

Review WS Standard 8

$$1) \cos x + \sin x \tan x = \sec x$$

$$\cos x + \sin x \cdot \frac{\sin x}{\cos x} = \sec x$$

$$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = \sec x$$

$$\frac{\cos^2 x + \sin^2 x}{\cos x} = \sec x$$

$$\boxed{\frac{1}{\cos x} = \sec x}$$

$$2) \frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$$

$$\frac{2 \tan x}{\sec^2 x} = \sin 2x$$

$$\frac{\frac{2 \sin x}{\cos x}}{\frac{1}{\cos^2 x}} = \sin 2x$$

$$\frac{2 \sin x}{\cos x} \cdot \frac{\cos^2 x}{1} = \sin 2x$$

$$\boxed{2 \sin x \cdot \cos x = \sin 2x}$$

$$3) 6 \sin x \cdot \cos x = 3 \sin 2x$$

$$3(2 \sin x \cdot \cos x) = 3 \sin 2x$$

$$\boxed{2 \sin x \cdot \cos x = \sin 2x}$$

$$4) \sin(\theta + 60^\circ) - \cos(\theta + 30^\circ) = \sin \theta$$

$$\sin \theta \cos 60^\circ + \cos \theta \sin 60^\circ - (\cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ) = \sin \theta$$

$$\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta - \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right) = \sin \theta$$

$$\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta - \frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta = \sin \theta$$

$$\boxed{\sin \theta = \sin \theta}$$

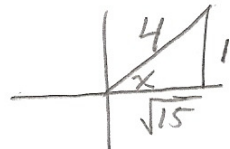
$$5) \cos 75^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos(45^\circ + 30^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} \right) - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\boxed{\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}}$$

6) a)  $\sin x = \frac{1}{4}$

b) $\cos x = \frac{\sqrt{15}}{4}$

c) $\tan x = \frac{1}{\sqrt{15}}$

d) $\sin 2x = 2 \sin x \cos x$
 $= 2 \left(\frac{1}{4} \right) \left(\frac{\sqrt{15}}{4} \right)$

$$\boxed{\sin 2x = \frac{\sqrt{15}}{8}}$$

e) $\cos 2x = \cos^2 x - \sin^2 x$
 $\cos 2x = \left(\frac{\sqrt{15}}{4} \right)^2 - \left(\frac{1}{4} \right)^2$
 $= \frac{15}{16} - \frac{1}{16}$
 $= \frac{14}{16}$

$$\boxed{\cos 2x = \frac{7}{8}}$$

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$$\begin{aligned}
 (6f) \quad \sin\left(x - \frac{\pi}{3}\right) &= \sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} \\
 &= \frac{1}{4} \left(\frac{1}{2}\right) - \frac{\sqrt{15}}{4} \left(\frac{\sqrt{3}}{2}\right) \\
 &= \frac{1}{8} - \frac{3\sqrt{5}}{8}
 \end{aligned}$$

$$\boxed{\sin\left(x - \frac{\pi}{3}\right) = \frac{1 - 3\sqrt{5}}{8}}$$

$$\begin{aligned}
 g) \quad \cos\left(\frac{1}{2}x\right) &= \sqrt{\frac{1 + \cos x}{2}} \\
 &= \sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}}
 \end{aligned}$$

$$\cos\left(\frac{1}{2}x\right) = \sqrt{\frac{\frac{4 + \sqrt{15}}{4}}{2}}$$

$$\boxed{\cos\left(\frac{1}{2}x\right) = \sqrt{\frac{4 + \sqrt{15}}{8}}}$$

$$h) \sin x = \frac{1}{4}$$

$$x = \sin^{-1}\left(\frac{1}{4}\right)$$

$$\boxed{x = 0.253}$$

$$7) \text{ Prove: } \frac{1}{2}(1 + \cos 2A) = \cos^2 A$$

$$\frac{1}{2}(1 + 2\cos^2 A - 1) = \cos^2 A$$

$$\frac{1}{2}(2\cos^2 A) = \cos^2 A$$

$$\boxed{\cos^2 A = \cos^2 A}$$

$$8) \text{ a) } 4 \sin \theta \cos \theta = \sqrt{3}$$

$$2(2 \sin \theta \cos \theta) = \sqrt{3}$$

$$2 \sin \theta \cos \theta = \frac{\sqrt{3}}{2}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2} \quad \text{for } 0 \leq \theta \leq 720$$

