Solutions to Standard 8 Practice #2

① $$\frac{\sin 2x}{2\cos^2 x} = \tan x$$

$$\frac{2\sin x\cos x}{2\cos^2 x} = \tan x$$

$$\frac{\sin x}{\cos x} = \tan x$$

$$\tan x = \tan x \checkmark$$

② $$\frac{\sin t}{1-\cos t} = \frac{1+\cos t}{\sin t}$$

$$\sin^2 t = (1-\cos t)(1+\cos t)$$

$$\sin^2 t = 1 - \cos^2 t$$

$$\sin^2 t = \sin^2 t \checkmark$$

③ $$\cos^4 t - \sin^4 t = 1 - 2\sin^2 t$$

Factor

$$(\cos^2 t + \sin^2 t)(\cos^2 t - \sin^2 t) = 1 - 2\sin^2 t$$

$$1\ (\cos^2 t - \sin^2 t) = 1 - 2\sin^2 t$$

$$\cos 2t = \cos 2t \checkmark$$

④ $$\sec x - \cos x = \sin x \tan x$$

$$\frac{1}{\cos x} - \cos x = \sin x \cdot \frac{\sin x}{\cos x}$$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} \checkmark$$

(5) $\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

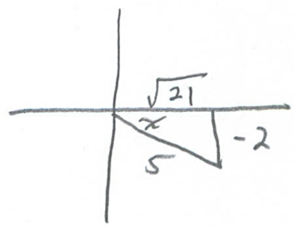
$\sin(45^\circ - 30^\circ) =$

$\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ =$

$\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} =$

$\dfrac{\sqrt{6}}{4} - \dfrac{\sqrt{2}}{4} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ ✓

(6) $\sin x = \dfrac{-2}{5}$



a) $\cos x = \boxed{\dfrac{\sqrt{21}}{5}}$

b) $\tan x = \dfrac{-2}{\sqrt{21}} = \boxed{\dfrac{-2\sqrt{21}}{21}}$

c) $\sin 2x = 2 \sin x \cos x$

$= 2\left(\dfrac{-2}{5}\right)\left(\dfrac{\sqrt{21}}{5}\right)$

$\boxed{\sin 2x = \dfrac{-4\sqrt{21}}{25}}$

d) $\tan \dfrac{x}{2} = \dfrac{1 - \cos x}{\sin x}$

$= \dfrac{1 - \frac{\sqrt{21}}{5}}{-\frac{2}{5}}$

$= -\dfrac{5}{2}\left(1 - \dfrac{\sqrt{21}}{5}\right)$

$= \dfrac{-5}{2} + \dfrac{\sqrt{21}}{2}$

$\boxed{= \dfrac{-5+\sqrt{21}}{2}}$

e) $\cos\left(x + \dfrac{\pi}{4}\right) = \cos x \cos \dfrac{\pi}{4} - \sin x \sin \dfrac{\pi}{4}$

$= \dfrac{\sqrt{21}}{5} \cdot \dfrac{\sqrt{2}}{2} - \left(\dfrac{-2}{5}\right) \cdot \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{42}}{10} + \dfrac{2\sqrt{2}}{10}$

or $= \dfrac{\sqrt{42} + 2\sqrt{2}}{10}$

f) $\tan\left(x - \dfrac{\pi}{4}\right) = \dfrac{\tan x - \tan \frac{\pi}{4}}{1 + \tan x \cdot \tan \frac{\pi}{4}}$

$\begin{matrix}\sqrt{21} \\ \sqrt{21}\end{matrix} = \left[\dfrac{\frac{-2}{\sqrt{21}} - 1}{1 + \frac{-2}{\sqrt{21}}}\right]$

$\boxed{= \dfrac{-2 - \sqrt{21}}{\sqrt{21} - 2}}$

g) $x = \sin^{-1}\left(\dfrac{-2}{5}\right)$

$\boxed{x \approx 336.4^\circ}$

h) $\sin^2 x + \cos^2 x = \boxed{1}$

7 (a) $2\cos x + \sin x = 0$

$2\cos x = -\sin x$

$2 = \frac{-\sin x}{\cos x}$

$-2 = \tan x$

$x = \tan^{-1}(-2)$

$x = -63.43$

$\boxed{x = 116.6^\circ + 180^\circ n}$

(b) $4\cos^2\theta - 4\sin^2\theta = 2$

$\cos^2\theta - \sin^2\theta = \frac{1}{2}$

$1 - \sin^2\theta - \sin^2\theta = \frac{1}{2}$

$1 - 2\sin^2\theta = \frac{1}{2}$

$\cos 2\theta = \frac{1}{2}$

$2\theta = \cos^{-1}(\frac{1}{2})$

$2\theta = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

$\boxed{\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ}$

$\boxed{\begin{array}{l} \theta = 30^\circ + 180^\circ n \\ \theta = 150^\circ + 180^\circ n \end{array}}$

(c) $\sin 2x = \sin x$

$2\sin x \cos x = \sin x$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \qquad 2\cos x - 1 = 0$

