic equation is the best fit for the data over this interval.

   $y = 0.0094x^4 - 0.17x^3 + 0.90x^2 - 1.18x + 28.14$

   The data seems to have a "W" shape over this interval which can be represented by a quartic equation. The R^2-value for my quartic regression is 0.6975, which is greater than the R^2-value for the other regressions.

Guiding Questions for Share Phase, Questions 2 through 5

- Does the data after the tenth day appear somewhat linear?
- What is the coefficient of determination for the linear function representing the data after the tenth day?
- What are the domains for each piece of the piecewise function? How do you know?
- Is the linear regression equation or the quartic regression equation used to predict the salinity of the estuary on the 30th day?
- Does the linear regression equation used to calculate the salinity of the estuary on the 30th day result in a reasonable answer? What is a reasonable answer?
- Is the linear regression equation or the quartic regression equation used to predict the salinity of the estuary 5 days before the data in the table was recorded?
- What value was used in the quartic equation to calculate the salinity of the estuary 5 days before the data in the table was recorded?
- Does the quartic regression equation used to calculate the salinity of the estuary 5 days before the data in the table was recorded result in a reasonable answer? What is a reasonable answer?

2. Consider the data after the tenth day. Determine the regression equation that is the best fit for the data over this interval. Explain your reasoning.

Answers will vary.

A linear equation seems to be the best fit for the data over this interval.

$y = -0.09x + 30.89$

The data seems to be decreasing at a fairly constant rate. The R^2-value is 0.9398, which is close to 1. The R^2-values for the other regressions are slighter greater, but the leading coefficient is so close to 0 that the curve is practically a line.

3. Use your answers to Questions 1 and 2 to write a piecewise function that models the salinity over the 24-day period. Then, graph the function on the scatter plot.

$$f(x) = \begin{cases} 0.0094x^4 - 0.17x^3 + 0.90x^2 - 1.18x + 28.14, & 0 < x \le 10 \\ -0.09x + 30.89, & 10 < x \le 24 \end{cases}$$

See graph.

4. Predict the salinity of the estuary on the 30^{th} day. Does your prediction seem reasonable?

Answers will vary.

I can extend the domain of the second part of the piecewise function to include the 30^{th} day.

$-0.09(30) + 30.89 = 28.19$

The salinity would be 28.19 parts per thousand, which seems reasonable.

5. Predict the salinity of the estuary 5 days before the data in the table was recorded. Does your prediction seem reasonable?

Answers will vary.

I can extend the domain of the first part of the piecewise function to include 5 days before, or -5.

$y = 0.0094(-5)^4 - 0.17(-5)^3 + 0.90(-5)^2 - 1.18(-5) + 28.14 = 83.665$

The salinity would be 83.665 parts per thousand, which does not seem reasonable according to the data given.

Problem 3

A table of values describes the average price of a gallon of regular unleaded gas from the years 1980 through 2008. Students will use the data to create a scatter plot, divide the scatter plot into pieces, and write a piecewise function to represent the situation. The function is used to make predictions.

Grouping

Have students complete Questions 1 through 5 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 and 2

- What scale was used on the *x*-axis when you created the scatter plot?
- What scale was used on the *y*-axis when you created the scatter plot?

PROBLEM 3 Fill It Up!

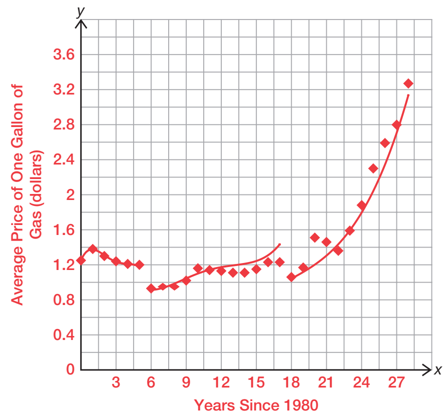
The table shows the average price of a gallon of regular unleaded gas from the years 1980 through 2008.

Years Since 1980	0	1	2	3	4	5	6	7	8	9
Average Gas Price (dollars)	1.25	1.38	1.30	1.24	1.21	1.20	0.93	0.95	0.95	1.02

Years Since 1980	10	11	12	13	14	15	16	17	18	19
Average Gas Price (dollars)	1.16	1.14	1.13	1.11	1.11	1.15	1.23	1.23	1.06	1.17

Years Since 1980	20	21	22	23	24	25	26	27	28
Average Gas Price (dollars)	1.51	1.46	1.36	1.59	1.88	2.30	2.59	2.80	3.27

1. Create a scatter plot of the data on the grid shown.



2. Describe the type of function(s) that best models this data. Explain your reasoning.

A piecewise function would best model this data because I can see different patterns over different intervals of the domain.

Guiding Questions for Share Phase, Questions 3 through 5

- The scatter plot was divided into how many pieces?
- Where in the domain does the data seem to naturally break apart?
- What does linear data look like?
- Are any of the observable patterns in the scatter plot linear in nature? How do you know?
- What does quadratic data look like?
- Are any of the observable patterns in the scatter plot quadratic in nature? How do you know?
- What does cubic data look like?
- Are any of the observable patterns in the scatter plot cubic in nature? How do you know?
- What does quartic data look like?
- Are any of the observable patterns in the scatter plot quartic in nature? How do you know?
- What are the domains of the three pieces of the piecewise function?
- What is a reasonable answer for the prediction?

3. Consider using a piecewise function to model this data. Determine the intervals for the domain, and the type of polynomial function for each interval. Explain your reasoning.

Answers will vary.

The data points seem to break at 6 years and again at 16 years, so I will determine domain intervals of $0 \le x < 6$, $6 \le x < 16$, and $16 \le x \le 28$.

Over the interval $0 \le x < 6$ the data appear to be quartic. Over the interval $6 \le x < 16$, the data appear to be quartic. Over the interval $16 \le x \le 28$, the data appear to be cubic.

4. Write a piecewise function to model the data. Then, graph the piecewise function on the grid in Question 1.

Answers will vary.

$$\text{Sample answer: } f(x) = \begin{cases} -0.006x^4 + 0.070x^3 - 0.271x^2 + 0.336x + 1.250, & 0 \le x < 6 \\ 0.00072x^4 - 0.0306x^3 + 0.469x^2 - 3.024x + 7.884, & 6 \le x < 16 \\ 0.000323x^3 - 0.00102x^2 + 0.265x - 4.407, & 16 \le x \le 28 \end{cases}$$

The coefficient of determination for each part of the piecewise function are 0.998, 0.937, and 0.977 respectively.

5. Use your piecewise function to predict the price of gas in the year 2020. Does your prediction seem reasonable? Explain your reasoning.

Answers will vary.

I can extend the domain of the third part of the piecewise function to include the year 2020, which is 40 years after 1980.

$0.000323(40)^3 - 0.00102(40)^2 - 0.265(40) + 4.407 = 12.835$

According to this model, gas will cost \$12.84 per gallon in 2020. This may be reasonable, if gas prices tend to increase at such a high rate.

Problem 4

A table of values describes the average life expectancy of a person from the years 1910 to 1920. Students will use the data to create a scatter plot, divide the scatter plot into pieces, and write a piecewise function to represent the situation. The function is used to make predictions.

Grouping

Have students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- What scale was used on the *x*-axis when you created the scatter plot?
- What scale was used on the *y*-axis when you created the scatter plot?

