Math Analysis Release Items 2015

Open Ended Question

(a) Are there any real solutions to the given equation? If yes, find all possible solutions. If not, justify why not.

$(a^4 - 2a^2b^2 + b^4)^{x-1} - \dfrac{(a-b)^{2x}}{(a+b)^2} = 0$ where $a > b > 0$ are two real numbers.

The distance between a point $P(x, y)$ and point $Q(0, 3)$ is $\frac{3}{2}$ of the distance between the point $P(x, y)$ and the line $y = -2$. Find the equation of the curve on which the point P lies.

(a) $4x^2 - 5y^2 - 60y = 0$ (b) $4x^2 + 5y^2 - 60y + 9 = 0$ (c) $4x^2 = 5y + 9$
(d) $x^2 - y^2 - 12y - 9 = 0$ (e) $4x^2 - 5y^2 - 60y - 27 = 0$

The vertex of the parabola $y = -2x^2 + 4x + 6$ is

(a) $(-1, 3)$ (b) $(1, -8)$ (c) $(1, 8)$ (d) $(-1, 0)$ (e) $(3, 0)$

Consider the graph of the equation $9x^2 + 4y^2 = 36$. Find the equation of the graph obtained by rotating the given graph by an angle $90°$ in the counter clockwise direction.

(a) $\frac{x^2}{36} + y^2 = 1$ (b) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (c) $\frac{x^2}{9} + \frac{y^2}{4} = 1$
(d) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (e) None of the above

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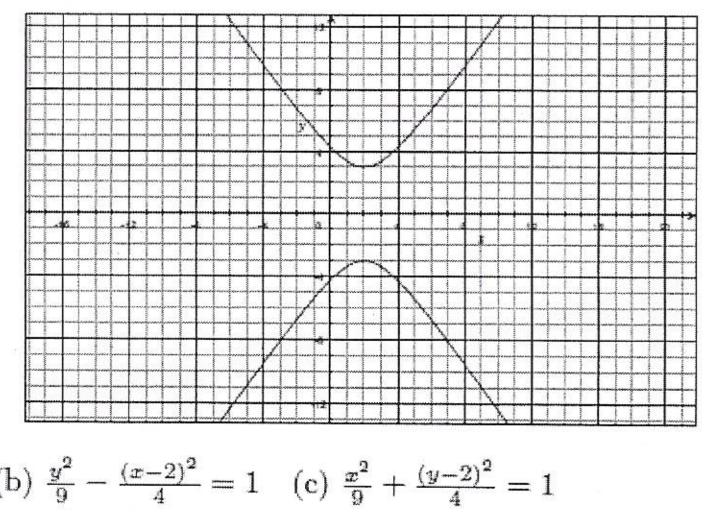
The domain of a rational function is all real numbers not equal to $-2, 2$ and 5 and the function eventually grows to positive infinity. Which of the following is a possible expression for this rational function?

(a) $f(x) = \dfrac{x^3+1}{x(x^2-4)(x-5)}$ (b) $f(x) = \dfrac{x^5+2x-1}{(x^2-4)(x-5)}$ (c) $f(x) = \dfrac{x^2+1}{(x^2-4)(x-5)}$
(d) $f(x) = -\dfrac{x^4+1}{(x^2-4)(x-5)}$ (e) $f(x) = \dfrac{x^5+1}{x(x^2-4)(x+5)}$

Which equation represents the graph?



(a) $(x-2)^2 = 3y$ (b) $\frac{y^2}{9} - \frac{(x-2)^2}{4} = 1$ (c) $\frac{x^2}{9} + \frac{(y-2)^2}{4} = 1$
(d) $\frac{(y-2)^2}{4} + \frac{x^2}{9} = 1$ (e) $y^2 = 3(x-2)$

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(4) Which of the following complex numbers is represented by a point on the real axis of the complex plane?

(a) $e^{i\frac{\pi}{2}}$ (b) $\cos 2 + i \sin 2$ (c) 5 (d) $1 + 2i$ (e) $e^{\frac{\pi}{4}i}$

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(5) A square S has sides of length $x$cm. A new rectangle R is formed from S by increasing the lengths of S by 5 cm and keeping the width at x cm. How much more area does R have than S?

