Review WS Standard 8

1) $\cos x + \sin x \tan x = \sec x$

$\cos x + \sin x \cdot \frac{\sin x}{\cos x} = \sec x$

$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = \sec x$

$\frac{\cos^2 x + \sin^2 x}{\cos x} = \sec x$

$\boxed{\frac{1}{\cos x} = \sec x}$

2) $\frac{2\tan x}{1 + \tan^2 x} = \sin 2x$

$\frac{2\tan x}{\sec^2 x} = \sin 2x$

$\frac{\frac{2\sin}{\cos x}}{\frac{1}{\cos^2 x}} = \sin 2x$

$\frac{2\sin x}{\cos x} \cdot \frac{\cos^2 x}{1} = \sin 2x$

$\boxed{2\sin x \cdot \cos x = \sin 2x}$

3) $6 \sin x \cdot \cos x = 3 \sin 2x$

$3(2\sin x \cdot \cos x) = 3\sin 2x$

$\boxed{2\sin x \cdot \cos x = \sin 2x}$

4) $\sin(\theta+60^\circ)-\cos(\theta+30^\circ)=\sin\theta$

$\sin\theta\cos60^\circ+\cos\theta\sin60^\circ-(\cos\theta\cos30-\sin\theta\sin30^\circ)=\sin\theta$

$\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta-\left(\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)=\sin\theta$

$\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta-\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta=\sin\theta$

$\boxed{\sin\theta=\sin\theta}$

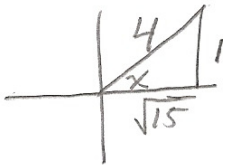
5) $\cos75^\circ=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos(45^\circ+30^\circ)=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos45\cos30-\sin45\sin30^\circ=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}}$

6) a) 

$\sin x=\frac{1}{4}$

b) $\cos x=\frac{\sqrt{15}}{4}$

c) $\tan x=\frac{1}{\sqrt{15}}$

d) $\sin2x=2\sin x\cos x$

$=2\left(\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right)$

$\boxed{\sin2x=\frac{\sqrt{15}}{8}}$

e) $\cos2x=\cos^2x-\sin^2x$

$\cos2x=\left(\frac{\sqrt{15}}{4}\right)^2-\left(\frac{1}{4}\right)^2$

$=\frac{15}{16}-\frac{1}{16}$

$=\frac{14}{16}$

$\boxed{\cos2x=\frac{7}{8}}$

f) next page

(6 f) $\sin\left(x - \frac{\pi}{3}\right) = \sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}$

$= \frac{1}{4}\left(\frac{1}{2}\right) - \frac{\sqrt{15}}{4}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{8} - \frac{3\sqrt{5}}{8}$

$$\boxed{\sin\left(x - \frac{\pi}{3}\right) = \frac{1 - 3\sqrt{5}}{8}}$$

g) $\cos\left(\frac{1}{2}x\right) = \sqrt{\frac{1 + \cos x}{2}}$

$= \sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}}$

$\cos\left(\frac{1}{2}x\right) = \sqrt{\frac{\frac{4 + \sqrt{15}}{4}}{2}}$

$$\boxed{\cos\left(\frac{1}{2}x\right) = \sqrt{\frac{4 + \sqrt{15}}{8}}}$$

h) $\sin x = \frac{1}{4}$

$x = \sin^{-1}\left(\frac{1}{4}\right)$

$$\boxed{x = 0.253}$$

7) Prove: $\frac{1}{2}(1+\cos 2A) = \cos^2 A$

$\frac{1}{2}(1+2\cos^2 A - 1) = \cos^2 A$

$\frac{1}{2}(2\cos^2 A) = \cos^2 A$

$\boxed{\cos^2 A = \cos^2 A}$

8) a) $4\sin\theta\cos\theta = \sqrt{3}$

$2(2\sin\theta\cos\theta) = \sqrt{3}$

$2\sin\theta\cos\theta = \frac{\sqrt{3}}{2}$

$\sin 2\theta = \frac{\sqrt{3}}{2}$ for $0 \le \theta \le 720$

$2\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2\theta = 60°, 120°, 420°, 480°$

$\boxed{\theta = 30°, 60°, 210°, 240°}$

b) $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ $\quad$ $\cos x - 1 = 0$

$2\cos x = -1$ $\quad$ $\cos x = 1$

$\cos x = \frac{-1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$

8 c) $1 - 2\sin^2\theta = \frac{1}{2}$

$-2\sin^2\theta = -\frac{1}{2}$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$\theta = 30°, 150°, 210°, 330°$

d) $\frac{\sin x}{1+\cos x} = \sqrt{3}$

$\tan\frac{x}{2} = \sqrt{3}$

$\frac{x}{2} = \tan^{-1}\sqrt{3}$

$\frac{x}{2} = \frac{\pi}{3}, \frac{4\pi}{3}$

$x = \frac{2\pi}{3}, \frac{8\pi}{3}$

9) $\cos x = 0$

$x = \cos^{-1} 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos 2x = 0$

$2x = \cos^{-1} 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\cos x \neq \cos 2x$
therefore it is false

(10) $\cos 4\theta - \sin 2\theta = 0 \qquad \theta \in [0, 180]$

$\cos(2[2\theta]) - \sin 2\theta = 0$

$1 - 2\sin^2 2\theta - \sin 2\theta = 0$

$-1 + 2\sin^2 2\theta + \sin 2\theta = 0$

$2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$(2\sin 2\theta - 1)(\sin 2\theta + 1) = 0$

$2\sin 2\theta - 1 = 0 \qquad \sin 2\theta + 1 = 0$

$\sin 2\theta = \frac{1}{2} \qquad \sin 2\theta = -1$

$2\theta = \sin^{-1}\left(\frac{1}{2}\right) \qquad 2\theta = \sin^{-1}(-1)$ For $0 \le \theta < 360$

$2\theta = 30°, 150° \qquad 2\theta = 270$

$\theta = 15°, 75° \qquad \theta = 135°$

$\boxed{\theta = 15°, 75°, 135°}$

(11) $\sqrt{3}\sin x - \cos x = 1$

$\sqrt{3}\sin x = 1 + \cos x$

$\sqrt{3} = \frac{1 + \cos x}{\sin x}$

$\frac{1}{\sqrt{3}} = \frac{\sin x}{1 + \cos x}$

$\frac{\sqrt{3}}{3} = \tan\frac{x}{2}$

$\frac{x}{2} = \tan^{-1}\frac{\sqrt{3}}{3}$

$\frac{x}{2} = \frac{\pi}{6}$ For all real #

$\boxed{x = \frac{\pi}{3} + \pi n}$

(12) $2\cos x - 2\cos^3 x = \sin x \cdot \sin 2x$

$2\cos x(1 - \cos^2 x) = \sin x \cdot \sin 2x$

$2\cos x \cdot \sin^2 x = \sin x \cdot 2\sin x \cos x$

$\boxed{2\cos x \cdot \sin^2 x = 2\cos x \sin^2 x}$