# 2014 Mathletics Contest Calculus AB

1. Evaluate the limit
$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x + 9}$$

   a) DNE    b) $\frac{1}{6}$    c) $\frac{1}{7}$    d) 1    e) none of these

2. A dairy farmer with 20 cows gets an average of 3 gallons of milk from each cow per day. The farmer noticed that for each additional cow added to the herd, the average production drops by 0.2 gallons per cow. How many cows should the farmer have in order to maximize total milk production?

   a) 16    b) 17    c) 20    d) 27    e) none of these

3. For what value of $x$ does the graph of $y = \dfrac{1}{\sqrt{x}}$ have a tangent line parallel to the line $x + 16y = 5$?

   a) 2    b) $\dfrac{1}{\sqrt{2}}$    c) 4    d) $\sqrt{2}$    e) None of these

4. If $f(x) = \cos^2(4 + 3x)$, then $f'(1)$ is equal to which of the following?

   a) $-6\cos(7)\sin(7)$    b) $2\cos(7)$    c) $-2\cos(7)\sin(7)$    d) $6\sin(7)$    e) none of these

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5. The position of a particle moving along a line is given by
$s(t) = 2t^3 - 24ct^2 + 90c^2t + 7$, $t \geq 0$, with $c$ a positive constant. For what values of $t$ is the speed of the particle increasing?

   a) $3c < t < 4c$ only    b) $t > 4c$ only    c) $t > 5c$ only

   d) $0 < t < 3c$ and $t > 5c$    e) none of these

6. What is the sum of all critical values of $\ln(x) + \ln(y) = 2x$?

    $a)$ 0     $b)$ $\dfrac{1}{2}$     $c)$ $\dfrac{\ln(2)}{2}$     $d)$ $\dfrac{2e-1}{2}$     $e)$ None of these

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7. What is the maximum value of $f(x) = \sin(\ln(x))$ on the interval $[1, 10]$?

    $a)$ 0     $b)$ $e^{\frac{\pi}{2}}$     $c)$ $e^{-\frac{\pi}{2}}$     $d)$ $\frac{\pi}{2}$     $e)$ none of these

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8. The line tangent to $h(x) = x^3 - 5x^2 - x + 3$ at the point $(1, -2)$ has an $x$–intercept of

    $a)$ 0     $b)$ 0.75     $c)$ 3     $d)$ 6     $e)$ none of these

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9. Find the sum of the critical values of the function $f(x) = e^x \cos(x)$ over the interval $(0, 2\pi)$.

    $a)$ $4\pi$     $b)$ $\pi$     $c)$ $\frac{3\pi}{2}$     $d)$ 0     $e)$ none of these

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10. Find the inflection point of $y = (x + 1)\tan^{-1}(x)$.

    $a)$ 1     $b)$ $\dfrac{-\pi}{4}$     $c)$ 0     $d)$ $\dfrac{\pi}{4}$     $e)$ It has no inflection point

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