(a) $5x^2 cm^2$ (b) $6x^2 cm^2$ (c) $4x^2 cm^2$ (d) $5x cm^2$

(e) none of the above

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(10) If $f(x) = \tan x$ and $g(x) = x^2 + x - 1$, then which of the following is $f(g(x))$?

(a) $\tan(x^2) + \tan x - 1$ (b) $\tan(x^2 + x - 1)$ (c) $\tan^2(x) + \tan x - 1$

(d) $\tan x^2 + x$ (e) None of the above

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(11) If $y = x^2 - 3x + 7$ then to complete the square you add and subtract which one of the following values?

(a) $\frac{3}{2}$ (b) $\frac{9}{4}$ (c) $\frac{3}{7}$ (d) $\sqrt{3/2}$ (e) none

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(16) The solutions of the equation $\sin x - \sqrt{3}\cos x = 0$ that lie in the interval $[0, 2\pi]$ are

(a) $\frac{\pi}{6}$ and $\frac{11\pi}{6}$ (b) $\frac{\pi}{6}$ and $\frac{7\pi}{6}$ (c) $\frac{\pi}{3}$ and $\frac{4\pi}{3}$ (d) $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

(e) none of the above

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(17) The long-run behavior of $r(t) = \dfrac{1 + 2t^5}{3t^2 + \sqrt{3}t}$.

(a) $y = \frac{1}{\sqrt{3}t}$ (b) $y = \frac{2t^4}{\sqrt{3}}$ (c) $y = 2t^5$ (d) $y = \frac{2}{3}$ (e) $\frac{2t^3}{3}$

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(28) For positive real numbers m, n, and r, which of the following are true?

I. $\log(m - n) = \log m - \log n$

II. $\log\left(\frac{m}{n}\right) = \dfrac{\log m}{\log n}$

III. $\log m^{-1} = \frac{1}{\log m}$

IV. $\log(m^r n) = r \log m + \log n$

V. $\log_m 1 = 0$

(a) I, II and III (b) I, II, V (c) IV and V (d) II and III (e) only IV

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(37) Which of the following is ordered least to greatest?

(a) 3^{-30}, 2^{-20}, 30^{-3}, 20^{-2}, 1^{-500}

(b) 1^{-500}, 20^{-2}, 30^{-3}, 2^{-20}, 3^{-30}

(c) 1^{-500}, 2^{-20}, 3^{-30}, 20^{-2}, 3^{-3}

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(38) An angle of $\frac{5\pi}{3}$ radians is the same as an angle of

(a) 180° (b) 420° (c) 240° (d) 30°

(e) none of the above

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(45) The solution(s) of the equation $\ln x + \ln(x - 3) = 0$ is (are)

(a) $x = \frac{3}{2}$ (b) $x = 1$ and $x = 2$ (c) $x = \frac{-3 \pm \sqrt{13}}{2}$ (d) $x = \frac{3 \pm \sqrt{13}}{2}$ (e) No solution

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Math Analysis Release Questions

(2) Which of the following is NOT a vertical asymptote of the function $y = \dfrac{x^2 - 3x}{5x(4x^2 - 1)}$?

(a) $x = \frac{1}{2}$ (b) $x = -\frac{1}{2}$ (c) $x = 0$ (d) $y = \frac{1}{2}$
(e) none of the above

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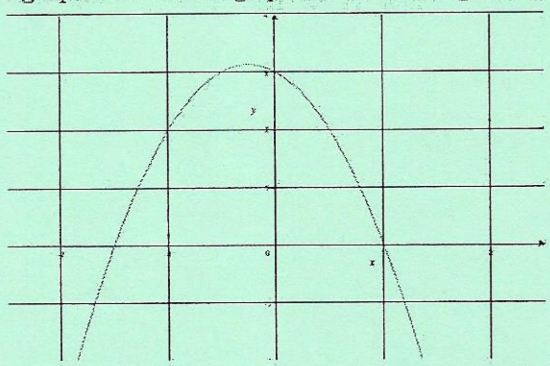
(4) Which one of the following equations has the graph shown in the figure below?