PROBLEM 4 Live Long and Strong

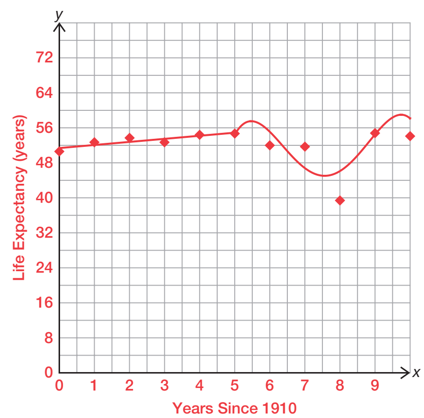
Life expectancy is a prediction of the number of years that a person will live. Life expectancies often vary significantly over time and across different groups such as country, gender, and race.

The table shows the average life expectancy of a person from the years 1910 through 1920.

Years Since 1910	0	1	2	3	4	5
Life Expectancy (years)	50.6	52.7	53.7	52.7	54.4	54.7

Years Since 1910	6	7	8	9	10
Life Expectancy (years)	52.0	51.2	39.4	54.8	54.1

1. Create a scatter plot of this data on the grid shown.



Guiding Questions for Share Phase, Questions 2 and 3

- The scatter plot was divided into how many pieces?
- Where in the domain does the data seem to naturally break apart?
- Are any of the observable patterns in the scatter plot linear in nature? How do you know?
- Are any of the observable patterns in the scatter plot quadratic in nature? How do you know?
- Are any of the observable patterns in the scatter plot cubic in nature? How do you know?
- Are any of the observable patterns in the scatter plot quartic in nature? How do you know?
- What are the domains of the two pieces of the piecewise function?
- Do you have knowledge of any medical epidemics that would have shifted the data over time?

Talk the Talk

Students list the advantages and disadvantages of using a piecewise function instead of a single function type to model data.

Grouping

Have students complete the Question with a partner. Then have students share their responses as a class.

2. Write a piecewise function to model the data. Then, graph the piecewise function on the grid. Explain your reasoning.

Answers will vary.

The data points seem to increase at a fairly constant rate through 1915, and then they decrease and increase, so I will determine domain intervals of $0 \le x \le 5$ and $5 < x \le 10$.

Over the interval $0 \le x \le 5$, the data appear to be linear. Over the interval $5 < x \le 10$, the data appear to be quadratic or quartic.

Sample answer:

$$f(x) = \begin{cases} 0.7029x + 51.3762, & 0 \le x \le 5 \\ -0.6333x^4 + 19.2278x^3 - 213.2417x^2 + 1021.3544x - 1729.1952, & 5 < x \le 10 \end{cases}$$

3. Write a brief report that explains the patterns shown by the data in terms of life expectancy from 1910 through 1920. Do some research and use facts to support your claims.

Answers will vary, but reports should mention the Influenza Epidemic of 1918.

Talk the Talk

Write a brief summary about what you've learned about using piecewise functions to model real-world data. Include advantages and disadvantages of using a piecewise function instead of a single function type to model data.

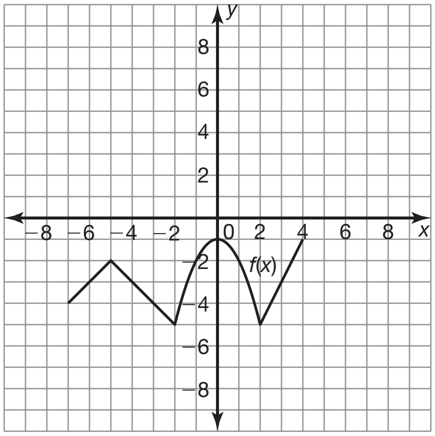
Piecewise functions can sometimes more accurately model data, instead of trying to fit the data using one type of function.

There can be more than one way to represent piecewise functions depending on how a person decides to determine the intervals of the domain.

Be prepared to share your solutions and methods.

Check for Students' Understanding

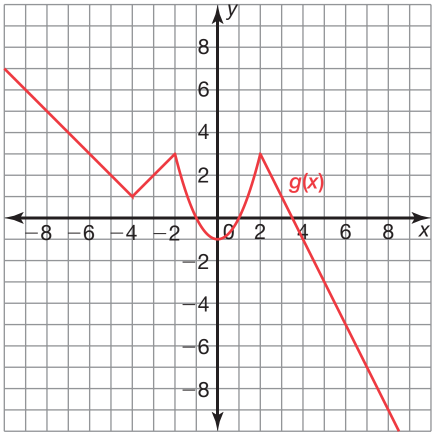
1. Write the piecewise function $f(x)$ given its graph.



$$f(x) = \begin{cases} x + 3 & -7 \leq x \leq -5 \\ -x - 7 & -5 \leq x \leq -2 \\ -x^2 - 1 & -2 \leq x \leq 2 \\ 2x - 9 & 2 \leq x \leq 4 \end{cases}$$

2. Sketch the piecewise function.

$$g(x) = \begin{cases} -x - 3 & x \leq -5 \\ x + 5 & -5 \leq x \leq -2 \\ -x^2 - 1 & -2 \leq x \leq 2 \\ -2x + 7 & 2 \leq x \end{cases}$$



7.4

Modeling Gig

Modeling Polynomial Data

LEARNING GOALS

In this lesson, you will:

- Model a problem situation with a polynomial function.
- Solve problems using a regression equation.

ESSENTIAL IDEAS

- Polynomial functions are used to model problem situations.
- Regression equations are used to solve problem situations.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

A-CED Creating Equations

Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A-REI Reasoning with Equations and Inequalities

Represent and solve equations and inequalities graphically

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F-LE Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

S-ID Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

Overview

The monthly profits of a calculator company, the average cost of a movie ticket over time, and the price of tickets for a fundraiser dependent on the number of students participating are the situations which can be modeled using polynomial functions. Students will use a graphing calculator to determine a regression equation that best fits the data and then use the regression equation to make predictions and answer questions related to the problem situation.

Warm Up

The table of values describes the average monthly precipitation in Pittsburgh, Pennsylvania, during the years 1971 through 2000.

Month	Average Monthly Precipitation (inches)
1	2.7
2	2.37
3	3.17
4	3.01
5	3.8
6	4.12
7	3.96
8	3.38
9	3.21
10	2.25
11	3.02
12	2.86

1. What type of regression equation appears to fit the data?

Answers will vary.

A quartic regression equation appears to fit the data because the data decreases, increases, decreases, and increases again.

2. Write a regression equation that best fits the data.

$y = 0.003x^4 - 0.082x^3 + 0.633x^2 - 1.484x + 3.582$

The coefficient of determination is approximately 0.801.

3. Determine the coefficient of determination for your regression equation.

The coefficient of determination for the regression equation is approximately 0.801.

4. How do you interpret the coefficient of determination for your regression equation?

A coefficient of determination equal to 0.801 is a good fit to represent the interval for 12 months.

7.4

Modeling Gig

Modeling Polynomial Data

LEARNING GOALS

In this lesson, you will:

- Model a problem situation with a polynomial function.
- Solve problems using a regression equation.

Americans watch a lot of movies. The first ever movie was made in the 1870s, but it wasn't until the early 20th century that movie theaters were invented. Americans lined up to pay a quarter to see black and white productions with no sound. As audio-visual technology advanced, so did the quality of movie productions. Changes in technology not only improved the quality of movies, they also led to changes in the entire industry.

