Calculus AB 2015 Release Items

1. The function $f(x) = x^3 - 3k^2x + 3$ (k is a constant) has two horizontal tangent lines. Find the distance between these two tangent lines.

   a) k^2 b) $2k^2$ c) $3k^2$ d) $4k^3$ e) none of these

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2. For what value of x does the graph of $y = \dfrac{1}{\sqrt{x}}$ have a tangent line parallel to the line $x + 16y = 5$?

   a) 2   b) $\dfrac{1}{\sqrt{2}}$ c) 4   d) $\sqrt{2}$ e) None of these

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3. Which of the following statements about the function $f(x) = x^4 - ax^3$, $a > 0$ is true?

   (a) The function has no relative extrema.

   (b) The graph of the function has one point of inflection and two relative extrema.

   (c) The graph of the function has two points of inflection and one relative extremum.

   (d) The graph of the function has two points of inflection and two relative extrema.

   (e) none of these

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4. The slopes of $f(x) = \frac{x^3}{3} + 3x^2 + 12x$ and $g(x) = -8\ln(x)$ are equal at what value of x?

   a) -2 b) 0   c) 0.461   d) 1   e) none of these

5. Show all work for this problem. Find the equations of all the lines containing the point $(1, 4)$ that are tangent to the graph of $y = x^3 - 10x^2 + 6x - 2$ and find their points of tangency.

4. Given that $g(x) = \arctan(\frac{1}{x})$, evaluate $\lim\limits_{h \to 0} \dfrac{g(2+h) - g(2)}{h}$

(a) $4/5$ (b) $1/2$ (c) $-1/5$ (d) $\frac{\ln(2)}{2}$

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5. The line tangent to $h(x) = x^3 - 5x^2 - x + 3$ at the point $(1, -2)$ has an x-intercept of

(a) 0 (b) 0.75 (c) 3 (d) 6

9. The smallest possible value of x where $f(x) = \frac{x}{x^2+a}$ has an inflection point is

(a) $\sqrt{a}$ (b) $\sqrt{2a}$ (c) $\sqrt{3a}$ (d) $a\sqrt{2}$

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10. Which of the following is incorrect?

(a) $\frac{d}{dx}(2^x) = 2^x \ln(2)$ (b) $\frac{d}{dx}(\log_4(x)) = \frac{1}{x \ln(4)}$

(c) $\frac{d}{dx}(x^x) = x^x[1 + \ln(x)]$ (d) $\frac{d}{dx}(a^{u(x)}) = a^{u(x)} \ln(a)$

14. For what value(s) of a does $f(x) = x^3 + ax^2 + bx + 2$ have a local maximum at $x = -3$ and a local minimum at $x = -1$?

(a) 6 (b) 9 (c) both 6 and 9

(d) there are no such values

30. Let $f(x) = \dfrac{c}{x} + x^2$ Determine all values of c for which $f(x)$ has a relative minimum, but no relative maximum.

(a) $c > \sqrt[3]{2}$ (b) $|c| > \sqrt[3]{2}$ (c) all values of c

(d) there are no such values of c

3. What is the sum of the values of x where the line tangent to the curve

   $x^2 + y^2 = 3 - xy$ is horizontal ?

   (a) -1 (b) 0   (c) 1   (d) 3

   (e) none of these

5. A dairy farmer with 20 cows gets an average of 3 gallons of milk from each cow per day. The farmer noticed that for each additional cow added to the herd, the average production drops by 0.2 gallons per cow. How many cows should the farmer have in order to maximize total milk production?

   (a) 16   (b) 17   (c) 20   (d) 27

   (e) none of these

9. The position of a particle moving along a line is given by

   $s(t) = 2t^3 - 24ct^2 + 90c^2t + 7$, $t \geq 0$, with c a positive constant. For what values of t is the speed of the particle increasing?

   (a) $3c < t < 4c$ only   (b) $t > 4c$ only   (c) $t > 5c$ only

   (d) $0 < t < 3c$ and $t > 5c$ (e) none of these

13. Evaluate

$$\lim_{x \to \infty} x^{\frac{k}{x}}$$

   (a) 0   (b) 1   (c) DNE   (d) k

   (e) none of these

15. If $f(x)$ has an inflection point at $x = 3$ and $f(x)$ is differentiable at $x = 3$ then $f'(x)$ has a local extrema at $x = 3$. This statement is

   (a) Always true   (b) Sometimes true   (c) Never True

   (d) Cannot be determined from the information given

2014 Mathletics Contest Calculus AB

1. Evaluate the limit

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x + 9}$$

   a) DNE   b) $\frac{1}{6}$ c) $\frac{1}{7}$ d) 1   e) none of these

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   a) 16   b) 17   c) 20   d) 27   e) none of these

3. For what value of x does the graph of $y = \dfrac{1}{\sqrt{x}}$ have a tangent line parallel to the line $x + 16y = 5$?

   a) 2   b) $\dfrac{1}{\sqrt{2}}$ c) 4   d) $\sqrt{2}$ e) None of these

4. If $f(x) = \cos^2(4 + 3x)$, then $f'(1)$ is equal to which of the following?

   a) $-6\cos(7)\sin(7)$ b) $2\cos(7)$ c) $-2\cos(7)\sin(7)$ d) $6\sin(7)$ e) none of these

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   a) $3c < t < 4c$ only   b) $t > 4c$ only   c) $t > 5c$ only

   d) $0 < t < 3c$ and $t > 5c$ e) none of these

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