$$2\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$2\theta = 60^\circ, 120^\circ, 420^\circ, 480^\circ$$

$$\boxed{\theta = 30^\circ, 60^\circ, 210^\circ, 240^\circ}$$

$$\text{b) } \cos 2x = \cos x$$

$$2\cos^2 x - 1 = \cos x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$2\cos x + 1 = 0 \quad \cos x - 1 = 0$$

$$2\cos x = -1 \quad \cos x = 1$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$$

$$8c) 1 - 2 \sin^2 \theta = \frac{1}{2}$$

$$= 2 \sin^2 \theta = -\frac{1}{2}$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

$$d) \frac{\sin x}{1 + \cos x} = \sqrt{3}$$

$$\tan \frac{x}{2} = \sqrt{3}$$

$$\frac{x}{2} = \tan^{-1} \sqrt{3}$$

$$\boxed{\frac{x}{2} = \frac{\pi}{3}, \frac{4\pi}{3}}$$

$$x = \frac{2\pi}{3}, \frac{8\pi}{3}$$

$$9) \cos x = 0$$

$$x = \cos^{-1} 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos 2x = 0$$

$$2x = \cos^{-1} 0$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$\cos x \neq \cos 2x$
therefore it is false

$$\textcircled{10} \quad \cos 4\theta - \sin 2\theta = 0 \quad \theta \in [0, 180]$$

$$\cos(2[2\theta]) - \sin 2\theta = 0$$

$$1 - 2\sin^2 2\theta - \sin 2\theta = 0$$

$$-1 + 2\sin^2 2\theta + \sin 2\theta = 0$$

$$2\sin^2 2\theta + \sin 2\theta - 1 = 0$$

$$(2\sin 2\theta - 1)(\sin 2\theta + 1) = 0$$

$$2\sin 2\theta - 1 = 0 \quad \sin 2\theta + 1 = 0$$

$$\sin 2\theta = \frac{1}{2} \quad \sin 2\theta = -1$$

$$2\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$2\theta = \sin^{-1}(-1)$$

For $0 \leq \theta < 360$

$$2\theta = 30^\circ, 150^\circ$$

$$2\theta = 270^\circ$$

$$\theta = 15^\circ, 75^\circ$$

$$\theta = 135^\circ$$

$$\boxed{\theta = 15^\circ, 75^\circ, 135^\circ}$$

$$\textcircled{11} \quad \sqrt{3} \sin x - \cos x = 1$$

$$\sqrt{3} \sin x = 1 + \cos x$$

$$\sqrt{3} = \frac{1 + \cos x}{\sin x}$$

$$\frac{1}{\sqrt{3}} = \frac{\sin x}{1 + \cos x}$$

$$\frac{\sqrt{3}}{3} = \tan \frac{x}{2}$$

$$\frac{x}{2} = \tan^{-1} \frac{\sqrt{3}}{3}$$

$$\frac{x}{2} = \frac{\pi}{6}$$

For all real #

$$\boxed{x = \frac{\pi}{3} + \pi n}$$

$$(12) \quad 2 \cos x - 2 \cos^3 x = \sin x \cdot \sin 2x$$

$$2 \cos x (1 - \cos^2 x) = \sin x \cdot \sin 2x$$

$$2 \cos x \cdot \sin^2 x = \sin x \cdot 2 \sin x \cos x$$

$$\boxed{2 \cos x \cdot \sin^2 x = 2 \cos x \sin^2 x}$$