The Video Home System (also known as VHS), developed in the 1970s, allowed consumers to rent or purchase movies and watch them on their TVs at home. With the invention of the remote control, people could even fast forward to their favorite parts, without even getting off the couch! Video stores were a huge business, leading way to newer technology that allowed customers to stream movies from their computers at home.

How do you think movies will change throughout the 21st Century? Do you think people will still go to "old-fashioned" movie theaters to watch movies?

Problem 1

A table of values that describes the relationship between the price of various models of graphing calculators and the monthly profit earned from the sales of the calculators is given. Students recognize patterns in the data and will use a graphing calculator to determine the regression equation of best fit. Using the coefficient of determination students conclude a quadratic regression equation best fits the data. Students then answer questions related to profit loss and gain.

Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4

- Does the profit increase as the price of each calculator increases?
- Does the profit decrease as the price of each calculator increases?
- Does the profit reach a maximum? When?
- Is the function increasing or decreasing?
- Which type of function best models this data? How do you know?
- Does the coefficient of determination support your choice?

PROBLEM 1 Strut Down the Statwalk

The CALC_U-Now Company sells a variety of calculators. The table shows the relationship between the price of various models of graphing calculators and the monthly profit earned from the sale of the calculators.

1. Analyze the data in the table of values.

Price of Calculators (dollars)	Monthly Profit (dollars)
65	15,950
70	17,600
75	19,060
80	19,300
85	19,290
90	19,240
95	18,000
100	17,150
105	15,300

   a. What patterns do you notice in the data?

   The profit increases as the price of each calculator increases, the profit reaches a maximum, and then decreases.

   b. Describe the polynomial function that best models this data. Explain your reasoning.

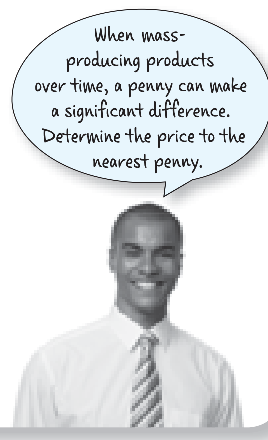
   A quadratic function models the data. The way profit increases, reaches a maximum, and then decreases resembles a quadratic function.

2. Use a graphing calculator to determine the regression function that best models this data.

   The function $y = -9.49x^2 + 1592.80x - 47{,}367.33$ models the data. A quadratic function has the coefficient of determination closest to 1.

3. Write inequalities to represent the prices for which CALC-U-Now would lose money? Explain your reasoning.

   The company loses money for $x < 38.63$ or $x > 129.22$. I determined the x-intercepts and the values for which the output is negative.



4. CALC-U-Now must make budget cuts! As a financial contractor, you must determine which calculator price will generate the most profit. Write a statement to support your decision including all relevant mathematics.

   CALC-U-Now will make a profit of $19,466 if it charges $83.92 for each calculator. I determined the regression equation, graphed it on my calculator, and located the vertex. This is the maximum value.

- Using the regression equation, is the output ever a negative value? When?
- What does a negative output value mean with respect to this problem situation?
- What does the vertex of the parabola mean with respect to this problem situation?
- Is the vertex of the parabola associated with a maximum profit? Why?

Problem 2

A table of values that describes the average price of a movie ticket over a period of time is given. Students recognize patterns in the data and will use a graphing calculator to determine the regression equation of best fit. Using the cubic regression equation, students make predictions. Students then answer questions related to the end behavior of the regression equation and conclude that even though the equation fits the values in the table, it is not a good fit for the values beyond the table.

Grouping

Have students complete Questions 1 through 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Questions 1 through 4

- Does the price of a movie ticket increase as the time increases?
- Is the function increasing or decreasing?
- Does the function increase at a constant rate?
- Which type of function best models this data? How do you know?
- Does the coefficient of determination support your choice?
- Using the regression equation, is the output ever a negative value? When?

PROBLEM 2 **3,2,1…. Polynomial Modeling Action!**

Inflation has influenced the price of a movie ticket over the years. The first movie theater opened in the year 1900, charging $.05 per ticket. The data provided shows how the average price of a movie ticket has increased over the years.

Years	Average Price of a Movie Ticket (dollars)
1900	0.05
1948	0.36
1958	0.68
1971	1.65
1983	3.15
1995	4.35
2003	6.03
2007	6.88
2009	7.50

1. Determine a regression function that best models this data.
$y = 0.00000777x^3 - 0.000224x^2 - 0.000794x + 0.0465$

2. Use your regression equation to predict when the average price of a movie ticket will reach $15.00. Explain your reasoning.
The average price of a movie ticket will reach $15.00 in the year 2036. I graphed the horizontal line $y = 15$ and determined the point of intersection.

3. Use your regression equation to predict the cost of a movie ticket in the year 2100. Explain your reasoning.
In the year 2100, the price of a movie ticket will be approximately $53.10.

- What does a negative output value mean with respect to this problem situation?
- Are the predictions that result from the cubic regression equation reasonable? What is reasonable?
- Does the cubic regression equation work for the years beyond the table of values?
- What is the end behavior of a cubic equation? Does the end behavior make sense in this problem situation? Why not?

4. Jessica and Lindsay disagree over how to model this situation with a polynomial function.

> Jessica
> A cubic function is the most appropriate model. The coefficient of determination is closest to 1.

> Lindsay
> A piece-wise function is most appropriate for this situation.

Who's correct? Explain your reasoning.

Jessica is correct for the data that is provided, as well as for $x > 109$. Lindsay is correct when considering years prior to the opening of the first theater or beyond 2009.

Problem 3

A table of values describes the cost of ticket to a school fundraiser, which is dependent on the number of participants. Students will write the principal a letter summarizing the potential profits of the fundraiser depending on the number of participants. To do this, they first determine a quadratic regression equation and identify the vertex of the parabola as the point which tells them the maximum money that can be raised and the cost of the ticket that will generate this money.

Grouping

Have students complete the Problem with a partner. Then have students share their responses as a class.

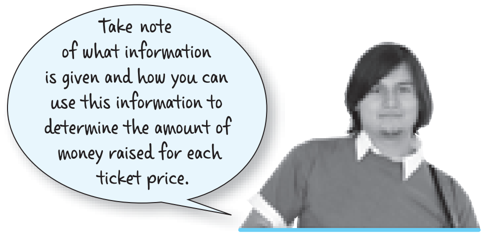
Guiding Questions for Share Phase, Problem 3

- What patterns do you see in the data?
- Is the function increasing or decreasing?
- Does the amount of money raised steadily increase as the ticket price increases?
- When does the amount of money raised reach a maximum?
- What type of function best models this situation?
- How did you determine the money raised for each ticket price?

PROBLEM 3 "Polynomial Models" for $500, Please!

The Math Club sponsors an event each year to raise money for their trip to the Quiz Bowl. As the president of the Math Club, you propose having a movie night fundraiser. You survey the students to see how many students will attend. The number of students varies depending on the ticket price.

Ticket Price (dollars)	Students Who Will Attend
1.25	120
1.75	105
2.25	95
2.75	83
3.25	77
3.75	64
4.25	58
4.75	40
5.25	30



Write a short letter to your principal about your findings. Include details about the exact ticket price that raises the most money as well as the approximate number of students who will attend.

In order to raise money for the Math Club, I propose that we hold a movie night. From the table of values, you can see that the amount of money raised steadily increases as the ticket price increases. It reaches a maximum at approximately $3.25 before steadily decreasing.

