

Trigonometry

Angle Measurement

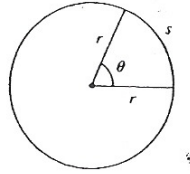
$$\pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$s = r\theta$$

(θ in radians)



Right-Angle Trigonometry

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

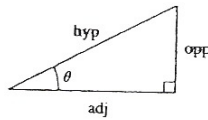
$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$



Trigonometric Functions

$$\sin \theta = \frac{y}{r}$$

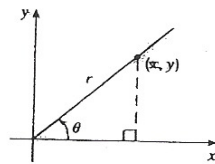
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

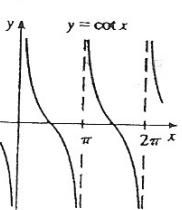
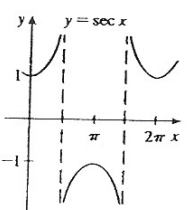
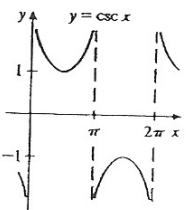
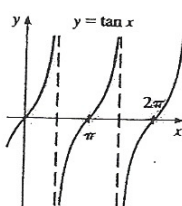
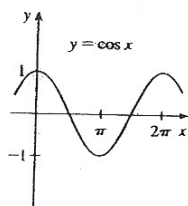
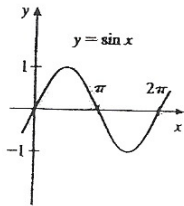
$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



Graphs of Trigonometric Functions



Trigonometric Functions of Important Angles

| θ | radians | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ |
|------------|---------|---------------|---------------|---------------|
| 0° | 0 | 0 | 1 | 0 |
| 30° | $\pi/6$ | $1/2$ | $\sqrt{3}/2$ | $\sqrt{3}/3$ |
| 45° | $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 |
| 60° | $\pi/3$ | $\sqrt{3}/2$ | $1/2$ | $\sqrt{3}$ |
| 90° | $\pi/2$ | 1 | 0 | — |

Fundamental Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

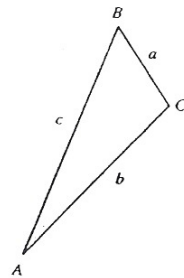
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Formulas

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$