$\boxed{x = 180^\circ n \qquad \begin{array}{l} x = 60^\circ + 360^\circ n \\ x = 300^\circ + 360^\circ n \end{array}}$

(d) $\cos 2x + 5\sin x = -2$

$1 - 2\sin^2 x + 5\sin x = -2$

$-2\sin^2 x + 5\sin x + 3 = 0$

$2\sin^2 x - 5\sin x - 3 = 0$

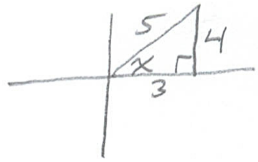
$(2\sin x + 1)(\sin x - 3) = 0$

$2\sin x + 1 = 0 \qquad \sin x - 3 = 0$

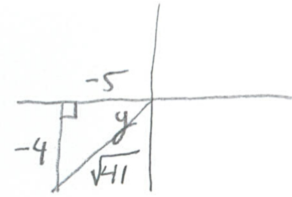
$\sin x = -\frac{1}{2} \qquad \cancel{\sin x = 3}$

$\boxed{\begin{array}{l} x = 150^\circ + 360 n \\ x = 210^\circ + 360 n \end{array}}$

8 $\cos x = 0.6$



$\tan y = \frac{4}{5}$



a $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$= \frac{4}{5}\left(\frac{-5}{\sqrt{41}}\right) + \frac{3}{5}\left(\frac{-4}{\sqrt{41}}\right)$$

$$= \frac{-20}{5\sqrt{41}} + \frac{-12}{5\sqrt{41}}$$

$$\boxed{= \frac{-32}{5\sqrt{41}}}$$

b $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$$= \frac{\frac{4}{3} - \left(\frac{4}{5}\right)}{1 + \left(\frac{4}{3}\right)\left(\frac{4}{5}\right)}$$

$$= \frac{\frac{20}{15} - \frac{12}{15}}{1 + \frac{16}{15}}$$

$$= \frac{\frac{8}{15}}{\frac{31}{15}} = \frac{8}{15} \cdot \frac{15}{31} = \frac{8}{31}$$

$$\boxed{\tan(x-y) = \frac{8}{31}}$$

c $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$$= \frac{3}{5} \cdot \left(\frac{-5}{\sqrt{41}}\right) - \left(\frac{4}{5}\right)\left(\frac{-4}{\sqrt{41}}\right)$$

$$\boxed{= \frac{1}{5\sqrt{41}} \text{ or } \frac{5\sqrt{41}}{205}}$$

⑨ a) $A = 30^\circ$ $B = 45^\circ$

a) $\cos^2 A - \sin^2 A$

$= (\cos 30)^2 - (\sin 30^\circ)^2$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} - \frac{1}{4}$

$\boxed{= \frac{1}{2}}$

b) $\sin A \cos B + \cos A \sin B$

$\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\boxed{= \frac{\sqrt{2} + \sqrt{6}}{4}}$

c) $\cos 2B + \sin 2A$

$\cos^2 B - \sin^2 B + 2 \sin A \cos A$

$(\cos 45^\circ)^2 - (\sin 45^\circ)^2 + 2 \sin 30^\circ \cos 30^\circ$

$0 + 2\left(\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2}$

$\boxed{= \frac{\sqrt{3}}{2}}$

*There is a <u>much</u> easier way ☺

d) $3 \sin^2 A + 4 \sin A - 2$

$= 3(\sin 30^\circ)^2 + 4 \sin 30^\circ - 2$

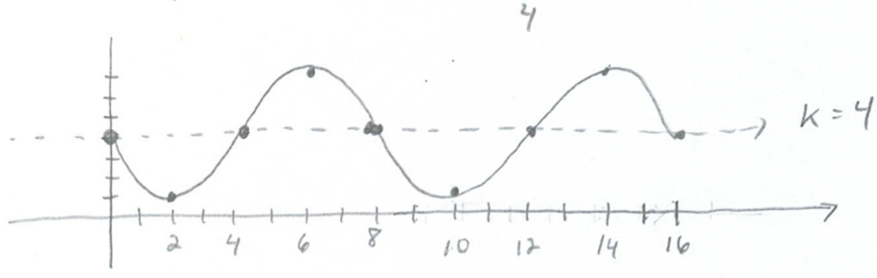
$= 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 2$

$= \frac{3}{4} + 2 - 2$

$\boxed{= \frac{3}{4}}$

⑩ $y = 4 - 3 \sin \frac{\pi}{4} x$ period $= \frac{2\pi}{\frac{\pi}{4}} = 8$ Amp $= 3$ $K = 4$

ⓐ



ⓑ $y(4.8) \approx 5.763$

ⓒ $x = 0.929, 3.071, 8.929$ when $y = 2$

⑪ $\cos 3\theta = 0$

$3\theta = \cos^{-1}(0)$

$3\theta = 90^\circ, 270^\circ, 450^\circ, 630^\circ, 810^\circ, 990^\circ$

$\boxed{\theta = 30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ, 330^\circ}$