Ticket Price	Students Who Will Attend	Money Raised
$1.25	120	150.00
$1.75	105	183.75
$2.25	95	213.75
$2.75	83	228.25
$3.25	75	243.75
$3.75	64	240.00
$4.25	58	246.50
$4.75	40	190.00
$5.25	30	157.50

To determine the exact value, I modeled the data with a quadratic regression equation. The vertex occurs at (3.34, 244) which means the maximum money raised will be $244 if we charge $3.34 per ticket.

Be prepared to share your solutions and methods.

- What is the coefficient of determination for the quadratic regression equation?
- What are the coordinates of the vertex of the parabola?
- What is the significance of the vertex of the parabola with respect to this problem situation?

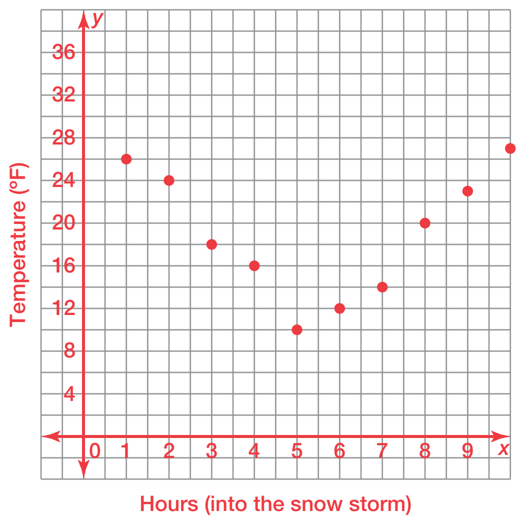
7

Check for Students' Understanding

The table of values describes the temperatures recorded during a 10-hour snow storm.

Hours (into the snow storm)	Temperature (°F)
1	26
2	24
3	18
4	16
5	10
6	12
7	14
8	20
9	23
10	27

1. Create a scatter plot.



2. Determine a quadratic regression equation to best fit the data.

$y = 0.742x^2 - 8.118x + 35.067$

The coefficient of determination is 0.910.

3. Using your regression equation, what are the coordinates of the vertex of the parabola and what do they mean with respect to this problem situation?

The coordinates of the vertex of the parabola are approximately (5.5, 13). The temperature reached the lowest, 13°F, 5.5 hours into the snow storm.

4. What was the lowest temperature during the snow storm?

If the quadratic regression equation is used, the lowest temperature was 13°F. If the table of values is used, the lowest temperature was 10°F.

7.5

The Choice Is Yours

Comparing Polynomials in Different Representations

LEARNING GOALS

In this lesson, you will:

- Compare polynomials using different representations.
- Analyze key characteristics of polynomials.

ESSENTIAL IDEAS

- Polynomial functions can be represented using a graph, table of values, equation, or description of its key characteristics.
- The transformational function is $g(x) = Af(B(x) - C) + D$, where the A-value vertically stretches or compresses the graph, the B-value horizontally stretches or compresses the graph, the C-value horizontally translates the graph right or left, and the D-value vertically translates the graph up or down.

COMMON CORE STATE STANDARDS FOR MATHEMATICS

F-IF Interpreting Functions

Analyze functions using different representations

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Overview

Polynomial functions can be represented using a graph, table of values, equation, or description of its key characteristics. Students will compare two functions written in different forms to determine which function has: a greater number of zeros, a greater degree, a degree divisible by 2, an odd degree, greater output as x approaches negative infinity or positive infinity, a greater y-intercept, a greater average rate of change over a specified interval, a greater relative minimum, an axis of symmetry with a greater x-value, a greater minimum, a lower minimum, a greater output for a given input, and a greater input for a given output.

Warm Up

Determine the number of zeros associated with each function.

1. $f(x) = x^3 - 2x^2 + 5x - 7$

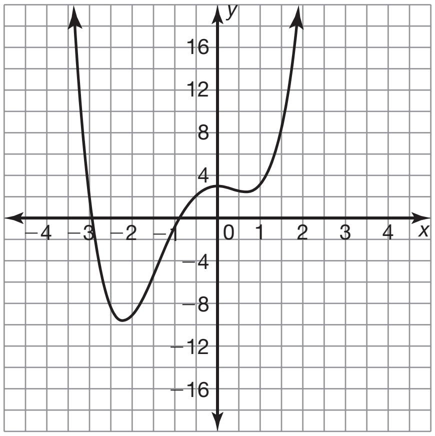
   This function has 3 zeros.

2.

x	$f(x)$
−3	45
−2	20
−1	5
0	0
1	5
2	20
3	45

This function has 2 zeros.

3.



This function has 4 zeros.

7.5

The Choice Is Yours

Comparing Polynomials in Different Representations

LEARNING GOALS

In this lesson, you will:

- Compare polynomials using different representations.
- Analyze key characteristics of polynomials.

Infinity refers to something that goes on without end. The set of natural numbers {1, 2, 3, . . .} and the set of integers {. . . −2, −1, 0, 1, 2 . . .} are examples of infinite sets because they continue without end. Another example of an infinite set is the set of rational numbers between 0 and 1.

Seeing different infinite sets of numbers begs the question: do all infinite sets have the same quantity of numbers in them? The set of natural numbers are only positive, while the set of integers are positive and negative. Does this mean that the set of natural numbers has fewer numbers than the set of integers?

How do you compare the size of these sets of numbers? Is it possible for one infinite set to be greater than another infinite set?

Problem 1

A worked example shows students how to compare a function written as an equation to a function drawn on coordinate plane. Students will compare two functions written in different forms to determine which function has a greater number of zeros, which function has a greater degree, which function has a degree divisible by 2, which function is an odd degree, or which function has the greater output as x approaches negative infinity or positive infinity.

Grouping

Ask a student to read the information and worked example. Discuss as a class.

PROBLEM 1 The Best of Both Representations

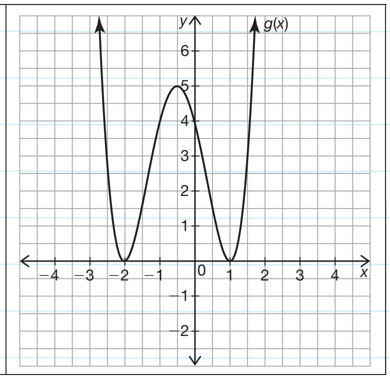
Recall that you can represent a polynomial using a graph, table of values, equation, or description of its key characteristics. The ability to compare functions using different representations is an important mathematical habit. This skill allows you to model problems in different ways, solve problems using a variety of methods, and more easily identify patterns. At times you may need to compare functions when they are in different representations.

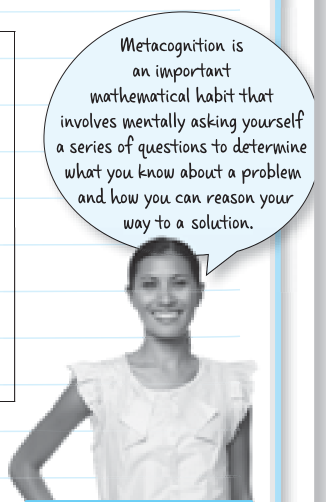
When comparing two functions in different forms, it may be helpful to ask yourself a series of questions. Examples include:

- *What information is given?*
- *What is the degree of each function?*
- *What do I know about all functions of this degree?*
- *What key characteristics do I need to know?*
- *How do the functions compare?*

Consider two polynomial functions $f(x)$ and $g(x)$. Which polynomial has a greater number of real zeros? Justify your choice.

