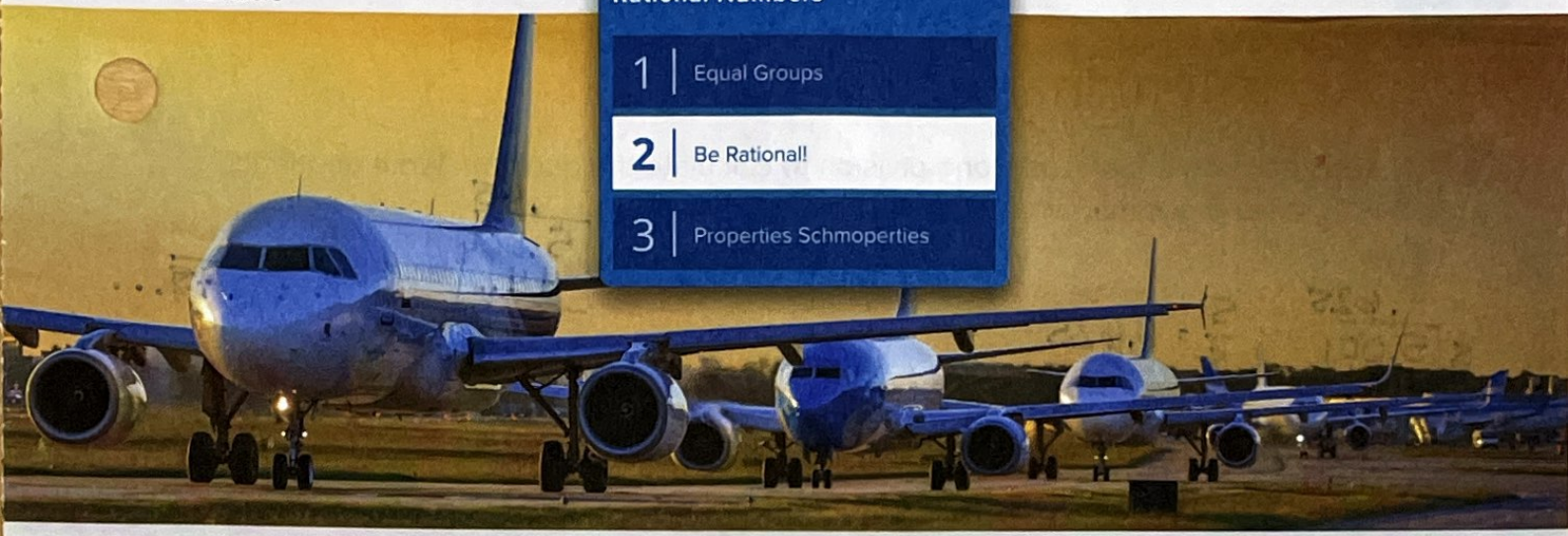

LESSON 2

Be Rational!

Quotients of Integers

Learning Goals

- Determine that every quotient of integers is a rational number, provided the divisor is not zero, and use long division to represent those quotients as terminating or repeating decimals.
- Know that the decimal form of a rational number terminates in 0s or eventually repeats.
- Write equivalent forms of signed rational numbers.
- Solve real-world problems using operations with signed numbers.

KEY TERMS

- terminating decimals
- non-terminating decimals
- repeating decimals
- bar notation
- non-repeating decimals
- percent error

REVIEW (1–2 minutes)

› Classify each number into as many sets of numbers as it belongs: natural, whole, integer, and rational.

1. -3 integer, rational
2. $\frac{1}{2}$ rational
3. 0 whole, integer, rational
4. 5 natural, whole, integer, rational

You have learned the rule to determine the sign of a quotient.

Does a quotient change when the negative sign is on the divisor instead of the dividend?

Are You a Terminator?

1 For each pair of numbers, use long division to calculate the quotient. Write quotients in fractional and decimal form.

ⓐ 5 ÷ 8

.625
8 | 5.000
-48
20
-16
40

5/8 = 0.625

ⓑ 5 ÷ 11

.45
11 | 5.000
-44
60
-55
50

5/11 = 0.45...

ⓒ 7 ÷ 9

.7
9 | 7.00
-63
70

7/9 = 0.7...