(a) $y = -2x^2 + 3$ (b) $y = -2x^2 - x - 3$ (c) $y = -2x^2 - x + 3$
(d) $y = (x - 3)(x - 2)$ (e) $y = (x - 1)(x + \frac{3}{2})$

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(7) If $f(x) = \frac{1}{x^3}$ and $g(x) = f(-\frac{x}{3})$, then which of the following is true?

(a)$g(x) = 27f(x)$ (b) $g(x) = -27x^3$ (c) $g(x) = -27f(x)$
(d) $g(x) = -\frac{1}{27}f(x)$ (e) none of the above

(11) The equation $x^2 + y^2 + 2x + 10y + 25 = 0$ describes a circle with

(a) center $(-1, -5)$ and radius 1 (b) center $(1, -5)$ and radius 25
(c) center $(1, 5)$ and radius 25 (d) center $(1, 5)$ and radius 1
(e) none of the above

(34) Using properties of logarithm function expand $\ln \sqrt[5]{\dfrac{4x^2 - 1}{4x^2 + 1}}$.

(a) $5\ln(4x^2 - 1) - 5\ln(4x^2 + 1)$
(b)$5\ln(2x - 1) + 5\ln(2x + 1) - 5\ln(4x^2 + 1)$
(c) $\frac{1}{5}\ln(2x - 1) + \frac{1}{5}\ln(2x + 1) - \frac{1}{5}\ln(4x^2 + 1)$
(d) $\frac{1}{5}\ln(2x - 1) - \frac{1}{5}\ln(2x + 1) + \frac{1}{5}\ln(4x^2 + 1)$

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(36) For a given function $f(x)$, which of the following are always true?

I. If $f(a) = f(b)$ then $a = b$ for any a, b in the domain.
II. A vertical line can intersect the graph of $y = f(x)$ at most once.
III. If $f(x) = f(-x)$ for all x in the domain of f, then the graph of $y = f(x)$ is symmetric with respect to y axis.
IV. For any real number a, $f(ax) = af(x)$.
V. If $a = b$ then $f(a) = f(b)$ for any a, b in the domain.

(a) I, II and III (b) II, III and V (c) IV and V (d) II and III (e) only V

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Mathletics Contest 2014
Mathematical Analysis

2. The Domain of function $f(x) = 4 - \log_4 (1-x^2)$

(A) $x > 0$ (B) $x < 4$ (C) $x < 1$ (D) $-1 < x < 1$

4. Let $P = \left(\frac{8}{17}, -\frac{15}{17}\right)$ be the point on the unit circle corresponding to an angle θ.

The value of $\csc\theta$

(A) $\frac{15}{17}$ (B) $-\frac{17}{15}$ (C) $\frac{8}{17}$ (D) $-\frac{8}{17}$

8. If $2\sin^2 x = 2 - \cos x$. The value of x cannot be

(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) $\frac{\pi}{6}$

10. Which of the following is an exponential function?

(A) π^{-x} (B) 3π (C) πx^{-1} (D) π^{-1}

17. For a real number x between 0 and $\frac{\pi}{2}$, the product of $\tan(x)$ and $\tan(\frac{\pi}{2} - x)$ is

(A) 0 (B) $\tan[x(\frac{\pi}{2} - x)]$ (C) $\frac{\pi}{2}$ (D) 1

20. $\frac{\tan(x)+\sin(x)}{1+\cos(x)}$ is equivalent to

(A) $\tan(x)$ (B) $\sec(x)$ (C) $\sin(x)$ (D) $1+\cos(x)$

23. $\log(x + y) = \log(x) + \log(y)$ is true

(A) never (B) if $x+y = 1$ (C) if $xy = 0$ (D) if $y = x(y-1)$

Open Ended Problem:

41. Find all positive values of **x** that satisfy $\log_{\tan(x)} 2014 < \log_{\tan(x)} 1007$.