$f(x) = -2(x - 1)^3$





- The Fundamental Theorem of Algebra states that the number of zeros must be equal to the degree of the function. Therefore, $f(x)$ has 3 zeros.
- The function $f(x)$ has a real zero at 1 (multiplicity 3), so all zeros are real.
- The graph of $g(x)$ shows each zero has multiplicity 2, for a total of 4 real zeros.

The function $g(x)$ has 4 real zeros while $f(x)$ has 3. Therefore the correct choice is $g(x)$.

Grouping

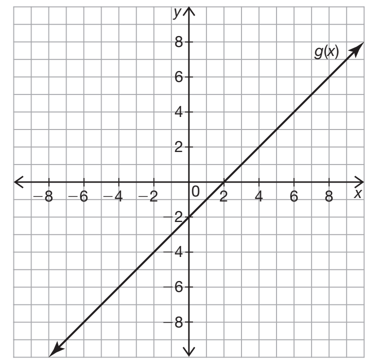
Students complete Question 1 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- Does $f(x)$ intersect the x-axis? How do you know?
- Where does $f(x)$ intersect the x-axis?
- Does $f(x)$ intersect the x-axis each time the sign of the output changes?
- How many times does $f(x)$ intersect the x-axis?

1. Toby compared the table of values for $f(x)$ and the graph of $g(x)$ to determine which polynomial function has the greater number of real zeros.

x	$f(x)$
−2	3
−1	−2
0	−5
1	−6
2	−5
3	−2
4	3



Toby

Function $g(x)$ has the greater number of real zeros. The graph has 1 zero at $x = 2$ while the table of values has no output value of 0, and therefore no zeros.

Is Toby correct? Explain your reasoning.

Toby is not correct.

Function $f(x)$ is a polynomial function with at least 2 zeros. I know that the function must cross the x-axis when the sign of the output values changes. Zeros occur between $x = -2$ and $x = -1$ as well as between $x = 3$ and $x = 4$.

Grouping

Have students complete Questions 2 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 2

- Why must a function with at least 1 absolute maximum and 1 absolute minimum have a degree greater than 2?
- How do you know the second function is quadratic?
- How are the two functions represented?
- Do imaginary zeros always come in pairs? Why?
- If a function has one imaginary zero, must it have two imaginary zeros?
- Do the values in the table imply $m(x)$ is a quadratic function? How?
- Does the function $p(x)$ have a common difference? What does this imply?
- Can a function have an odd number of imaginary zeros? Why not?

2. Analyze each pair of representations. Then, answer each question and justify your reasoning.

a. Which function has a greater degree?

A polynomial function $h(x)$ has 1 absolute maximum and 1 relative maximum.	$j(x) = -40(x - 7)^2 + 30x^2 - 17x + 1$

The function $h(x)$ has a greater degree.

A function with at least 1 absolute maximum and 1 absolute minimum must have degree greater than 2. The second function is quadratic.

b. Which function has a greater degree?

x	$m(x)$
−2	9
−1	3
0	1
1	3
2	9

A polynomial function $n(x)$ has a real zero and an imaginary zero.

The function $n(x)$ has a greater degree.

The function $m(x)$ is quadratic. Imaginary zeros are always in pairs, so the second choice must have at least 3 zeros.

c. Which function has a degree divisible by 2?

x	$p(x)$
−2	2
−1	4
0	6
1	8
2	10

The function $q(x)$ has only imaginary solutions.

The function $q(x)$ has degree divisible by 2.

The function $p(x)$ has a common first difference, so it is linear. Imaginary solutions are always in pairs. The Fundamental Theorem of Algebra states that the number of solutions is equal to the degree of the function. Therefore, the degree of $q(x)$ must be even.

Guiding Questions for Share Phase, Question 2, part (d) through Question 3

- Which of the two functions approaches infinity as x approaches infinity?
- How do the a-values help determine the end behavior?
- If the a-value is less than zero, do the output values approach positive or negative infinity? How do you know?
- How do you know the function $s(x)$ never intersects the x-axis?
- As x approaches infinity, does the function $s(x)$ approach positive or negative infinity? How do you know?
- As x approaches infinity, does the function $t(x)$ approach positive or negative infinity? How do you know?
- Are all functions with an absolute maximum or an absolute minimum considered even functions? Why?
- What degree polynomial is $g(x)$? How do you know?

d. Determine which function has the greater output as x approaches infinity.

An odd function $r(x)$ with $a < 0$.	$k(x) = x^6 + x^4 + 3x^2 + 5x - 10{,}000$

Function $k(x)$ has the greater output as x approaches infinity.

Function $k(x)$ is an even function with $a > 0$. Therefore, it approaches positive infinity as x approaches infinity. Function $r(x)$ is an odd function with $a < 0$, so the output values approach negative infinity.

e. Determine which function has the greater output as x approaches negative infinity.

$t(x) = -3(x - 4)^8 + 130$	A quartic function $s(x)$ with y-intercept (0, 5) and all imaginary roots.

Function $t(x)$ has the greater output as x approaches negative infinity.

Function $s(x)$ never crosses the x-axis. It approaches positive infinity as x approaches infinity. The negative a-value in function $t(x)$ results in the output approaching negative infinity as x approaches infinity.

3. Sam and Otis disagree when they compared the two functions shown to determine which one has an odd degree.

The function $f(x)$ has an absolute maximum value.	$g(x) = x^4(3 - x)(2x^2 + 3)(x^4 + 4)$

Sam

The function $f(x)$ has an odd degree because odd functions approach positive infinity as x either increases or decreases. This means $f(x)$ has a maximum value.

Otis

The function $g(x)$ has an odd degree. When I multiplied the factors, I got a term with a highest exponent of 11: $x^4(-x)(2x^2)(x^4) = -2x^{11}$. Therefore, $g(x)$ is odd.

Who is correct? Justify your reasoning.

Otis is correct.

Expanding $g(x)$ results in a polynomial with degree 11. Function $f(x)$ is an even function because it has an absolute maximum.

Problem 2

A worked example shows students how to compare a function written as an equation to a function represented by a table of values. Students will compare two functions written in different forms to determine which function has: a greater y-intercept, a greater average rate of change over a specified interval, a greater relative minimum, an axis of symmetry with a greater x-value, a greater minimum, a lower minimum, a greater output for a given input, and a greater input for a given output.

Grouping

Ask a student to read the information and worked example. Discuss as a class.

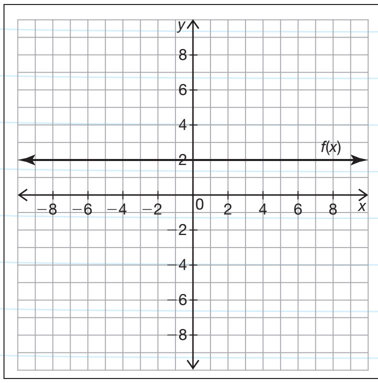
PROBLEM 2 Two Representations Are Better Than One

Many problems in mathematics are unique, without specific step-by-step algorithms that lead to an answer. In Problem 1, *The Best of Both Representations*, you mentally asked yourself a series of metacognitive questions to compare functions in different representations. As you consider additional questions in this lesson, it may be helpful to compare the problems to ones that you have already completed.