ⓓ 6 ÷ 2

3
2 | 6

6/2 = 3

2 Which types of numbers are the quotients in Question 1? **Use the definitions of the different number classifications to explain why this makes sense.**

All the quotients are rational numbers because they can be written in fractional form

3 How many decimal places did you need to go to in the long division for each quotient? **Why?**

a) thousandths

b) thousandths

c) hundredths

d) ones.

I could either determine they would repeat or I got a remainder of zero.

Classifying Decimals

HABITS OF MIND
- Attend to precision.

You can classify decimals into two categories: *terminating* and *non-terminating*.

A **terminating decimal** has a finite number of digits, meaning that after a finite number of decimal places, all following decimal places have a value of 0. Terminating decimals are rational numbers.

A **non-terminating decimal** is a decimal that continues infinitely without ending in a sequence of zeros.

1. Classify the decimals in Question 1 of the Getting Started as terminating or non-terminating decimals.

Terminating: 0.625, 3

Non-terminating: 0.7, 0.45

2. Determine which unit fractions are terminating and which are non-terminating. **Explain your reasoning for each.**

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{9}$ $\frac{1}{10}$

Terminating: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{10}$

Non-terminating: $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{9}$

You can rewrite the terminating decimals with a denominator that is a power of ten.

You can further divide non-terminating decimals into two categories: *repeating* and *non-repeating*.

A **repeating decimal** is a decimal in which a digit, or a group of digits, repeat(s) infinitely. Repeating decimals are rational numbers.

You can use **bar notation** to indicate the digits that repeat in a repeating decimal. In the quotient of 3 and 7, the sequence 428571 repeats. The numbers that lie underneath the bar are the numbers that repeat.

DID YOU KNOW?
You call the bar a vinculum.

$$\frac{3}{7} = 0.\boxed{428571}\boxed{428571}4\ldots = 0.\overline{428571}$$

A **non-repeating decimal** continues without terminating and without repeating a sequence of digits. Non-repeating decimals are not rational numbers.

DID YOU KNOW?
Pi (π) is one of the most well-known non-repeating decimals.

3. Classify the non-terminating decimals in Question 1 of the Getting Started as repeating or non-repeating decimals. If they are repeating decimals, rewrite them using bar notation.

$0.\overline{7}$, $0.\overline{45}$

4. Use your results in Question 2 to make a conjecture about other fractions. Which fractions will have repeating decimal representations? **Use examples to support your conjecture.**

TAKE NOTE...
If you can identify a counterexample to your conjecture, revise your conjecture.

A decimal always repeats when the denominator of a fraction in lowest terms has a prime factor other than 2 or 5.

5. Will a fraction with a denominator of 6 always have a repeating decimal representation? **Explain your reasoning.**

No. If the fraction is not in simplest form it could be a terminating decimal

$\frac{3}{6} = \frac{1}{2} = 0.5$

Equivalent Rational Numbers

You can use multiple representations to express equivalent rational numbers.

HABITS OF MIND
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

1 Write a new equivalent fractional representation and write an equivalent decimal representation.

$-\frac{4}{5}$	$=$ $\frac{-4}{5}$	$=$ $\frac{4}{-5}$	$=$ -0.8	$5\overline{)4.0}$ → .8
$\frac{-2}{3}$	$=$ $\frac{2}{-3}$	$=$ $-\frac{2}{3}$	$=$ $-0.\overline{6}$	$3\overline{)2.0}$ → .6, −18, 2
$\frac{11}{-4}$	$=$ $-\frac{11}{4}$	$=$ $\frac{-11}{4}$	$=$ -2.75	$4\overline{)11.00}$ → 2.75, −8, 30, −28, 20
$-\frac{39}{60}$	$=$ $\frac{39}{-60}$	$=$ $\frac{-39}{60}$	$=$ -0.65	$60\overline{)39.00}$ → .65, −360, 300, −300, 0
$\frac{-7}{22}$	$=$ $\frac{7}{-22}$	$=$ $-\frac{7}{22}$	$=$ $-0.3\overline{18}$	$22\overline{)7.0000}$ → .318, −66, 40, −22, 180, −176, 4

2 Classify each decimal you wrote in Question 1 as repeating or terminating.

Repeating: $-0.6, -0.3\overline{18}$

Terminating: $-0.8, -2.75, -0.65$

TOPIC 2