Pre-Calculus
1st Semester Final Review
Extended Problem Practice

Name ______________
Date ______________

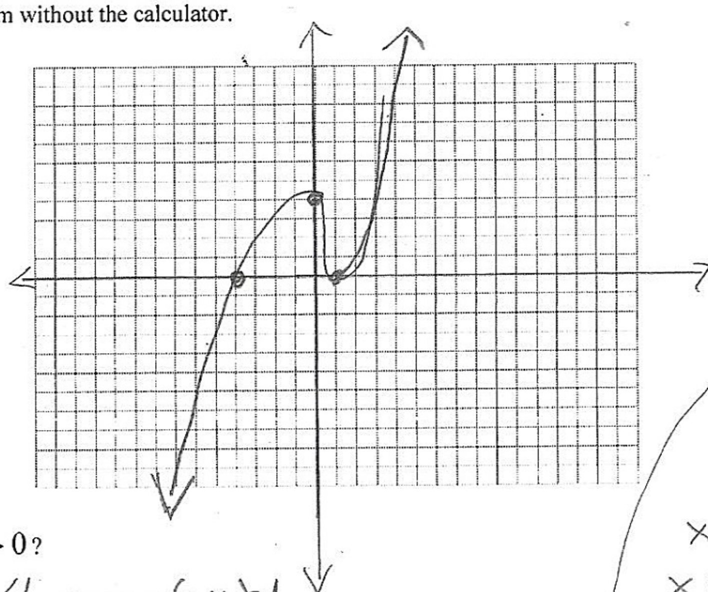
1. Let $P(x) = x^3 + 2x^2 - 7x + 4$

a. $x = 1$ is a root of $P(x)$. Completely factor $P(x)$.

b. Graph $P(x)$, clearly label all zeros and the y-intercept. You will not be able to accurately find the exact maximum or minimum without the calculator.

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -7 & 4 \\ & & 1 & 3 & -4 \\ \hline & 1 & 3 & -4 & \boxed{0} \end{array}$$

$P(x) = (x-1)(x^2+3x-4)$
$= (x-1)(x+4)(x-1)$
$= (x-1)^2(x+4)$
$x = 1, -4$



c. Where is $P(x) > 0$?

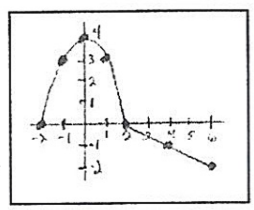
$x > -4$ except $x \neq 1$

d. Find the three values where $P(x)$ intersects the line $Q(x) = 4 - 4x$. Set $P(x) = Q(x)$.

$x^3 + 2x^2 - 7x + 4 = 4 - 4x$
$x^3 + 2x^2 - 3x = 0$
$x(x^2 + 2x - 3) = 0$
$x(x+3)(x-1) = 0$
$x = 0, -3, 1$

$\boxed{(0,4)(-3,16)(1,0)}$

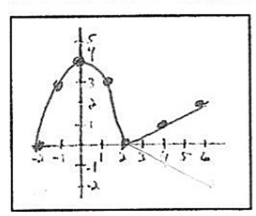
2. The graph of $f(x)$ (shown below) for the given domain is $f(x) = \begin{cases} 4 - x^2, -2 \le x < 2 \\ -\frac{1}{2}x + 1, 2 \le x \le 6 \end{cases}$



a. List the domain and range of $f(x)$.

Domain $-2 \le x \le 6$ Range $-2 \le y \le 4$

b. Sketch a graph of $|f(x)|$.



c. If $g(x) = x^3 - 7x + 5$, then find $g(f(2))$.

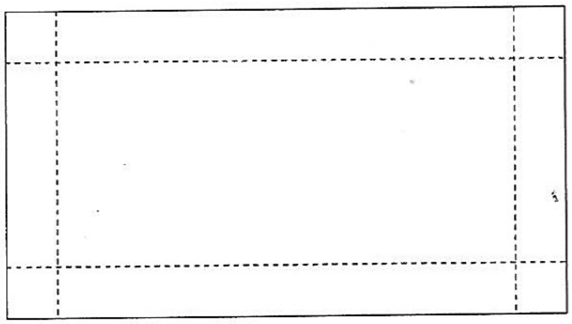
$f(2) = 0$ $\boxed{g(f(2)) = 5}$

d. Tell why this function is <u>not</u> a one to one function and give specific set of points to illustrate your answer.

This is not a one to one function because it does not pass the horizontal line test. The points (-1, 3) and (1, 3) illustrate this because y repeats itself.

3. Free Response-Calculator Allowed

A box is to be made from a 20" x 30" rectangular piece of cardboard by cutting squares from the corners and folding up the sides.



a. Find the volume of the box created by cutting squares with sides of 2".

$V = 2(16)(26) = \boxed{832 \text{ in}^3}$

b. Write the height, length, width and volume of the box as functions of x as the side of the corner square.

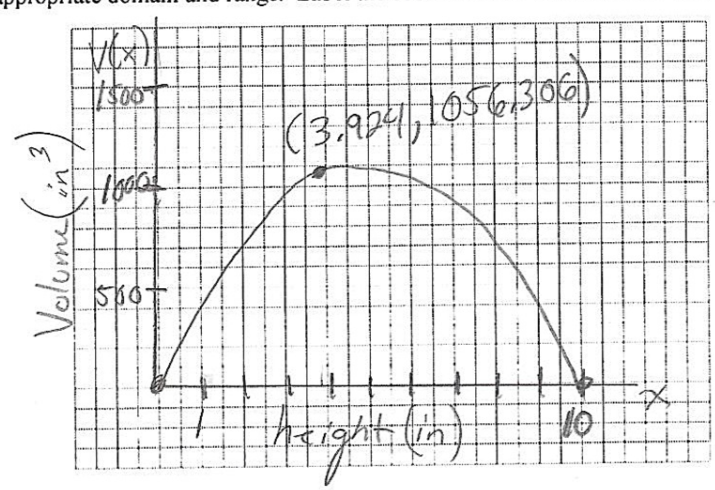
$h =$ x

w $30-2x$

$l =$ $20-2x$

$V(x) =$ $x(20-2x)(30-2x)$

c. Sketch the graph of $V(x)$ over an appropriate domain and range. Label the coordinates of the maximum volume.



d. List the dimensions and volume of the largest box possible.

$\boxed{V(3.924) = 1056.306 \text{ in}^3}$

$h = x = 3.924$ in

$l = 12.152$ in

$w = 22.152$ in

$\boxed{3.924'' \times 12.152'' \times 22.152''}$