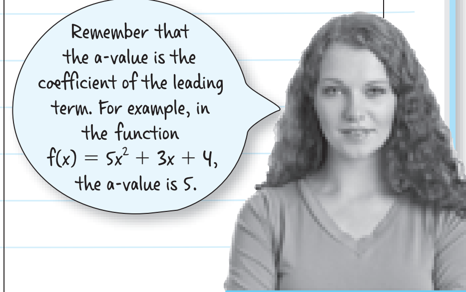
Ask yourself:

- *How is this problem the same or different than the previous ones that I have already solved?*
- *What do I know about the function that is given? What can I conclude that is not directly stated?*

Consider the representations shown. Which function has a greater y-intercept? Justify your reasoning.



A function $g(x)$ has an a-value less than zero and all roots have a multiplicity of 2.



Solution:

This problem is similar to previous problems in that you must consider functions with restrictions on the a-value and functions with multiple roots. The problem is also similar in that you must consider an output value for a given input. In this case, the input is 0.

In function $f(x)$, the output value is 2 for any given input. Analyzing function $g(x)$, the multiplicity 2 tells you that the function is even, and the negative a-value indicates that the function opens downward. The multiplicity of the roots also tells you that the function does not cross the x-axis. Instead, it reflects at a given point where the double root occurs.

Comparing the two functions, you know that function $g(x)$ is always below the x-axis and function $f(x)$ is above the x-axis. Therefore, $f(x)$ has a greater y-intercept.

Grouping

Students complete Questions 1 through 3 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 1

- How is the y-intercept of function $g(x)$ calculated?
- What is the y-intercept of function $g(x)$?
- Did Tina multiply the binomials in $g(x)$ before identifying the y-intercept?

1. Isaac and Tina disagree over which function has a greater y-intercept.

$g(x) = 2(x - 2)(x + 2)(x - 3) - 4$

x	$h(x)$
−2	−2
−1	0
0	4
1	10
2	18

Isaac

Function g(x) has a greater y-intercept. I calculated the y-intercept by substituting 0 for x. This value is greater than (0, 4) shown in the table for the function h(x).

Tina

Function h(x) has a greater y-intercept. The y-intercept of h(x) is (0, 4) and the y-intercept of g(x) is (0, −4).

Who is correct? Justify your reasoning.

Isaac is correct.

The y-intercept of function $h(x)$ is (0, 4). The y-intercept of function $g(x)$ is calculated by substituting 0 into the function, producing $2(-2)(2)(-3) - 4 = 20$. Tina falsely assumed the y-intercept is the constant that is subtracted and did not multiply the binomials in $g(x)$.

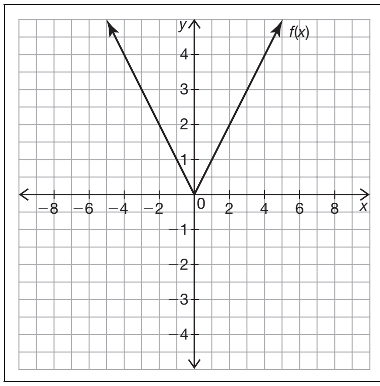
7

Guiding Questions for Share Phase, Question 2, parts (a) and (b)

- What is the average rate of change associated with the function $f(x)$?
- How did you determine the average rate of change for $k(x)$?
- What is the average rate of change associated with the function $j(x)$?
- Is the average rate of change associated with the function $j(x)$ positive or negative?

2. Analyze each pair of representations. Then, answer each question and justify your reasoning.

   a. Which function has a greater average rate of change for the interval $(-4, 4)$?



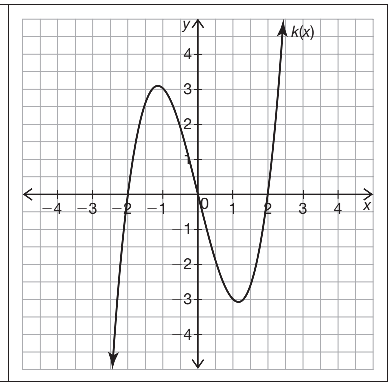
$g(x) = x$

The function $g(x)$ has a greater average rate of change.

The function $f(x)$ has an average rate of change of 0 while the function $g(x)$ has an average rate of change of 1 unit.

   b. Which function has a greater average rate of change for the interval $(-1, 1)$?

x	$j(x)$
−2	4
−1	1
0	0
1	1
2	4



Function $j(x)$ has a greater average rate of change.

The average rate of change for the function $j(x)$ is 0. The function $k(x)$ has a negative average rate of change.

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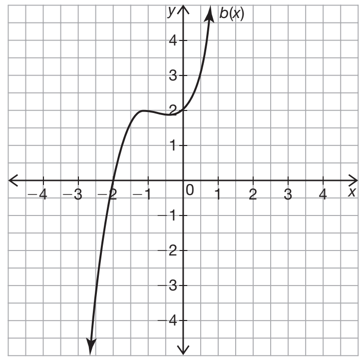
Guiding Questions for Share Phase, Question 2, parts (c) and (d)

- Is the relative minimum of function $b(x)$ located above or below the x-axis?
- Is the relative minimum of function $a(x)$ located above or below the x-axis? How do you know?
- What is the axis of symmetry of the function $m(x)$?
- What formula is used to determine the axis of symmetry?
- What is the axis of symmetry of the function $n(x)$?

c. Which function has a greater relative minimum?

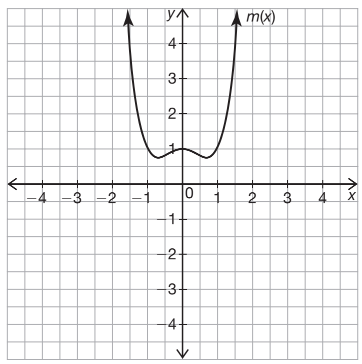
A cubic function $a(x)$ with $a > 0$ and 3 distinct real roots.



Function $b(x)$ has a greater relative minimum.

The relative minimum of function $b(x)$ is above the x-axis, while the relative minimum of function $a(x)$ occurs below the x-axis.

d. Which function's axis of symmetry has a greater x-value?



$n(x) = x^2 - 3x + 1$

Function $n(x)$ has a greater axis of symmetry.

The axis of symmetry of function $m(x)$ is $x = 0$. The axis of symmetry of function $n(x)$ is $x = \frac{-b}{2a} = \frac{3}{2}$.

Guiding Questions for Share Phase, Question 3

- Is the output of $m(x)$ negative? How do you know?
- What is the minimum value for the function $d(x)$?

Grouping

- Ask a student to read the information. Discuss as a class.
- Students complete Question 4 with a partner. Then have students share their responses as a class.

Guiding Questions for Share Phase, Question 4, part (a)

If the function $g(x)$ is shifted vertically up 1 unit, does that result in a greater output for a given input value?

3. Emilio studied the table of values and description of the key characteristics to determine which function has a greater minimum.

x	$d(x)$
−2	5
−1	2
0	1
1	2
2	5

A quartic function $m(x)$ has $a < 0$ and 2 pairs of real zeros (multiplicity 2).

Emilio

Function $d(x)$ has a greater minimum. This function is a parabola opening up, with its vertex at (0, 1). Function $m(x)$ opens down because $a < 0$. Since the real zeros have multiplicity 2, I know any real zeros occur when the function reflects off the x-axis. Therefore, the output values of $m(x)$ never reach a point greater than $y = 0$.

Is Emilio correct? Justify your reasoning.

Yes. Emilio's reasoning is correct because the output of $m(x)$ must be negative while the minimum value for function $d(x)$ is (0, 1).

Recall that a basic function is a function in its simplest form. The basic function of *a* is $f(x) = x^n$ for any natural number *n*. Transformations of the basic functions are performed by changing the A-, B-, C-, and D-values in the form $g(x) = Af(B(x - C)) + D$. Remember, each value describes different transformations of the graph: the A-value vertically stretches or compresses the graph, the B-value horizontally stretches or compresses the graph, the C-value horizontally shifts the graph right or left, and the D-value vertically shifts the graph up or down.

4. Analyze the transformations of the basic functions. Then answer each question and justify your reasoning.

   a. Which function has a greater output for a given input?

The basic quadratic function $f(x) = x^2$.	$g(x) = f(x - 2) + 1$

   The function $g(x)$ has a greater output for a given input.

   The transformation translates the function up 1, so the output values increase.

7

Guiding Questions for Share Phase, Question 4, parts (b) and (c)

- If the function $k(x)$ is shifted vertically up 2 units, does that result in a lower minimum value?
- If the function $g(x)$ is shifted to the right 5 units, does that result in a greater x-value?

b. Which function has a lower minimum?

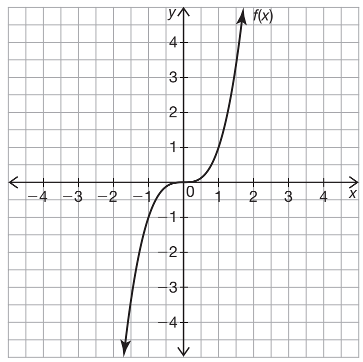
x	$j(x)$
−2	16
−1	1
0	0
1	1
2	16

$k(x) = 5f(x - 4) + 2$

Function $j(x)$ has a lower minimum.

The transformation in $k(x)$ translates the minimum output up 2 units.

c. Which function has the greater input for a given output value?



$g(x) = 3f(x - 5) + 1$

Function $g(x)$ has the greater input for a given output.

For a given y-value, the function $g(x)$ is translated to the right 5 units, so the x-value is greater.

Be prepared to share your solutions and methods.

Check for Students' Understanding

1. Create an equation for a function that has 5 zeros and a y-intercept at (0, −7).

   Answers will vary.

   $f(x) = x^5 - 2x^2 + 5x - 7$

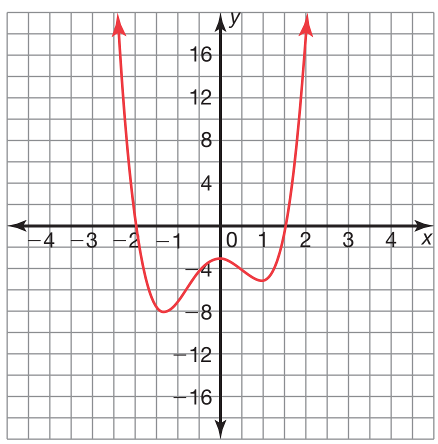
2. Create a table for a function that has 2 imaginary zeros, and passes through the point (0, 8).

   Answers will vary.

x	$f(x)$
−3	17
−2	12
−1	9
0	8
1	9
2	12
3	17

3. Create a graph for a function that has 2 real zeros, 2 imaginary zeros, and 2 relative minimums.

   Answers will vary.



7

Chapter 7 Summary

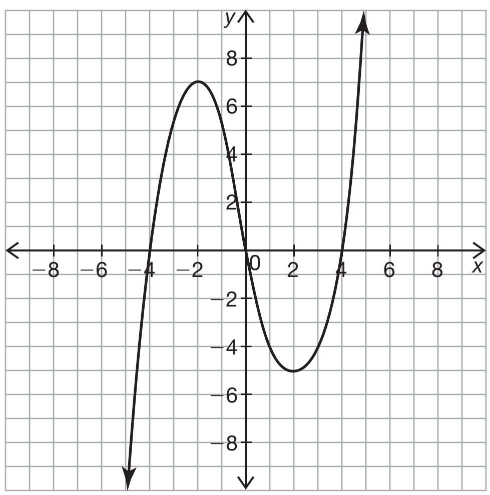
KEY TERMS

- regression equation (7.2)
- coefficient of determination (7.2)
- piecewise function (7.3)

7.1 Determining When a Polynomial Function is Greater Than and When it is Less Than 0 Using Its Roots

To determine when a polynomial function is greater than or less than 0, first determine the zeros of the function or where the function equals 0. Then use the graph to determine whether the function is greater than or less than 0 for the intervals between the zeros.

Example



$f(x) < 0$ when $\begin{Bmatrix} x < -4 \\ 0 < x < 4 \end{Bmatrix}$

$f(x) > 0$ when $\begin{Bmatrix} -4 < x < 0 \\ x > 4 \end{Bmatrix}$

7.1 Determining the Solution to Polynomial Inequalities Using a Graphing Calculator

You can use a graphing calculator to solve higher order polynomials that are not easily factorable.

Step 1: Press **Y** = and input the expression.

Step 2: Scroll to the left of the Y_1, when your cursor is blinking over the diagonal line, press **ENTER** 2 times, you will see the area above the diagonal shaded (this represents $y \geq$ expression). If you Press **ENTER** 1 more time, you will see the area below the diagonal line shaded (this represents $y \leq$ expression).

Step 3: Make sure your viewing window is appropriate and press **GRAPH**.

Step 4: To determine the particular values of x that makes the inequality true, press 2nd, **CALC, 2:ZERO**. Scroll to the appropriate bounds to determine the zeros.

Step 5: Determine if x must be greater than or less than the roots depending on the inequality sign for your solution.

Example

$12 < x^2 - 2x + 3$

$x < -2.16$ or $x > 4.16$

7.1 Determining the Solution to Polynomial Inequalities Algebraically and Graphically

When solving polynomial inequalities treat the inequality as an equation and solve. Factor or use the quadratic formula to determine the x-intercepts. Then choose a test point between each interval created by the roots or graph the equation to determine which values satisfy the inequality. The section(s) that provide a true solution for the test point is the solution to the inequality.

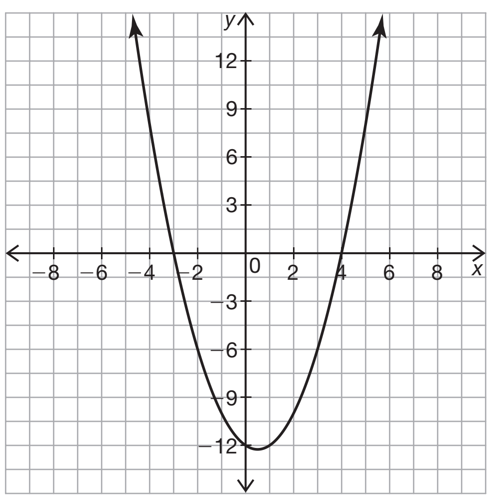
Example

$x^2 - x - 12 < 0$

$(x + 3)(x - 4) = 0$

$x = -3, 4$

$x^2 - x - 12 < 0$ when $-3 < x < 4$



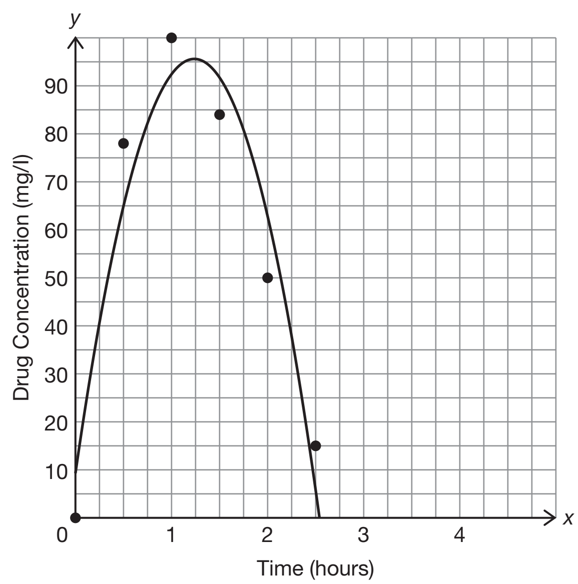
7.2 Determining the Appropriate Regression Equation to Model a Problem

Analyze data as a scatter plot to identify any patterns in the data. Based on how the data increases or decreases, determine the type of polynomial that best fits the data. Then, use a graphing calculator to determine the regression equation. A regression equation is a function that models the relationship between 2 variables in a scatter plot. Generally, there is 1 curve or degree of polynomial that will best fit the data. The coefficient of determination measures the strength of the relationship between the original data and the regression equation. The value ranges from 0 to 1 with a value of 1 indicating a perfect fit between the curve and the original data.

Example

The table shows the concentration of medication in a patient's blood as time passes.

Time (hours)	Concentration (mg/l)
0	0
0.5	78
1	100
1.5	84
2	50
2.5	15



The data could be represented by a quadratic equation.

Regression equation:
$y = -56.357x^2 + 139.464x + 9.321$

Coefficient of determination: 0.924

The curve is a pretty good fit for the data.

7.2 Predicting Outcomes Using a Regression Equation

The regression equation is often used to make predictions about past and future events. Substitute various inputs into the regression equation to determine the likely outputs. Or, use intersecting lines to determine inputs.

Example

The medicine is considered at its most effective when the concentration in the blood is at least 60 mg/l. About for how long after administering is the medicine most effective? Use the regression equation: $g(x) = -56.357x^2 + 139.464x + 9.321$ where x is time in hours and $g(x)$ is the concentration of the medicine in the blood in mg/l.

The drug is most effective between 0.44 hour after administering and 2.03 hours after administering.

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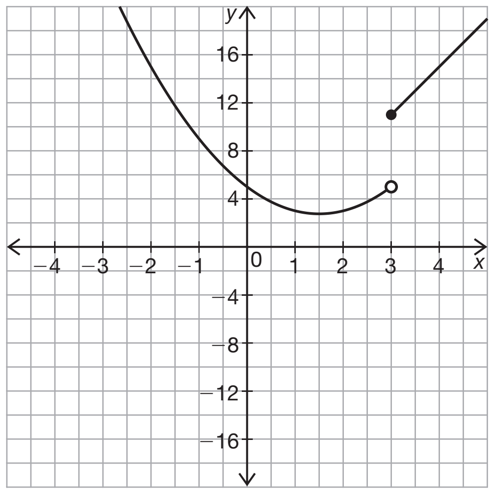
7.3 Graphing a Piecewise Function

A piecewise function includes different functions that represent different parts of the domain. Sometimes a single polynomial function is not the best model for a set of data. Data with breaks in the patterns can be better modeled by separating the data into pieces where each piece is modeled by a different polynomial function. Each equation can be graphed for its domain. Open points are associated with $<$ and $>$ and closed points are associated with $\leq$ and $\geq$.

Example

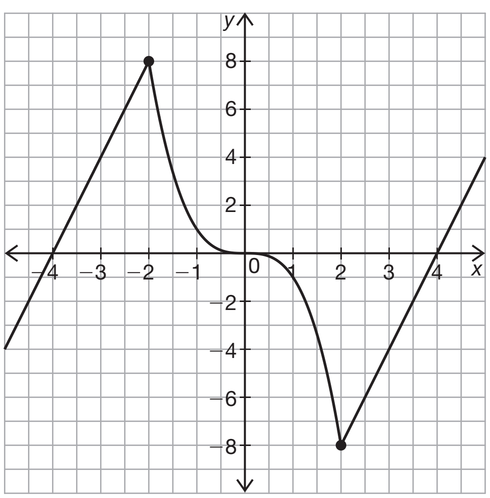
$$d(x) = \begin{cases} x^2 - 3x + 5, & x < 3 \\ 4x - 1, & x \geq 3 \end{cases}$$



7.3 Writing a Piecewise Function Based on Its Graph

A regression equation can be determined for each interval on the graph of a piecewise function given with the appropriate domain.

Example



$$c(x) = \begin{cases} 4x + 16, & x \leq -2 \\ -x^3, & -2 < x < 2 \\ 4x - 16, & x \geq 2 \end{cases}$$

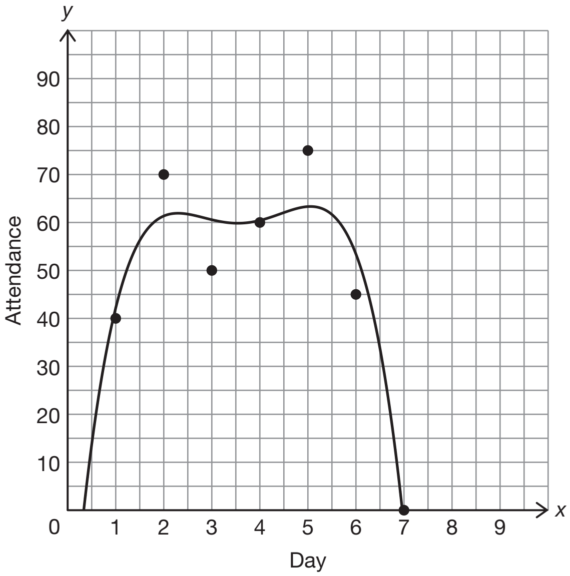
7

7.4 Modeling a Problem Situation with a Polynomial Function

Examine data in a table or scatter plot for patterns to determine what type of polynomial would best match the data. Use a graphing calculator to determine a regression equation to best model the data.

Example

Day	Attendance
1	40
2	70
3	50
4	60
5	75
6	45
7	0



The data increases, decreases, increases, and decreases. A quartic function would best model the data.

Regression equation: $y = -0.758x^4 + 11.01x^3 - 57.083x^2 + 124.72x - 35.714$

7.4 Solving Problems Using Polynomial Regression Equations

Use the regression equation for the data to answer questions and make predictions about the data. The vertex, intersection of lines, or table of values can each be useful for solving problems about the data.

Example

Attendance at a local museum fluctuates throughout the week according to the regression equation $f(x) = -0.758x^4 + 11.01x^3 - 57.083x^2 + 124.72x - 35.714$, where x is the day and $f(x)$ is attendance. If the days 1–7 correspond to days of the week Monday through Sunday, how many people should the museum plan to expect on a typical Wednesday?

The museum can expect about 60–61 people on a typical Wednesday.

7.5 Comparing Polynomials Using Different Representations

Polynomials can be represented using a graph, table of values, equation, or description of key characteristics. When comparing 2 functions in different forms, important information to look for includes the degree of each function, the shape of the graph, the number and type of zeros, transformations of a basic function, etc.

Example

Which polynomial function has an even degree?

$a(x)$	$b(x)$
A polynomial function with 2 absolute maximums and 1 relative minimum.	$b(x) = 4(3 - 2x) + 3(x + 6)$

The function $a(x)$ has an even degree. A function with 3 turns must have a degree greater than 3. And, having absolute maximums means the end behavior of the function is to approach negative infinity as x approaches both negative and positive infinity. This indicates an even degree function. The function $b(x)$ is a linear function—the x-values are added, not multiplied.